# ECE 6604 Homework # 6 Solutions

1. Since the PAPR is determined through simulations, your results may differ <u>slightly</u> from the answers below

| $N$ | $\mu$ | $\sigma^2$ |
|---|---|---|
| 256 | 6.12 | 1.50 |
| 512 | 6.84 | 1.61 |
| 1024 | 7.54 | 1.62 |

All values are in fractional units.
Use 10log10(x) to convert to dB units.

5.10. Non-coherent FSK $P_b(\gamma_b) = \frac{1}{2} e^{-\gamma_b/2}$ $\gamma_b = \alpha^2 \frac{E_b}{N_0}$

a) $\alpha$ Rayleigh

$$p_{\gamma_b}(x) = \frac{1}{\bar{\gamma}_b} e^{-x/\bar{\gamma}_b}, \quad \bar{\gamma}_b = E[\alpha^2]\frac{E_b}{N_0}$$

$$P_b = \int_0^\infty P_b(x) p_{\gamma_b}(x) dx = \int_0^\infty \frac{1}{2} e^{-x/2} \frac{1}{\bar{\gamma}_b} e^{-x/\bar{\gamma}_b} dx = \frac{1}{2\bar{\gamma}_b} \int_0^\infty e^{-x\left(\frac{1}{2} + \frac{1}{\bar{\gamma}_b}\right)} dx$$

$$= \frac{1}{2 + \bar{\gamma}_b}$$

(b) $$p_{\gamma_b}(x) = \frac{1}{\bar{\gamma}_b} e^{-K - \frac{(K+1)x}{\bar{\gamma}_b}} I_0\left(2\sqrt{\frac{K(K+1)x}{\bar{\gamma}_b}}\right)$$

$$P_b = \int_0^\infty P_b(x) p_{\gamma_b}(x) dx = \int_0^\infty \frac{1}{2} e^{-x/2} p_{\gamma_b}(x) dx = \frac{1}{2} E\left[e^{-\gamma_b/2}\right]$$

Noting that $\phi_{\gamma_b}(j\nu) = E\left[e^{j\nu\gamma_b}\right]$ is the characteristic function, we can use $\phi_{\gamma_b}(j\nu)\big|_{\nu=-1/2}$ to find the result.

$$\phi_{\gamma_b}(j\nu) = \frac{1}{1 - j2\nu\sigma^2} \exp\left\{\frac{j\nu s^2}{1 - j\nu 2\sigma^2}\right\} \qquad 2\sigma^2 = \frac{\bar{\gamma}_b}{K+1} \qquad s^2 = \frac{K\bar{\gamma}_b}{K+1}$$

$$\therefore \quad E\left[e^{-\gamma_b/2}\right] = \phi_{\gamma_b}(j\nu)\big|_{\nu=-1/2} = \frac{2}{2+2\sigma^2} \exp\left\{-\frac{s^2}{2+2\sigma^2}\right\}$$

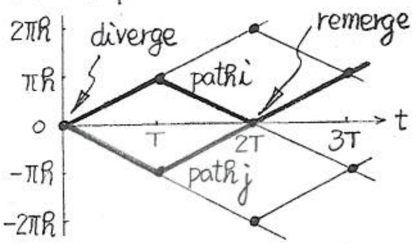
$$= \frac{2}{2 + \frac{\bar{\gamma}_b}{K+1}} \exp\left\{-\frac{\frac{K\bar{\gamma}_b}{K+1}}{2 + \frac{\bar{\gamma}_b}{K+1}}\right\}$$

$$= \frac{2(K+1)}{2(K+1) + \bar{\gamma}_b} \exp\left\{-\frac{K\bar{\gamma}_b}{2(K+1) + \bar{\gamma}_b}\right\}$$

Finally, $$P_b = \frac{1}{2} E\left[e^{-\gamma_b/2}\right]$$

$$= \frac{K+1}{2(K+1) + \bar{\gamma}_b} \exp\left\{-\frac{K\bar{\gamma}_b}{2(K+1) + \bar{\gamma}_b}\right\}$$

5.11 (a) The phase trellis




$\underline{x}^{(i)} = \{1, -1, x_2, x_3, \cdots\}$

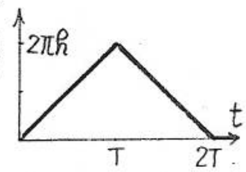
$\underline{x}^{(j)} = \{-1, 1, x_2, x_3, \cdots\}$

path $i$ & $j$ differ for $t = 0, T$

agree for $t \ge 2T$

Corresponding to the phase difference sequences :

