# ECE 6604 Assignment #7 Solutions

1./ The transmitted sequence with the addition of a guard interval is

$$X_k^g = X_{(k)_N} = A \sum_{n=0}^{N-1}{}' x_{0,n} e^{j\frac{2\pi nk}{N}}, \quad k=0,1,\ldots,N+G-1$$

The received sequence over a channel with discrete impulse response $\{g_m\}_{m=0}^{L}$ is

$$R_n^g = \sum_{m=0}^{L}{}' g_m X_{n-m}^g$$

After removal of the guard interval

$$R_n = R^g_{G+(n-G)_N} = \sum_{m=0}^{L}{}' g_m X_{(n-m)_N}, \quad 0 \le n \le N-1$$

Now form

$$Z_i = \frac{1}{N}\sum_{n=0}^{N-1}{}' R_n e^{-j\frac{2\pi n i}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}{}' \left(\sum_{m=0}^{L}{}' g_m X_{(n-m)_N}\right) e^{-j\frac{2\pi n i}{N}}$$

Change variable of summation. Let $l = n-m$, $n = l+m$

$$Z_i = \frac{1}{N}\sum_{m=0}^{L}{}' \sum_{l=-m}^{N-1-m}{}' g_m X_{(l)_N} e^{-j\frac{2\pi i(l+m)}{N}}$$

$$= \underbrace{\sum_{m=0}^{L}{}' g_m e^{-j\frac{2\pi i m}{N}}}_{\eta_i} \frac{1}{N}\sum_{l=-m}^{N-1-m}{}' X_{(l)_N} e^{-j\frac{2\pi i l}{N}}$$

$$= \eta_i \frac{1}{N}\sum_{l=-m}^{N-1-m}{}' X_{(l)_N} e^{-j\frac{2\pi i l}{N}}$$

$$= \eta_i \frac{1}{N}\sum_{l=0}^{N-1}{}' X_l e^{-j\frac{2\pi i l}{N}}$$

$$= \eta_i A x_i$$

b) Now the transmitted sequence with guard interval is

$$X_k^g = \begin{cases} X_k, & k = 0, \dots N-1 \\ 0, & k = N, \dots N+G-1 \end{cases}$$

The received sequence is

$$R_n = \begin{cases} \sum_{m=0}^{n}{}' g_m X_{n-m}, & n < L \\ \sum_{m=0}^{L} g_m X_{n-m}, & n \geq L \end{cases}$$

Now form
$$Z_i = \frac{1}{N} \sum_{n=0}^{N-1}{}' R_n e^{-j\frac{2\pi n i}{N}}$$

If $L=0$, there is no problem, since

$$R_n = g_0 X_n, \quad n = 0, \dots N-1$$

$$Z_i = \frac{1}{N} \sum_{n=0}^{N-1}{}' R_n e^{-j\frac{2\pi n i}{N}}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1}{}' g_0 X_n e^{-j\frac{2\pi n i}{N}}$$

$$= g_0 A x_i$$

However, if $L > 0$, then

$$Z_i = \frac{1}{N} \sum_{n=0}^{N-1}{}' \left( \sum_{m=0}^{L}{}' g_m X_{n-m} \right) e^{-j\frac{2\pi i n}{N}}$$

Change variable of summation. Let $\ell = n-m$, $n = \ell + m$

$$Z_i = \frac{1}{N} \sum_{m=0}^{L}{}' \sum_{\ell=-m}^{N-1-m}{}' g_m X_\ell e^{-j\frac{2\pi i(\ell+m)}{N}}$$

$$= \underbrace{\sum_{m=0}^{L}{}' g_m e^{-j\frac{2\pi i m}{N}}} \frac{1}{N} \sum_{\ell=-m}^{N-1-m}{}' X_\ell e^{-j\frac{2\pi i \ell}{N}}$$

$$= \quad \eta_i \quad \frac{1}{N} \sum_{\ell=0}^{N-1-m}{}' X_\ell e^{-j\frac{2\pi i \ell}{N}}$$

