EE6604
Personal & Mobile Communications

Week 15

Massive MIMO

Reading: 6.14

Massive MIMO Principle

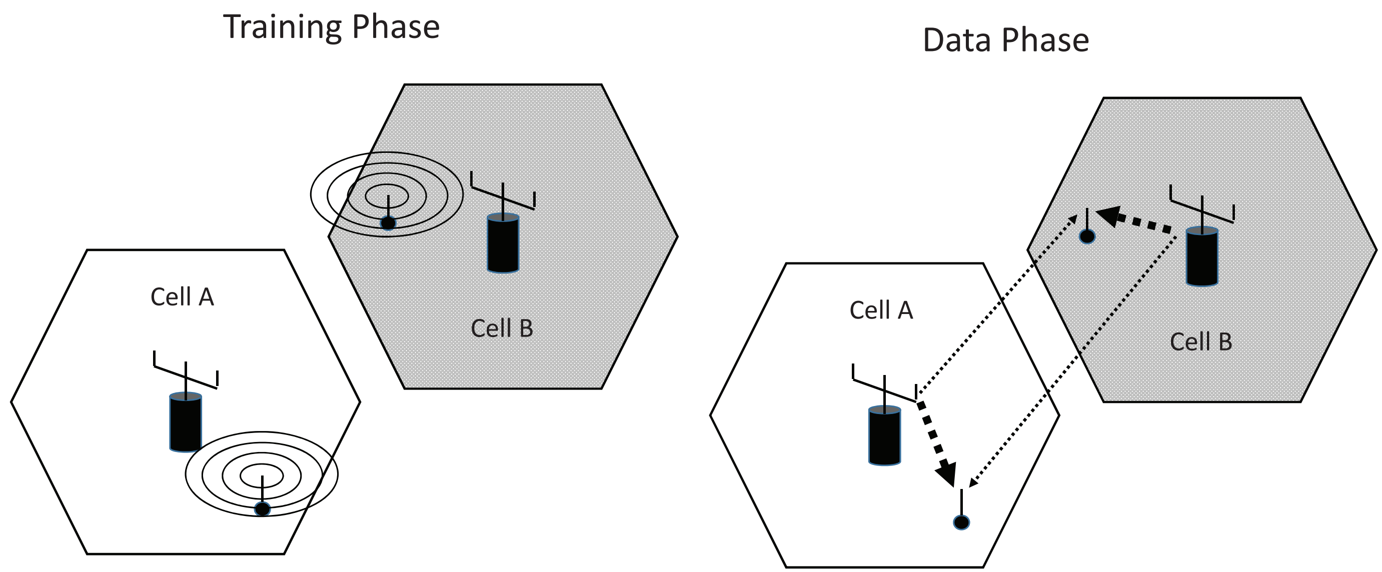
- Very large MIMO or massive MIMO systems use BS antenna arrays with an order of magnitude more elements than conventional MIMO systems, perhaps a hundred antenna elements or more, while the MSs use a single antenna.
- A large number of BS antennas allows the antenna beam pattern at the BS to be adjusted so as to sharply focus the radiated energy in a small region of space at the intended MS receiver antenna by adding together the signals transmitted from the multiple antennas constructively at the desired MS receiver, but destructively elsewhere at the other unintended MS receivers.
- Massive MIMO works on the principle of **favorable propagation**, which occurs when the channel vectors between the BS and each MS are nearly orthogonal as the number of BS antennas becomes very large.
- With a large number of BS antennas, L, the power that is transmitted by each BS antenna element is reduced by the factor $1/L$.
- The large number of BS antennas tends to reduced the depths of envelope fades through spatial averaging and makes the overall channel less time and frequency selective.

