ECE 6604 Homework #5 Solutions

1. 3 Compare the *peak-to-mean envelope power ratio* (PMEPR) of QPSK and $\pi/4$-DQPSK.

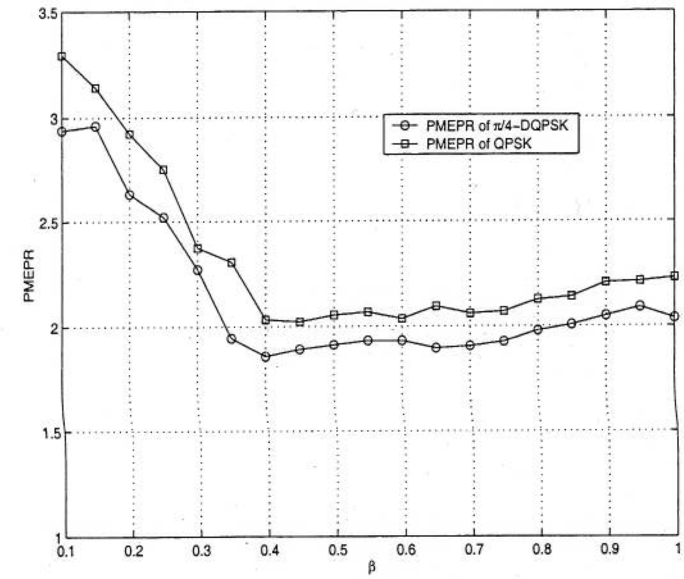

Figure 2: PMEPR Performance of $\pi/4$-DQPSK and QPSK

The *peak-to-mean envelope power ratio* of $\pi/4$-DQPSK is lower than that of QPSK. This indicates that $\pi/4$-DQPSK is less sensitive to the amplifier non-linearality than QPSK.

2: 2a) Note that $h_f(t)$ should integrate to $1/2$. So the peak value of $h_f(t)$ should have been $1/T$ and not $1/2T$. This would give a peak frequency deviation of $h \times \frac{1}{2} \times (M-1) = (M-1)/2T$.

b)

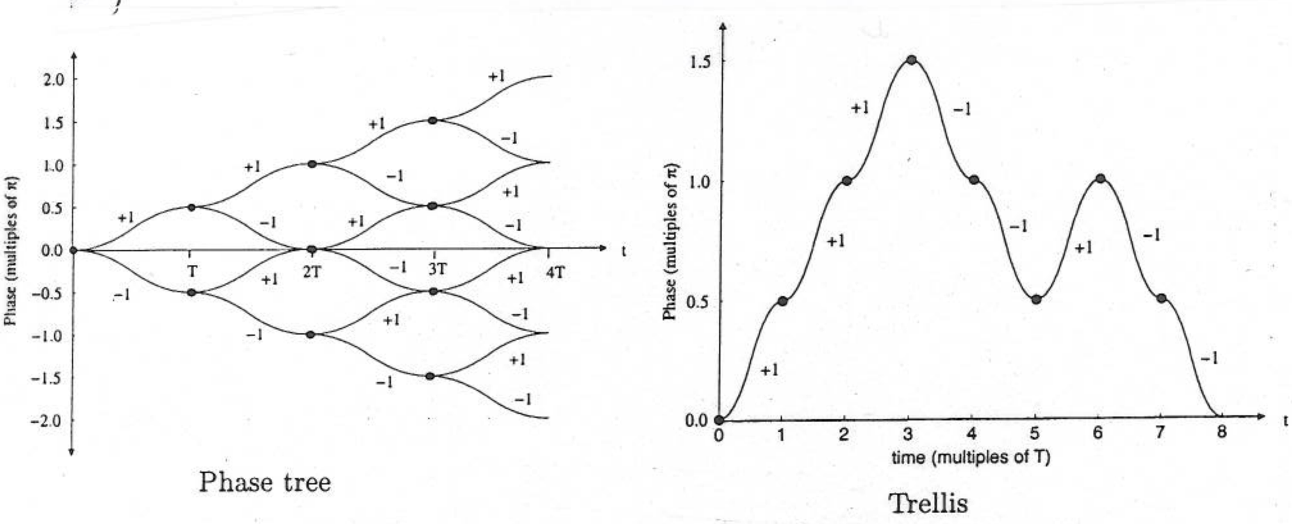

Figure 1: Phase tree and trellis for the sequence $\mathbf{x} = (+1, +1, +1, -1, -1, +1, -1, -1)$.

