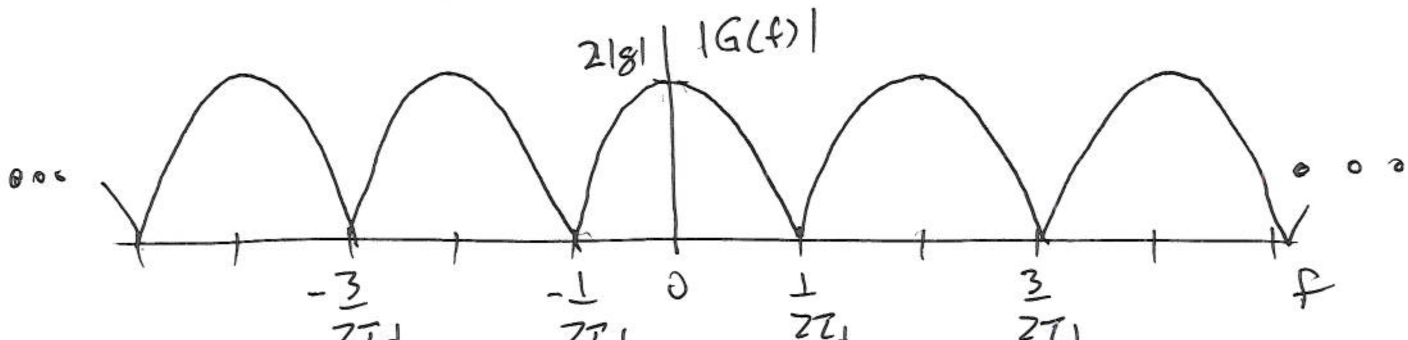
ECE 6604 Assignment #8 Solutions

1. The sub-carriers of the OFDM complex envelope (assuming the complex envelope is centered at 0 Hz) are placed at $2000k$ Hz, $k = \pm 1, \pm 3, \pm 5, \ldots$

The channel transfer function is

$$
\begin{aligned}
G(f) &= \mathcal{F}\{g(t,\tau)\} \\
&= g\left(1 + e^{-j2\pi f \tau_d}\right) \\
&= 2g e^{-j\pi f \tau_d}\left(\frac{e^{j\pi f \tau_d} + e^{-j\pi f \tau_d}}{2}\right) \\
&= 2g e^{-j\pi f \tau_d} \cos(\pi f \tau_d)
\end{aligned}
$$

$$|G(f)| = 2|g|\,|\cos(\pi f \tau_d)|$$



Now choose τ_d so the subcarriers are on the nulls of $|G(f)|$

$$\frac{k}{2\tau_d} = 2000k, \quad k \text{ odd}$$

$$\text{or} \quad \tau_d = .25 \times 10^{-3}$$

2.

$$\text{PAPR} = \frac{\max\limits_{0 \le t \le T} |\tilde{s}(t)|^2}{E\left[|\tilde{s}(t)|^2\right]}$$

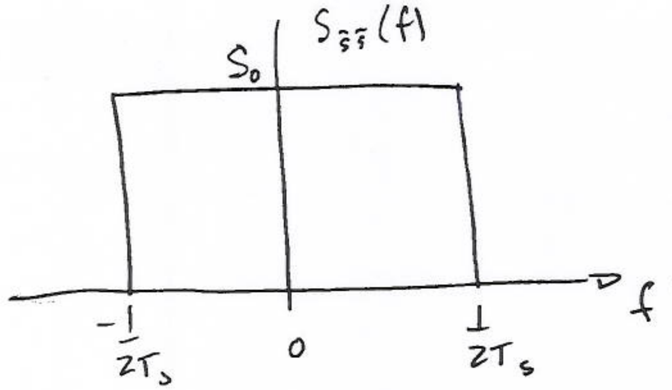
$$= \frac{\max\limits_{0 \le t \le T} |\tilde{s}(t)|^2}{N}$$

$$= \max_{0 \le t \le T} \left| \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{j 2\pi \Delta_f t} \right|^2$$

triangle inequality

$$\le \frac{1}{N} \left[\sum_{n=0}^{N-1} \left| x_n e^{j 2\pi \Delta f t} \right| \right]^2$$

$$\le N$$



a) pdf is Rayleigh

$$\frac{1}{2} E\left[|\tilde{s}(t)|^2\right] = \frac{S_0}{T_s} \qquad E\left[|\tilde{s}(t)|^2\right] = \frac{2S_0}{T_s}$$