Massive MIMO Principle

- To estimate the channels with massive MIMO, time-division duplex (TDD) operation is used with reverse link pilot sequences to enable the BS to estimate the reciprocal forward and reverse link channels.
- For the purpose of channel estimation a distinct pilot sequence is assigned to each MS in a cell, such that the pilots transmitted by the MSs in the same cell are orthogonal in time and frequency.
- The conjugate-transpose of the channel estimates are then used for linear precoding and combining, respectively, on the forward and reverse links.
- Neither the BS nor the MSs have any prior knowledge of the channels. Therefore, all channel state information (CSI) is acquired from reverse-link pilots which must be scheduled, along with forward- and reverse-link data transmissions, in a time interval (called a slot) over which the channel can be assumed to be constant.

Pilot Contamination

- With massive MIMO, there is no sharing of channel information between BSs and no power control is used. Since the number of orthogonal pilot sequences is limited, the pilot sequences are reused in the different cells of a multi-cell system according to some pilot re-use factor.
- When estimating the channels to the MSs it serves in the pilot period, a BS inadvertently learns the channel to MSs in other cells that reuse the same pilot sequence. This phenomenon is called **pilot contamination**.



Pilot contamination in massive MIMO. During the pilot phase (left) the BS overhears the pilot transmissions from MSs in other cells. During the data phase (right) the BS partially beamforms to the MSs it has overhead in other cells

Pilot Contamination Effects

- On the forward link channels, part of the energy sent to a particular MS will also be directed to an unintended MS in another co-channel cell during the data period.
- When the BS combines its reverse-link signals to receive the individual data transmissions of the MSs it serves, it is also coherently combining the signals from MSs in other co-channel cells.
- This out-of-cell interference on the forward and reverse links persists even when the number of BS antennas becomes very large and presents a fundamental performance limitation in massive MIMO systems.

Massive MIMO System Model

- Consider a massive MIMO cellular multi-access system consisting of a collection of cells, where the BSs at the center of each cell have L omnidirectional antennas and each BS serves K_{MS} MSs per cell, and where the MSs have a single omnidirectional antenna.

- The $L \times K_{\mathrm{MS}}$ matrix of channel gains between the jth BS and the K_{MS} MSs in cell ℓ is denoted as $\mathbf{H}_{j,\ell}$ which is assumed to have the form

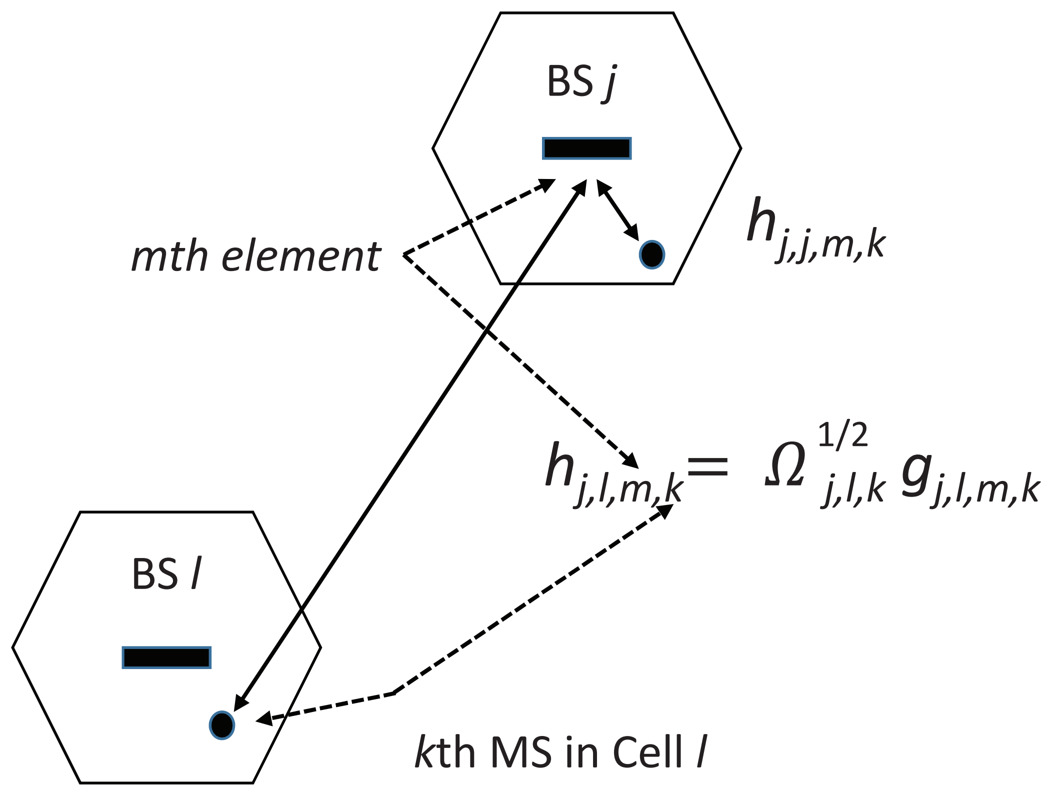
$$\mathbf{H}_{j,\ell} = \mathbf{G}_{j,\ell}\boldsymbol{\Omega}_{j,\ell}^{1/2} \ .$$

