

EE4061

Communication Systems

Lecture 18

Coherent Signal Detection

Optimum Coherent Detection

Suppose that we have M signals $s_1(t), s_2(t), \dots, s_M(t)$ that are defined over the time interval $0 \leq t \leq T$.

We select one of signals say $s_i(t)$ for transmission over an AWGN channel.

The received signal is

$$r(t) = \alpha s_i(t - t_o) + n(t)$$

where $n(t)$ is AWGN with power spectral density $N_o/2$ watts/Hz

Problem: By observing $r(t)$ determine which of the M signals was (most likely) transmitted.

Assume that the receiver knows α and t_o exactly. In this case, we can just assume that

$$r(t) = s_i(t) + n(t)$$

Correlation Detector

Any signal set $\{s_1(t), s_2(t), \dots, s_M(t)\}$ can be expressed in terms of a set of orthonormal basis functions $\{f_1(t), f_2(t), \dots, f_N(t)\}$ where N is the dimension of the signal space.

However, the basis functions *do not* span the noise space, i.e., the noise waveform $n(t)$ cannot be represented exactly in terms of the N basis functions. We have

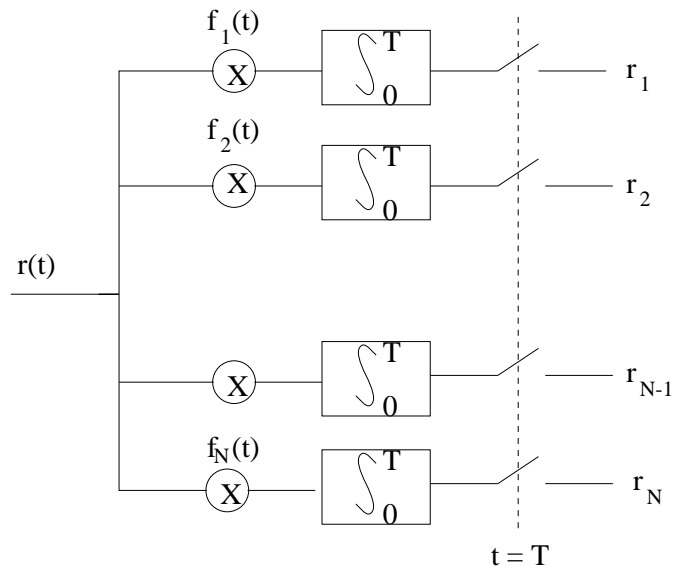
$$r(t) = \sum_{k=1}^N r_k f_k(t)$$

where

$$\begin{aligned} r_k &= \int_0^T r(t) f_k(t) dt \\ &= \int_0^T s_m(t) f_k(t) dt + \int_0^T n(t) f_k(t) dt \\ &= s_{mk} + n_k \end{aligned}$$

Hence, the projection of the received signal $r(t)$ onto the signal space yields the *received* vector $\mathbf{r} = (r_1, r_2, \dots, r_N)$.

Correlation Detector



Noise Statistics

The noise components n_k have mean

$$E[n_k] = \int_0^T E[n(t)]f_k(t)dt = 0$$

and covariance

$$\begin{aligned} E[n_j n_k] &= \int_0^T \int_0^T E[n(t)n(s)]f_j(t)f_k(s)dt ds \\ &= \frac{N_o}{2} \int_0^T \int_0^T \delta(t-s)f_j(t)f_k(s)dt ds \\ &= \frac{N_o}{2} \int_0^T f_j(t)f_k(t)dt \\ &= \frac{N_o}{2} \delta_{jk} \end{aligned}$$

Therefore, the r_k are independent Gaussian random variables with mean s_{mk} and variance $\frac{N_o}{2}$.

Joint Conditional Density

The vector \mathbf{r} has the joint conditional density function

$$\begin{aligned} p(\mathbf{r}|\mathbf{s}_m) &= \frac{1}{(\pi N_o)^{N/2}} \exp \left\{ -\frac{1}{N_o} \sum_{k=1}^N (r_k - s_{mk})^2 \right\} \\ &= \frac{1}{(\pi N_o)^{N/2}} \exp \left\{ -\frac{1}{N_o} \|\mathbf{r} - \mathbf{s}_m\|^2 \right\} \end{aligned}$$

which is a multivariate Gaussian distribution.