Note the lower limit of summation is $\ell = 0$, since $X_\ell = 0$, $\ell < 0$.

$$\neq \eta_i A x_i$$

2/ (a)

$$
\begin{aligned}
R_k &= \tilde{s}(kT_s + \Delta_t) \\
&= A\sum_{n=0}^{N-1} x_n e^{\frac{j2\pi kn}{N}} e^{\frac{j2\pi n\Delta_t}{NT_s}}.
\end{aligned}
$$

Taking FFT on the received samples,

$$
\begin{aligned}
Z_i &= \frac{1}{N}\sum_{k=0}^{N-1} R_k e^{-\frac{j2\pi ki}{N}} \\
&= \frac{A}{N}\sum_{k=0}^{N-1}\sum_{n=0}^{N-1} x_n e^{\frac{j2\pi kn}{N}} e^{\frac{j2\pi n\Delta_t}{NT_s}} e^{-\frac{j2\pi ki}{N}} \\
&= \frac{A}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} x_n e^{\frac{j2\pi k(n-i)}{N}} e^{\frac{j2\pi n\Delta_t}{NT_s}} \\
&= Ax_i e^{\frac{j2\pi n\Delta_t}{NT_s}}.
\end{aligned}
$$

Timing offset smaller than the guard interval results in a phase shift.

**(b)** Let us assume that M out of N samples come from different OFDM block due to timing offset. Then,

$$
R_k = \begin{cases} A\sum_{n=0}^{N-1} x_n e^{\frac{j2\pi kn}{N}} e^{\frac{j2\pi n\Delta_t}{NT_s}} & (0 \le k \le N-M-1) \\ A\sum_{n=0}^{N-1} x'_n e^{\frac{j2\pi kn}{N}} e^{\frac{j2\pi n\Delta_t}{NT_s}} & (N-M \le k \le N-1) \end{cases}
$$

Therefore,

$$
\begin{aligned}
Z_i &= \frac{1}{N}\sum_{k=0}^{N-1} R_k e^{-\frac{j2\pi ki}{N}} \\
&= \frac{A}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-M-1} x_n e^{\frac{j2\pi k(n-i)}{N}} e^{\frac{j2\pi n\Delta_t}{NT_s}} + \frac{A}{N}\sum_{n=0}^{N-1}\sum_{k=N-M}^{N-1} x'_n e^{\frac{j2\pi k(n-i)}{N}} e^{\frac{j2\pi n\Delta_t}{NT_s}} \\
&= Ax_i e^{\frac{j2\pi n\Delta_t}{NT_s}} + \frac{A}{N}\sum_{n=0}^{N-1}\sum_{k=N-M}^{N-1} (x'_n - x_n) e^{\frac{j2\pi k(n-i)}{N}} e^{\frac{j2\pi n\Delta_t}{NT_s}}.
\end{aligned}
$$

Timing offset greater than the guard channel introduces ISI.

3. This problem is similar to Homework # 6, Question 1.

Your solutions may differ slightly from those below, since they are generated from random vectors.

| $L$ | $\overline{PAPR}$ | $\sigma^2_{PARR}$ |
|---|---|---|
| 1 | 6.12 | 1.50 |
| 2 | 5.47 | 0.60 |
| 4 | 5.05 | 0.30 |

All values are in fractional units.
Use 10log10 to convert to dB units.

4. We employ Alamouti's scheme on a per subcarrier basis. That is, treat each subcarrier independently.

Transmit OFDM symbol

| | Period 1 | Period 2 |
|---|---|---|
| Ant 1 | $\underline{X}_1$ | $-\underline{X}_2^*$ |
| Ant 2 | $\underline{X}_2$, | $\underline{X}_1^*$ |

$\underline{X}_1 = (X_{11}, X_{12}, \dots X_{1N})$

$\underline{X}_2 = (X_{21}, X_{22}, \dots X_{2N})$

$X_{ij}$ = symbol from Ant $i$ and subcarrier $j$