- The elements of the matrix $\mathbf{G}_{j,\ell} = [g_{j,\ell}]_{m,k} = g_{j,\ell,m,k}$ are assumed to be zero mean complex Gaussian random variables with an envelope power of unity, i.e., $\mathrm{E}[|g_{j,\ell,m,k}|^2] = 1$, which is characteristic of Rayleigh fading.

- The matrix $\boldsymbol{\Omega}_{j,\ell}$ is a $K_{\mathrm{MS}} \times K_{\mathrm{MS}}$ diagonal matrix accounting for the effects of shadowing and path loss, where the k^{th} diagonal element of the matrix, denoted as $\Omega_{j,\ell,k}$ represents the received local mean envelope power, $\Omega_{j,\ell,k} = \mathrm{E}[|h_{j,\ell,m,k}|^2]$, between the jth BS and MS k in cell ℓ, assumed to the be same for all L antenna elements for the ℓth BS.

Massive MIMO System Model

- The signal model is shown below defines the propagation coefficient $h_{j,\ell,m,k}$ between the kth MS in Cell ℓ and the mth antenna element in Cell j.



Propagation coefficient between kth MS in Cell ℓ and the mth antenna element in Cell j.

Massive MIMO Propagation Model

- The $\Omega_{j,\ell,k}$ are log-normally distributed and distance dependent according to

$$\Omega_{j,\ell,k}(d) = \frac{\mu_{\Omega_p}(d_o)\epsilon}{(d/d_o)^\beta} \ ,$$

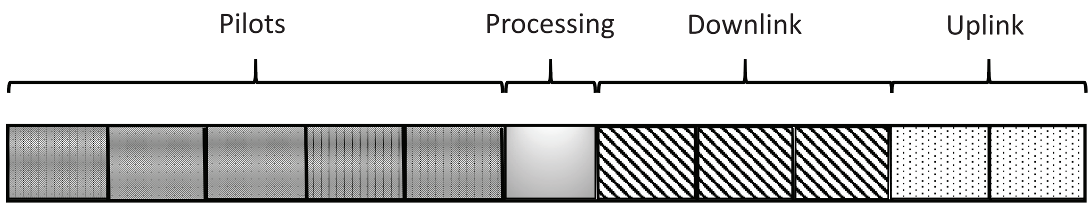
  - $\mu_{\Omega_p}(d_o) = \mathrm{E}[\Omega_p(d_o)]$ is the average received signal power at the reference distance d_o, β is the propagation path loss exponent.
  - ϵ is a log-normal random variable, such that $\epsilon_{(\mathrm{dB})}$ is a zero-mean Gaussian random variable with standard deviation equal to σ_Ω.

