

EE4601

Communication Systems

Lecture 15

Equalization

Equalization

- The cascade of the transmit filter $g(t)$, channel $c(t)$, receiver filter $h(t)$ yields the overall pulse

$$p(t) = g(t) * c(t) * h(t)$$

- The signal at the output of the matched filter is

$$y(t) = \sum_k a_k p(t - kT) + n(t)$$

and the sampled output is

$$\begin{aligned} y_n = y(nT) &= \sum_k a_k p_{n-k} + n_n \\ &= \sum_k p_k a_{n-k} + n_n \end{aligned}$$

- Assume a causal, finite-length, channel such that $p(t) = 0$ for $t < 0$ and $t > LT$.
- The discrete-time channel $p_n = p(nT)$, can be represented by the vector

$$\mathbf{p} = (p_0, p_1, \dots, p_L)$$

Equalization

- An equalizer is a digital filter that is used to mitigate the effects of inter-symbol interference that is introduced by a time dispersive channel.
- The tap co-efficients of the equalizer are denoted by the vector

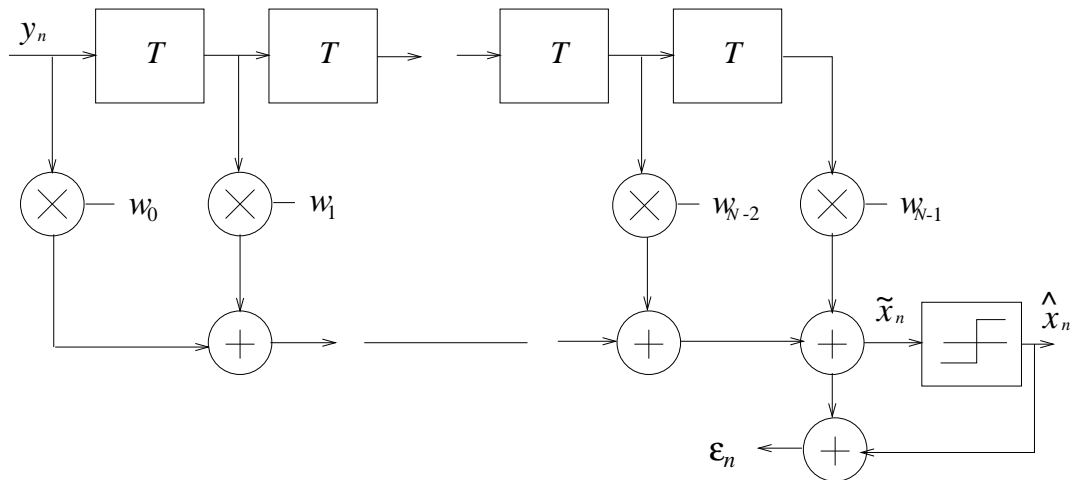
$$\mathbf{w} = (w_0, w_1, \dots, w_{N-1})^T$$

where N is the number of equalizer taps.

- If the equalizer is used to process the sampled outputs of the receiver matched filter, then the output of the equalizer is

$$\tilde{x}_n = \sum_{j=0}^{N-1} w_j y_{n-j}$$

Linear Transversal Equalizer



Overall Discrete-time Model

- The overall channel and equalizer can be represented by a overall digital filter with impulse response

$$\mathbf{q} = (q_0, q_1, \dots, q_{N+L-1})^T$$

where

$$\begin{aligned} q_n &= \sum_{j=0}^{N-1} w_j p_{n-j} \\ &= \mathbf{w}^T \mathbf{p}(n) \end{aligned}$$

with

$$\mathbf{p}(n) = (p_n, p_{n-1}, p_{n-2}, \dots, p_{n-N+1})^T$$

and $p_i = 0, i < 0, i > L$. That is, \mathbf{q} is the discrete convolution of \mathbf{p} and \mathbf{w} .

Perfect Equalization

- Let the component of \mathbf{p} of greatest magnitude be denoted by p_{d_1} . Note that we may have $d_1 \neq 0$.
- Let the number of equalizer taps be equal to $N = 2d_2 + 1$ where d_2 is an integer.
- Perfect equalization means that

$$\mathbf{q} = \mathbf{e}_d = \underbrace{(0, 0, \dots, 0)}_{d \text{ zeroes}}, 1, 0, \dots, 0, 0)^T$$

where d zeroes precede the “1” and d is an integer representing the overall delay, a parameter to be optimized.

