ECE6604 Personal & Mobile Communications

Assignment #8

Date Assigned: April 14, 2009

Date Due: April 21, 2009

1) Consider an OFDM system having $N = 1024$ sub-carriers that are spaced 4 kHz apart. Suppose that the waveform channel is a linear time-invariant channel consisting of two non-faded equal rays spaced τ_d seconds apart, i.e.,

$$g(t,\tau) = g\delta(\tau) + g\delta(\tau_d)$$

where $g = \alpha e^{j\phi}$ is the gain associated with each of the two channel taps.

Find the value of the delay τ_d in the above channel model that will yield the worst possible bit error rate performance for this OFDM system.

2) Consider an OFDM symbol

$$\tilde{s}(t) = \sum_{k=0}^{N-1} x_n e^{-j2\pi\Delta_f t} \quad 0 \leq t \leq T$$

where $\Delta_f = 1/T$, and $T = nT_s$. The data symbols x_n, $n = 0, \ldots, N-1$, are independent and each is chosen with equal probability from a BPSK symbol alphabet, such that $x_n \in \{-1, +1\}$. The PAPR of the OFDM symbol can be defined as follows:

$$\text{PAPR} = \frac{\max_{0\leq t\leq T} |\tilde{s}(t)|^2}{\text{E}[|\tilde{s}(t)|^2]}$$

By using the triangle inequality, show that PAPR $\leq N$.

3) An OFDM signal with a large number of sub-carriers N has a complex envelope that can be approximated as a zero-mean complex Gaussian random process. Assume an "ideal" OFDM signal spectrum, where the power spectrum is

$$S_{\tilde{s}\tilde{s}}(f) = \begin{cases} S_0 & |f| \leq 1/2T_s \\ 0 & \text{elsewhere} \end{cases}$$

where $T = NT_s$.

a) Using the above Gaussian approximation, what is the distribution of the magnitude of the complex envelope $\tilde{s}(t)$ at any time t.

b) Suppose that a power amplifier will clip the OFDM waveform if the complex envelope $\tilde{s}(t)$ will exceeds the level ΘR_{rms}, where R_{rms} is the rms envelope level $\sqrt{E[|\tilde{s}(t)|^2]}$. What is the probability that the OFDM waveform will be clipped at any time t?

b) How many times per second will the envelope be clipped?

4) The OFDM complex envelope with a carrier frequency offset Δf is

$$\tilde{s}(t) = A \sum_{n=0}^{N-1} x_n \exp\left\{ j2\pi \left(\frac{n}{NT_s} + \Delta f \right) t \right\} u_T(t)$$

The corresponding IFFT coefficients are

$$X_k = A \sum_{n=0}^{N-1} x_n \exp\left\{ j2\pi \left(\frac{nk}{N} + k\Delta f T_s \right) \right\} \ , \quad k = 0,\ 1,\ \ldots,\ N + G - 1$$

Show that the demodulated sequence (in absence of noise) can be written as

$$Z_l = \text{FFT}\{X_n\} = \eta_l x_l + c_l$$

where

$$\eta_l = A \left\{ \frac{\sin\left(\pi N T_s \Delta f\right)}{\pi N T_s \Delta f} \right\} e^{j\pi N T_s \Delta f}$$

and

$$c_l = A \sum_{n \neq l} a_n H(n, l)$$

is the random ICI term, where

$$H(n, l) = \left\{ \frac{\sin\left[\pi\left(n - l - NT_s\Delta f\right)\right]}{\pi\left[n - l - NT_s\Delta f\right]} \right\} e^{j\pi(n - l - NT_s\Delta f)}$$