Reverse Link Pilots

- Reverse-link pilots are required to estimate the channel for both forward and reverse data transmission.
- The same band of frequencies is reused in a multiplicity of cells according to a frequency reuse plan. Consequently, the reverse link pilots that are received at a particular base station will be contaminated by the reverse link pilots that are transmitted by MSs in other cells sharing the same time-frequency resource.
- It is assumed that a total of N base stations in the deployment share the same band of frequencies, and the same set of K_{MS} pilot signals, one for each MS in their cells.
- The forward and reverse link transmissions are assumed to be perfectly synchronized, which constitutes the worst case condition for pilot contamination.

Time-domain Aligned Pilot Structure

- The cells use the time-domain aligned pilot (TDAP) structure shown below.
- First, pilots are transmitted by the MSs.
- The BSs then processes the received pilots to extract channel estimates.
- The channel estimates are used by the BSs to precode the data transmitted on the downlink and to perform matched filtering on the uplink transmissions from the MSs.



Time domain aligned pilots for massive MIMO. In this example, the slot consists of 5 sub-slots for pilots, one sub-slot for pilot processing, 3 sub-slots for downlink transmissions and 2 slots for uplink transmissions.

Reverse Link Pilot Signal Model

- Let $\hat{\mathbf{H}}_{j,j}$ denote the estimate of the $L \times K_{\mathrm{MS}}$ channel matrix between the K_{MS} MSs in the jth cell and the L antennas at the jth BS. Accordingly,

$$\hat{\mathbf{H}}_{j,j} = \sqrt{\gamma_p} \sum_{\ell=1}^{N} \mathbf{H}_{j,\ell} + \mathbf{v}_j \ ,$$

  - $\mathbf{H}_{j,\ell}$ is the $L \times K_{\mathrm{MS}}$ propagation matrix between the K_{MS} MSs in the ℓth cell and the L antennas at the base station in the jth cell
  - $\mathbf{v}_j$ is an $L \times K_{\mathrm{MS}}$ receiver noise matrix, whose components are zero-mean unit-variance complex Gaussian random variables that are mutually uncorrelated under the assumption of orthogonal pilots and uncorrelated with the propagation matrices $\mathbf{H}_{j,\ell}$.
  - The quantity γ_p is the pilot signal-to-noise ratio (SNR), which will be shown later to be irrelevant as the effects of noise vanish when the number of uncorrelated BS antennas, L, becomes unbounded.

Reverse Link Data Received Signals

- The K_{MS} MSs in each cell transmit their data over the reverse channel to their respective BSs.
- The BSs use estimates of the channel matrices, $\hat{\mathbf{H}}_{j,j}$ to perform maximal ratio combining. Again, assuming perfect synchronization of the signals received from the N MSs sharing the same time-fequency resource, the $L \times 1$ received signal vector at the L antennas of the jth BS is

$$\tilde{\mathbf{r}}_j = \sqrt{\gamma_r} \sum_{\ell=1}^{N} \mathbf{H}_{j,\ell} \tilde{\mathbf{s}}_\ell + \tilde{\mathbf{n}}_j \ ,$$

  - $\tilde{\mathbf{s}}_\ell$ is the $K_{\mathrm{MS}} \times 1$ vector of K_{MS} modulation symbols transmitted from the K_{MS} MSs in the ℓth cell
  - $\tilde{\mathbf{n}}_j$ is the $L \times 1$ received noise vector whose components are zero-mean, mutually uncorrelated, and uncorrelated with the propagation matrices.
  - The parameter γ_r is the symbol-energy-to-noise ratio which, like the pilot SNR, will be shown later to be irrelevant as the number of uncorrelated BS antennas, L, becomes unbounded.