- Unfortunately, perfect equalization is difficult to achieve and does not always yield the best performance.

Zero Forcing Equalizer

- With a zero-forcing (ZF) equalizer, the tap coefficients \mathbf{w} are chosen to minimize the peak distortion of the equalized channel, defined as

$$D_p = \frac{1}{|q_d|} \sum_{\substack{n=0 \\ n \neq d}}^{N+L-1} |q_n - \hat{q}_n|$$

where $\hat{\mathbf{q}} = (\hat{q}_0, \dots, \hat{q}_{N+L-1})^T$ is the *desired equalized channel* and the delay d is a positive integer optimized to have the value $d = d_1 + d_2$.

- Lucky showed that if the initial distortion without equalization is less than unity, i.e.,

$$D = \frac{1}{|p_{d_1}|} \sum_{\substack{n=0 \\ n \neq d_1}}^L |p_n| < 1 ,$$

then D_p is minimized by those N tap values which simultaneously cause $q_j = \hat{q}_j$ for $d - d_2 \leq j \leq d + d_2$. However, if the initial distortion before equalization is greater than unity, the ZF criterion is not guaranteed to minimize the peak distortion.

Zero Forcing Equalizer

- For the case when $\hat{\mathbf{q}} = \mathbf{e}_d^T$ the equalized channel is given by

$$\mathbf{q} = (q_0, \dots, q_{d_1-1}, 0, \dots, 0, 1, 0, \dots, 0, q_{d_1+N}, \dots, q_{N+L-1})^T .$$

- In this case the equalizer forces zeroes into the equalized channel and, hence, the name “zero-forcing equalizer.”

Equalizer Tap Solution

- For a known channel impulse response, the tap gains of the ZF equalizer can be found by the direct solution of a simple set of linear equations. To do so, we form the matrix

$$\mathbf{P} = [\mathbf{p}(d_1), \dots, \mathbf{p}(d), \dots, \mathbf{p}(N + d_1 - 1)]$$

and the vector

$$\tilde{\mathbf{q}} = (\hat{q}_{d_1}, \dots, \hat{q}_d, \dots, \hat{q}_{N+d_1-1})^T .$$

- Then the vector of optimal tap gains, \mathbf{w}_{op} , satisfies

$$\mathbf{w}_{\text{op}}^T \mathbf{P} = \tilde{\mathbf{q}}^T \longrightarrow \mathbf{w}_{\text{op}} = (\mathbf{P}^{-1})^T \tilde{\mathbf{q}} .$$

Example

- Suppose that a system has the channel vector

$$\mathbf{p} = (0.90, -0.15, 0.20, 0.10, -0.05)^T ,$$

where $p_i = 0, i < 0, i > 4$. The initial distortion before equalization is

$$D = \frac{1}{|p_0|} \sum_{n=1}^4 |p_n| = 0.5555$$

and, therefore, the minimum distortion is achieved with the ZF solution.

- Suppose that we wish to design a 3-tap ZF equalizer. Since p_0 is the component of \mathbf{p} having the largest magnitude, $d_1 = 0$ and the optimal equalizer delay is $d = 1$. The desired response is $\hat{\mathbf{q}} = \mathbf{e}_1^T$ so that $\tilde{\mathbf{q}} = (0, 1, 0)^T$.

Example

- We then construct the matrix

$$\begin{aligned}\mathbf{P} &= [\mathbf{p}(0), \mathbf{p}(1), \mathbf{p}(2)] \\ &= \begin{bmatrix} 0.90 & -0.15 & 0.20 \\ 0.00 & 0.90 & -0.15 \\ 0.00 & 0.00 & 0.90 \end{bmatrix}\end{aligned}$$

and obtain the optimal tap solution

$$\mathbf{w}_{\text{op}} = (\mathbf{P}^{-1})^T \tilde{\mathbf{q}} = (0, 1.11111, -0.185185)^T .$$

The overall response of the channel and equalizer is

$$\mathbf{q} = (0.0, 1.0, 0.0, 0.194, 0.148, -0.037, -0.009, 0, \dots)^T .$$

- Hence, the distortion after equalization is

$$D_{\min} = \frac{1}{|q_0|} \sum_{n=1}^6 |q_n - \hat{q}_n| = 0.388 .$$