

For a narrowband channel

$$g(t) = \sum_{n=1}^N C_n e^{j\phi_n(t)}$$

where  $\phi_n(t) = \phi_n - 2\pi c z_n / \lambda_c + 2\pi f_{D,n} t$

and  $f_{D,n} t = f_m \cos(\theta_n) t$

we have,

$$g(t) = \underbrace{\sum_{n=1}^N C_n \cos \phi_n(t)}_{g_I(t)} + j \underbrace{\sum_{n=1}^N C_n \sin \phi_n(t)}_{g_Q(t)}$$

$$\phi_{g_I g_I}(\tau) = E [g_I(t) g_I(t+\tau)]$$

$$= E \left[ \sum_{n=1}^N C_n \cos \phi_n(t) \sum_{m=1}^N C_m \cos \phi_m(t+\tau) \right]$$

$$= E \left[ \sum_{n=1}^N \sum_{m=1}^N C_n C_m \cos \phi_n(t) \cos \phi_m(t+\tau) \right]$$

$$= \sum_{n=1}^N \sum_{m=1}^N C_n C_m E [\cos \phi_n(t) \cos \phi_m(t+\tau)]$$

For  $n \neq m$  in the double sum, the i.i.d. uniform random phases

$\phi_n - 2\pi c z_n / \lambda_c$ ;  $\phi_n, z_n$  are random and  $\phi_m - 2\pi c z_m / \lambda_c$

ensure that  $\phi_n(t)$  and  $\phi_m(t+\tau)$  are i.i.d. uniform random variables on  $[-\pi, \pi)$

$$\phi_{SIS}(z) = \sum_{n=1}^N C_n^2 E[\cos \phi_n(t) \cos \phi_n(t+z)]$$

$$E[\cos x] = 0 \\ x \sim U[-\pi, \pi)$$

$$+ \sum_n \sum_{m \neq n} E[\cos \phi_n(t)] E[\cos \phi_m(t+z)]$$

$$= \sum_{n=1}^N C_n^2 E\left[\frac{1}{2} \cos(2\pi f_{D,n} t)\right]$$

$$+ \frac{1}{2} \cos(2\phi_n - 4\pi c z_n / \lambda_c + 2\pi f_{D,n}(2t+z))$$

wrapped phase uniform random variable on  $[-\pi, \pi)$

$$= \frac{1}{2} \sum_{n=1}^N C_n^2 E[\cos(2\pi f_m t \cos \theta_n)]$$

$$+ \frac{1}{2} \sum_{n=1}^N C_n^2 E[\cos(2\phi_n - 4\pi c z_n / \lambda_c + 2\pi f_{D,n}(2t+z))]$$

$$= \frac{\Omega_p}{2} E[\cos(2\pi f_m t \cos \theta_n)]$$

where  $\sum_{n=1}^N C_n^2 = \Omega_p$