Reverse Link Data Processing

- The jth BS processes the received vector $\tilde{\mathbf{r}}_j$ by multiplying it by the conjugate-transpose of the channel estimate to perform maximal ratio combining.
- From the above

$$\begin{aligned} \bar{\mathbf{r}}_j &= \hat{\mathbf{H}}_{j,j}^H \tilde{\mathbf{r}}_j \\ &= \left(\sqrt{\gamma_p} \sum_{\ell_1=1}^{N} \mathbf{H}_{j,\ell_1} + \mathbf{v}_j \right)^H \left(\sqrt{\gamma_r} \sum_{\ell_2=1}^{N} \mathbf{H}_{j,\ell_2} \tilde{\mathbf{s}}_{\ell_2} + \tilde{\mathbf{n}}_j \right) \ . \end{aligned} \tag{1}$$

- The components of the vector $\bar{\mathbf{r}}_j$ consist of the sum of inner products between random vectors of length L. As the number of BS antennas becomes very large, the squared-length or L_2 norm of these vectors grows with L, while the inner products of uncorrelated vectors, by assumption of favorable propagation conditions, grow at a lesser rate.
- For very large L, only the products of identical terms in the bracketed expressions in the above expression are significant.

Reverse Link Data Analysis

- From the above, it follows

$$\frac{1}{L}\mathbf{H}^H_{j,\ell_1}\mathbf{H}_{j,\ell_2} = \mathbf{\Omega}^{1/2}_{j,\ell_1}\left(\frac{\mathbf{G}^H_{j,\ell_1}\mathbf{G}_{j,\ell_2}}{L}\right)\mathbf{\Omega}^{1/2}_{j,\ell_2} \ , \tag{2}$$

  where, once again, $\mathbf{\Omega}_{j,\ell_1}$ and $\mathbf{\Omega}_{j,\ell_2}$ are $K_{\mathrm{MS}} \times K_{\mathrm{MS}}$ diagonal matrices.

- As the number of BS antennas L grows without bound

$$\frac{\mathbf{G}^H_{j,\ell_1}\mathbf{G}_{j,\ell_2}}{L} \longrightarrow \mathbf{I}_{K_{\mathrm{MS}}\times K_{\mathrm{MS}}}\delta_{\ell_1,\ell_2} \tag{3}$$

  due to the assumption of favorable propagation conditions.

- Substituting, (3) and (2) into (1) gives

$$\frac{1}{L\sqrt{\gamma_p\gamma_r}}\bar{\mathbf{r}}_j \longrightarrow \sum_{\ell=1}^{N}\mathbf{\Omega}_{j,\ell}\tilde{\mathbf{s}}_\ell \ , \quad j = 1,\ldots,N \ . \tag{4}$$

- The kth element of the vector $\bar{\mathbf{r}}_j$ normalized by $1/L\sqrt{\gamma_p\gamma_r}$ is

$$\frac{1}{L\sqrt{\gamma_p\gamma_r}}\bar{r}_{j,k} \longrightarrow \Omega_{j,k,j}\tilde{s}_{j,k} + \sum_{\substack{\ell=1\\ \ell\neq j}}^{N}\Omega_{j,k,\ell}\tilde{s}_{\ell,k} \ , \quad k = 1,\ldots,K_{\mathrm{MS}} \tag{5}$$

  where $\Omega_{j,k,\ell}$ is the kth diagonal element of the $K_{\mathrm{MS}} \times K_{\mathrm{MS}}$ diagonal matrix $\mathbf{\Omega}_{j,\ell}$.

Reverse Link Data Analysis

- The noise terms due to $\mathbf{v}_j$ and $\tilde{\mathbf{n}}_j$ in (4) vanish as L goes to infinity, due to the multiplicative $1/L$ factor.
- The effects of fast envelope fading are completely eliminated, due to spatial (antenna) averaging, and the transmissions from the $K_{\rm MS}$ MSs in each cell do not interfere with each other.
- The second term in the R.H.S. of (5) represents co-channel interference received from MSs in other cells sharing the same pilot. Thus, with idealized (synchronized) massive MIMO, the interference-to-noise ratio is infinite and the signal-to-interference-plus noise ratio (SINR) becomes equal to the signal-to-interference ratio (SIR).
- In the R.H.S. of (5), both the desired symbol $\tilde{\mathbf{s}}_k$ and interfering symbols in the summation are multiplied by their respective local mean powers $\Omega_{j,k}$ that are received at the jth BS. Consequently, due to the vanishing effects of noise, the SIR at the jth BS involves the squares of the $\Omega_{j,k,j}$ and $\Omega_{j,k,\ell}$, and is given by

$$
{\rm SIR}_{r,k} = \frac{\Omega^2_{j,k,j}}{\sum_{\substack{\ell=1 \\ \ell\neq j}}^{N} \Omega^2_{j,k,\ell}} \ , \quad k = 1, \ldots, K_{\rm MS} \ ,
$$

where the assumption is made that the co-channel interferers sharing the same time-frequency resource block are uncorrelated, which is easily satisfied if their associated information symbol sequences are uncorrelated.