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A Guassian pulse-shaping filter has the transfer function

$$H(f) = \exp\left\{-(\frac{f}{B})^2\frac{\ln 2}{2}\right\}.$$

The impulse response of the filter is

$$\begin{aligned}
h(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(f)\exp j2\pi f t df \\
&= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp\{-(\frac{f}{B})^2\frac{\ln 2}{2} + j2\pi f t\}df \\
&= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp\{-\frac{1}{B^2}\frac{\ln 2}{2}[f - j2\pi t\frac{B^2}{\ln 2}]^2\}\cdot\exp\{-2\pi^2 t^2\frac{B^2}{\ln 2}\}df \\
&= \frac{1}{2\pi}\exp\{-2\pi^2 t^2\frac{B^2}{\ln 2}\}\int_{-\infty}^{+\infty} \exp\{-\frac{1}{B^2}\frac{\ln 2}{2}f^2\}\cdot df \\
&= \frac{1}{\sqrt{2\pi}}\exp\{-2\pi^2 t^2\frac{B^2}{\ln 2}\}\cdot B\sqrt{\frac{1}{\ln 2}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \exp\{-\frac{1}{2}s^2\}\cdot ds \\
&= \frac{1}{\sqrt{2\pi\ln 2}}B\exp\{-2\pi^2 t^2\frac{B^2}{\ln 2}\}.
\end{aligned}$$

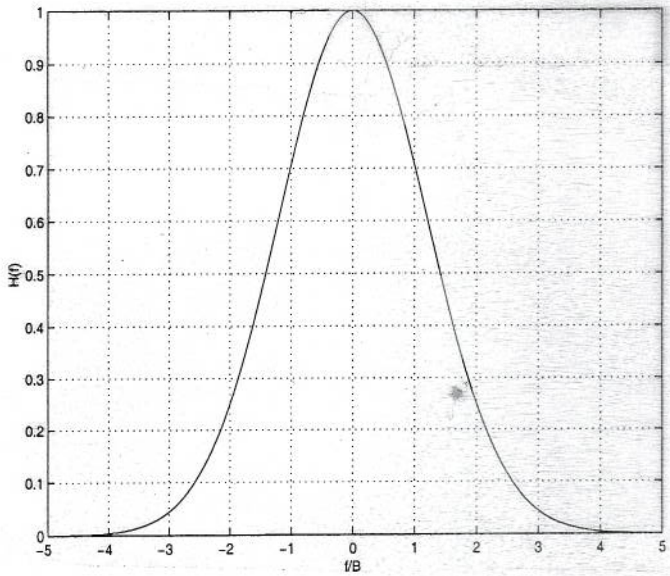
The frequence shaping pulse is

$$h_f(t) = \frac{1}{2T}\left[Q\left(\frac{t/T + 1/2}{\sigma}\right) - Q\left(\frac{t/T - 1/2}{\sigma}\right)\right],$$

