

ECE6604: Assignment 1

- Date Assigned: January 15, 2009.
- Date Due: January 27, 2009.

- 1) When a call is placed it will sometimes get blocked because there are no more channels available in the serving cell. One method to decrease the probability of call blocking is to allow queueing of the incoming calls. An incoming call that does not find an idle channel in the serving cell is allowed to remain in a queue for t_q seconds and is only dropped from the queue, i.e., get blocked, if no channel becomes available in that time.

Suppose the handoff queue is serviced using a “first-come-first-served” queueing discipline. If m is the total number of channels available in the serving cell and ρ is the total offered traffic, then the probability that an incoming call will be queued is given by the famous Erlang-C formula:

$$C(\rho, m) = \frac{\rho^m}{\rho^m + m! \left(1 - \frac{\rho}{m}\right) \sum_{k=0}^{m-1} \frac{\rho^k}{k!}},$$

which is derived under the standard Markovian assumptions of Poisson call arrivals and exponentially distributed call durations.

Given that an incoming call is queued, the probability that it will have to wait more than t_q seconds in the queue, i.e., get blocked, is

$$P(W > t_q) = \exp\left\{-\frac{(m - \rho)t_q}{\mu}\right\}$$

where μ is the mean call duration. Assuming that $\mu = 120$ s and $t_q = 5$ s, plot the probability of call blocking against the normalized offered traffic per channel $G_c = \rho/m$, for $m = 5, 10, 15$.

Note: In this problem you may need to find a recursive method to evaluate the sum in $C(\rho, m)$.

- 2) A wireless channel is characterized by the time-variant impulse response

$$g(t, \tau) = \left(1 - \frac{\tau}{T}\right) \cos(\Omega t + \phi_0), \quad 0 \leq \tau \leq T,$$

where $T = 0.05$ ms, $\Omega = 10\pi$, and $\phi_0 \in [-\pi, +\pi]$ is a constant.

- Determine the channel time-variant transfer function.
- Given the complex channel input signal

$$\tilde{s}(t) = \begin{cases} 1, & 0 \leq t \leq T_s \\ 0, & \text{otherwise} \end{cases},$$

where $0 \leq T_s \leq T$, determine the complex channel output signal, $\tilde{r}(t)$.

- For continuous digital transmission with symbol interval T_s , if the channel fading is frequency selective, specify the relation between T_s and T .

- 3) Consider a Rician fading channel with Rice factor K and average envelope power Ω_p . Derive an integral expression for the probability density function of the envelope phase in terms of K and Ω_p .

Note: There is a typo in the 3 expressions for $p_{RV}(r, v)$ on page 691 of the text. The multiplicative factor $\frac{1}{2\pi\sigma^2}$ should be $\frac{r}{2\pi\sigma^2}$; the r is missing.

- 4) A very useful model for a non-isotropic scattering environment assumes that the azimuth angle-of-arrival distribution is described by the von Mises pdf

$$p(\theta) = \frac{1}{2\pi I_0(k)} \exp[k \cos(\theta - \mu)] , \quad -\pi \leq \theta \leq \pi ,$$

where $\theta \in [-\pi, \pi)$, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, $\mu \in [-\pi, \pi)$ is the mean angle of arrival, and k controls the spread of scatterers around the mean. When $k = 0$, the distribution reduces to isotropic scattering, i.e.,

$$p(\theta) = \frac{1}{2\pi} , \quad -\pi \leq \theta \leq \pi .$$

- a) Assuming an isotropic receiver antenna, calculate the received Doppler power spectrum.
b) Under what conditions are the quadrature components $g_I(t)$ and $g_Q(t)$ uncorrelated?