$\Delta\phi_{ij}(t) = \{2, -2, 0, 0, \cdots\}$




$$S(t) = \mathrm{Re}\{v(t) e^{j2\pi f_c t}\} = A\cos[2\pi f_c t + \phi(t)] \ , \ \text{where } \phi(t) = \beta(T)\sum_{n=-\infty}^{k-1} x_n + x_k \beta(t - kT) \ , \ \beta(t) = \begin{cases} 0 & , t < 0 \\ \pi h t/T & ; 0 \le t \le T \\ \pi h & , t \ge T \end{cases}$$

$$\Rightarrow D_{min}^2 = \int_0^{2T} [S(t; \underline{x}^{(i)}) - S(t; \underline{x}^{(j)})]^2 dt$$

$$= \int_0^{2T} \{A\cos[2\pi f_c t + \phi_i(t)] - A\cos[2\pi f_c t + \phi_j(t)]\}^2 dt$$

$$= A^2 \int_0^{2T} \{\cos^2[2\pi f_c t + \phi_i(t)] + \cos^2[2\pi f_c t + \phi_j(t)] - 2\cos[2\pi f_c t + \phi_i(t)]\cos[2\pi f_c t + \phi_j(t)]\} dt$$

$$= A^2 \{T + T - \int_0^{2T} \cos[\Delta\phi_{ij}(t)]\, dt\} \qquad \langle f_c T \gg 1 \rangle$$

$$= 2A^2T - A^2 \left\{ \int_0^T \cos\left(\frac{2\pi h t}{T}\right) dt + \int_T^{2T} \cos\left(\frac{2\pi h(2T - t)}{T}\right) dt \right\}$$

$$= 2A^2T - 2A^2 \int_0^T \cos\left(\frac{2\pi h t}{T}\right) dt = 2A^2T\left(1 - \frac{\sin 2\pi h}{2\pi h}\right) = 4E_b[1 - Sa(2\pi h)] \qquad \left\langle E_b = \frac{A^2 T}{2} \right\rangle$$

(b) $P_b = K_{D_{min}} Q\left(\frac{D_{min}}{2\sigma_n}\right) = Q\left(\frac{\sqrt{4E_b[1 - Sa(2\pi h)]}}{2\sqrt{\frac{N_o}{2}}}\right) = Q\left(\sqrt{2[1 - Sa(2\pi h)]\gamma_b}\right)$ with $\gamma_b = \frac{E_b}{N_o}$

6.10. a) We will find the cdf of $\lambda$ as follows:

$$\begin{aligned}
F_\lambda(x) &= P(\lambda = \frac{s_0}{s_1} \leq x) = \int_0^\infty \int_0^{xs_1} p_{s_0}(s_0)p_{s_1}(s_1)ds_0ds_1 \\
&= \int_0^\infty \int_0^{xs_1} \frac{1}{s_0}e^{-s_0/\bar{s}_0}\frac{1}{s_1}e^{-s_1/\bar{s}_1}ds_0ds_1 \\
&= \frac{1}{s_1}\int_0^\infty e^{-s_1/\bar{s}_1}\left\{-e^{-s_0/\bar{s}_0}|_0^{xs_1}\right\}ds_1 = \frac{1}{s_1}\int_0^\infty e^{-s_1/\bar{s}_1}\left[1--e^{-xs_1/\bar{s}_0}\right]ds_1 \\
&= \frac{1}{s_1}\int_0^\infty e^{-s_1/\bar{s}_1 ds_1} - \frac{1}{s_1}\int_0^\infty e^{-s_1\left(\frac{1}{\bar{s}_1}+\frac{x}{\bar{s}_1}\right)}ds_1 \\
&= 1 - \frac{1}{1+\frac{\bar{s}_1}{\bar{s}_0}x} = \frac{x}{\frac{\bar{s}_0}{\bar{s}_1}+x} = \frac{x}{\bar{\lambda}+x}
\end{aligned}$$

The pdf of $\lambda$ is just the derivative of its cdf and is

$$p_\lambda(x) = \frac{\bar{\lambda}}{(x+\bar{\lambda})^2} \text{ for } \lambda \geq 0.$$

The mean value of $\lambda$ is

$$\mathrm{E}[\lambda] = \int_0^\infty x p_\lambda(x)dx = \int_0^\infty \frac{x\bar{\lambda}}{(x+\bar{\lambda})^2}dx = \infty.$$

**b)** When the selection diversity combining is used the output of the selection combiner will be

$$\lambda_b^s = \max\{\lambda_1, \ldots, \lambda_L\}$$

The cdf of the signal-to-interference ratio at the output of the selection combiner will be

$$F_{\lambda_b^s}(x) = \left(\frac{x}{x+\bar{\lambda}}\right)^L.$$

The pdf of $\lambda_b^s$ will be the derivative of its cdf, i.e.

$$p_{\lambda_b^s}(x) = L\left(\frac{x}{x+\bar{\lambda}}\right)^{L-1}\frac{\bar{\lambda}}{(x+\bar{\lambda})^2}$$