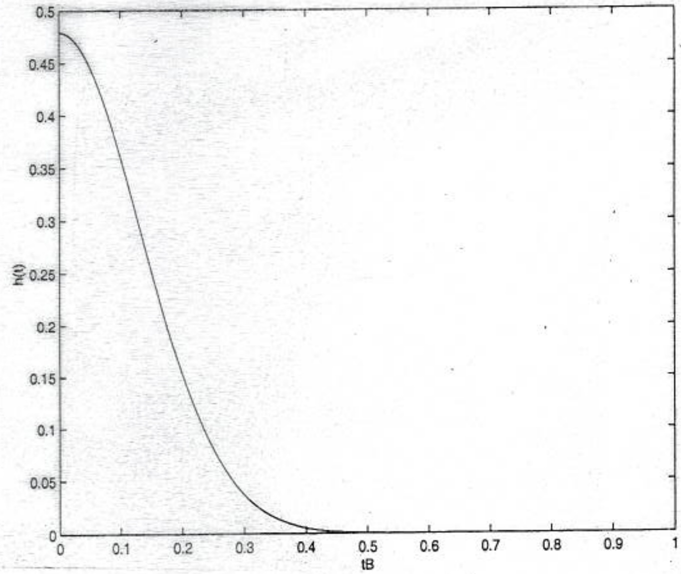
where

$$\sigma^2 = \frac{\ln 2}{4\pi^2(BT)^2}.$$

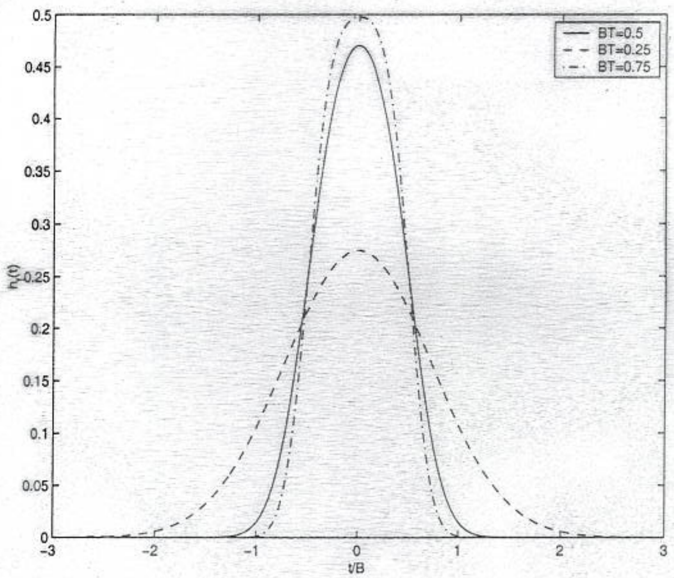
The frequency response and the impulse response of the filter, and the frequency shaping pulse $h_f(t)$ are shown in the respective figures.



Frequency Response $H(f)$



Impulse Response $h(t)$



Frequency Shaping Pulse $h_f(t)$

18 a) We need to calculate the autocorrelation function of the binary sequence $\mathbf{x}$, $x_i \in \{-1, +1\}$ such that $P(x_n = x_{n+1}) = 3/4$ and $P(x_n \neq x_{n+1}) = 3/4$. Consider $\phi_{xx}(0)$ and $\phi_{xx}(1)$,i.e.

$$\begin{aligned}\phi_{xx}(0) &= \mathrm{E}[x_n x_n] = 1 \text{ and}\\ \phi_{xx}(1) &= \mathrm{E}[x_n x_{n+1}] = (1)\cdot P(x_n = x_{n+1}) + (-1)\cdot P(x_n \neq x_{n+1}) = 3/4 - 1/4 = 1/2\end{aligned}$$

We claim that the autocorrelation function of this data sequence is $\phi_{xx}(m) = (\frac{1}{2})^{|m|}$ and we will prove it by induction. We have already shown that $\phi_{xx}(1) = 1/2$. Assuming $\phi_{xx}(m) = (\frac{1}{2})^{|m|}$, we will show $\phi_{xx}(m+1) = (\frac{1}{2})^{|m+1|}$. For $m \geq 2$, observe that

$$\begin{aligned}\phi_{xx}(m+1) &= \mathrm{E}[x_n x_{n+m+1}]\\ &= \mathrm{E}[x_{n+1}x_{n+m+1}]\cdot P(x_n = x_{n+1}) + -\mathrm{E}[x_{n+1}x_{n+m+1}]\cdot P(x_n \neq x_{n+1})\\ &= \phi_{xx}(m)\cdot\frac{3}{4} - \phi_{xx}(m)\cdot\frac{1}{4}\\ &= \frac{1}{2}\phi_{xx}(m) = \left(\frac{1}{2}\right)^{m+1}\end{aligned}$$

Similarly, we can show that $\phi_{xx}(m) = \left(\frac{1}{2}\right)^{-m}$ for $m \leq 0$.

b) The power spectrum $S_{xx}(f)$ of the binary data sequence $\mathbf{x}$ is

$$\begin{aligned}S_{xx}(f) &= \sum_{m=-\infty}^{\infty}\phi_{xx}(m)e^{-j2\pi fm} = \sum_{m=-\infty}^{\infty}\left(\frac{1}{2}\right)^{|m|}e^{-j2\pi fm}\\ &= -1 + \sum_{m=0}^{\infty}\left(\frac{1}{2}\right)^{m}e^{-j2\pi fm} + \sum_{m=0}^{\infty}\left(\frac{1}{2}\right)^{m}e^{j2\pi fm}\\ &= -1 + \frac{1}{1-\frac{1}{2}e^{-j2\pi f}} + \frac{1}{1-\frac{1}{2}e^{j2\pi f}}\\ &= \frac{2-\cos(2\pi f)}{5/4-\cos(2\pi f)} - 1 = \frac{3}{5-4\cos(2\pi f)}.\end{aligned}$$