$$= 2b_0$$

$$P_\alpha(x) = \frac{x}{b_0} e^{-x^2/2b_0} \qquad \Rightarrow b_0 = \frac{S_0}{T_s}$$

b) $P(\alpha > \theta R_{rms})$ $\qquad R_{rms} = \sqrt{2b_0}$

$$= e^{-\theta^2}$$

c) $$L_R = \int_0^{\infty} \dot{\alpha}\, p(R, \dot{\alpha})\, d\dot{\alpha}$$

$$= \sqrt{\frac{1}{2\pi b_2}}\, e^{-\dot{\alpha}^2/2b_2} \cdot \frac{\alpha}{b_0} e^{-\alpha^2/2b_0}$$

$$b_n = (2\pi)^n \int_{-f_m}^{f_m} S_{ss}^c(f) f^n\, df$$

$$b_0 = (2\pi)^0 S_0 \int_{-1/2T_s}^{1/2T_s} df = \frac{S_0}{T_s}$$

$$b_1 = 0$$

$$b_2 = (2\pi)^2 S_0 \int_{-1/2T_s}^{1/2T_s} f^2\, df$$

$$= (2\pi)^2 S_0 \frac{f^3}{3} \Big|_{1/2T_s}^{1/2T_s} = (2\pi)^2 \frac{2}{3} \left(\frac{1}{2T_s}\right)^3 S_0$$

$$L_R = \int_0^{\infty} \dot{\alpha}\, p(\theta R_{rms})\, p(\dot{\alpha})\, d\dot{\alpha}$$

$$= p(\theta R_{rms}) \int_0^{\infty} \frac{1}{\sqrt{2\pi b_2}}\, \dot{\alpha}\, e^{-\dot{\alpha}^2/2b_2}\, d\dot{\alpha}$$

$$= \sqrt{\frac{b_2}{2\pi}}\; p(\theta R_{rms})$$

$$= \sqrt{(2\pi)\frac{2}{3}\left(\frac{1}{2T_s}\right)^3 S_0}\;\; p(\theta R_{rms})$$

$$= \sqrt{(2\pi)\frac{2}{3}\left(\frac{1}{2T_s}\right)^3 S_0} \cdot \sqrt{\frac{2T_s}{S_0}}\; \theta e^{-\theta^2}$$

$$= \sqrt{(2\pi)\frac{2}{3}\left(\frac{1}{2T_s}\right)^2}\; \theta e^{-\theta^2}$$

4.

$$
\begin{aligned}
Z_l &= \sum_{k=0}^{N-1} X_k e^{-j\frac{2\pi lk}{N}} \\
&= A\sum_{k=0}^{N-1}\left(\sum_{n=0}^{N-1} x_n e^{j2\pi(\frac{nk}{N}+k\Delta f T_s)}\right)e^{-j\frac{2\pi lk}{N}} \\
&= A\sum_{n=0}^{N-1} x_n \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(n-l+\Delta f N T_s)} \\
&= Ax_l \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(\Delta f N T_s)k} + A\sum_{n=0,n\neq l}^{N-1} x_n \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(n-l+\Delta f N T_s)k} \\
&= \eta_l x_l + c_l
\end{aligned}
$$

$$
\begin{aligned}
\eta_l &= A\frac{1-e^{j2\pi\Delta f N T_s}}{1-e^{j2\pi\Delta f T_s}} \\
&\simeq A\frac{-e^{j\pi\Delta f N T_s}(e^{j\pi\Delta f N T_s}-e^{-j\pi\Delta f N T_s})}{1-[1+j\sin(2\pi\Delta f T_s)]}\ (when\ \Delta T_s << 1) \\
&= A\frac{-2j\sin(\pi\Delta f N T_s)}{-j\sin(2\pi\Delta f T_s)}e^{j\pi\Delta f N T_s} \\
&\simeq A\frac{\sin(\pi\Delta f N T_s)}{\pi\Delta f T_s}e^{j\pi\Delta f N T_s}
\end{aligned}
$$

Similarly,

$$c_l = A\sum_{n\neq l} x_n H(n,l)$$

$$H(n,l) \simeq A\frac{\sin[\pi(n-l+NT_s\Delta f)]}{\frac{\pi}{N}[n-l+\Delta f N T_s]}e^{\pi(n-l+\Delta f N T_s)}$$