Note that

$$r(t) = \sum_{k=1}^N r_k f_k(t) + n'(t)$$

where $n'(t)$ is the **remainder process**

$$n'(t) = n(t) - \sum_{k=1}^N n_k f_k(t)$$

Irrelevance

We have

$$\begin{aligned} \mathbb{E}[n'(t)r_k] &= \mathbb{E}[n'(t)]s_{mk} + \mathbb{E}[n'(t)n_k] \\ &= \mathbb{E}[n'(t)n_k] \\ &= \mathbb{E}\left[\left(n(t) - \sum_{j=1}^N n_j f_j(t)\right) n_k\right] \\ &= \int_0^T \mathbb{E}[n(t)n(\tau)] f_k(\tau) d\tau - \sum_{j=1}^N \mathbb{E}[n_k n_j] f_j(t) \\ &= \frac{1}{2} N_o f_k(t) - \frac{1}{2} N_o f_k(t) = 0 \end{aligned}$$

Hence, the vector \mathbf{r} is uncorrelated with $n'(t)$ and, therefore, $n'(t)$ is irrelevant since it does not contain any information about \mathbf{r} .

This is Wozencraft's irrelevance theorem which is certainly not irrelevant!

Matched Filter Receiver

Suppose that we filter the received signal $r(t)$ with a bank of matched filters having the impulse responses

$$h_k(t) = f_k(T - t) \quad , \quad 0 \leq t \leq T$$

and sample the filter outputs at time $t = T$.

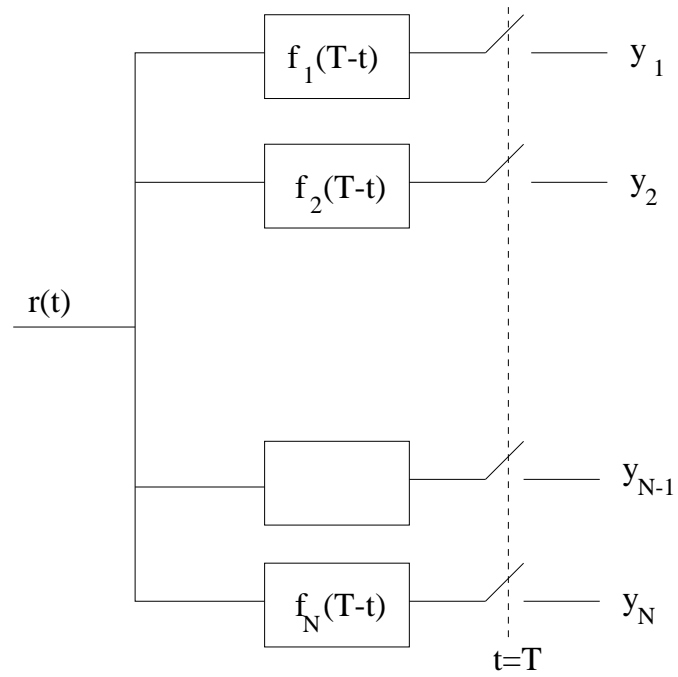
The filter outputs are

$$\begin{aligned} y_k(t) &= \int_0^t r(\tau) h_k(t - \tau) d\tau \\ &= \int_0^t r(\tau) f_k(T - t + \tau) d\tau \end{aligned}$$

$$y_k(T) = \int_0^T r(\tau) f_k(\tau) d\tau$$

Note that $y_k = r_k$, i.e., the matched filter outputs are identical to the correlator outputs.

Matched Filter Receiver



Minimum Distance Decisions

With minimum distance decisions, the receiver first calculates the vector \mathbf{r} . The receiver then decides in favour of the signal point \mathbf{s}_i that is closest in *Euclidean distance* or *squared Euclidean distance* to the received vector \mathbf{r} .

The minimum distance decision rule is

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s}_i} \|\mathbf{r} - \mathbf{s}_i\|^2$$

Since, the vector \mathbf{r} has the joint conditional density function

$$p(\mathbf{r}|\mathbf{s}_m) = \frac{1}{(\pi N_o)^{N/2}} \exp \left\{ -\frac{1}{N_o} \|\mathbf{r} - \mathbf{s}_m\|^2 \right\}$$

the choice of \mathbf{s}_i that minimizes $\|\mathbf{r} - \mathbf{s}_i\|^2$ also maximizes the likelihood $p(\mathbf{r}|\mathbf{s}_m)$.

This is also called a *maximum likelihood* receiver.