Forward Link Data Precoding

- The forward channel uses precoding such that each BS transmits a length-L vector from its L transmit antennas, which is proportional to the complex conjugate transpose of its propagation matrix as estimated from reverse link pilots.
- The ℓth BS transmits the $L \times 1$ vector, $\hat{\mathbf{H}}_{\ell,\ell}^H \tilde{\mathbf{s}}_\ell$, where $\tilde{\mathbf{s}}_\ell$ is a length-K_{MS} column vector of information symbols transmitted to the K_{MS} MSs served by BS ℓ.
- The K_{MS} MSs served by the the jth BS each receive their respective component of a length K_{MS} vector that is comprised of the signals transmitted from each of the BS in the N surrounding co-channel cells. That is,

$$\begin{aligned}
\bar{\mathbf{r}}_j &= \sqrt{\gamma_f} \sum_{\ell=1}^{N} \hat{\mathbf{H}}_{\ell,j}^T \mathbf{H}_{\ell,\ell}^H \tilde{\mathbf{s}}_\ell + \mathbf{w}_j \\
&= \sqrt{\gamma_f} \sum_{\ell_1=1}^{N} \hat{\mathbf{H}}_{\ell_1,j}^T \left(\sqrt{\gamma_p} \sum_{\ell_2=1}^{N} \mathbf{H}_{\ell_1,\ell_2} + \mathbf{v}_{\ell_1} \right)^H \tilde{\mathbf{s}}_{\ell_1} + \mathbf{w}_j \ , \qquad (6)
\end{aligned}$$

where $\mathbf{w}_j$ is a $K_{\mathrm{MS}} \times 1$ vector of uncorrelated noise components, and γ_f is a measure of the forward channel SNR which will be shown to be irrelevant as the number of BS antennas L grows without bound.

Forward Link Data Analysis

- As with the reverse channel, as the number of BS antennas L goes to infinity, (2) and (3) are once again used to yield

$$\frac{1}{L\sqrt{\gamma_p\gamma_f}}\bar{\mathbf{r}}_j \longrightarrow \sum_{\ell=1}^{N} \boldsymbol{\Omega}_{\ell,j}\tilde{\mathbf{s}}_\ell \ , \quad j = 1,\ldots,N \ . \tag{7}$$

- The kth MS in the jth cell receives

$$\frac{1}{L\sqrt{\gamma_p\gamma_f}}\bar{r}_{k,j} \longrightarrow \Omega_{j,k,j}\tilde{s}_{k,j} + \sum_{\substack{\ell=1 \\ \ell\neq j}}^{N} \Omega_{\ell,k,j}\tilde{s}_{k,\ell} \ , \quad j = 1,\ldots,N \ . \tag{8}$$

- As with the reverse link, the effective signal-to-interference ratio for the kth MS in the jth cell is

$$\mathrm{SIR}_{f,k} = \frac{\Omega^2_{j,k,j}}{\sum_{\substack{\ell=1 \\ \ell\neq j}}^{N} \Omega^2_{\ell,k,j}} \ , \quad k = 1,\ldots,K_{\mathrm{MS}} \tag{9}$$

- Although the forward and reverse SIRs look similar they are in fact different meaning that the forward and reverse channels exhibit link imbalance.
  - The numerator term in each case is the same. However, the denominators are different.
  - The set of propagation distances to the co-channel interferers are not the same for the forward and reverse links, meaning their respective area mean powers are different.