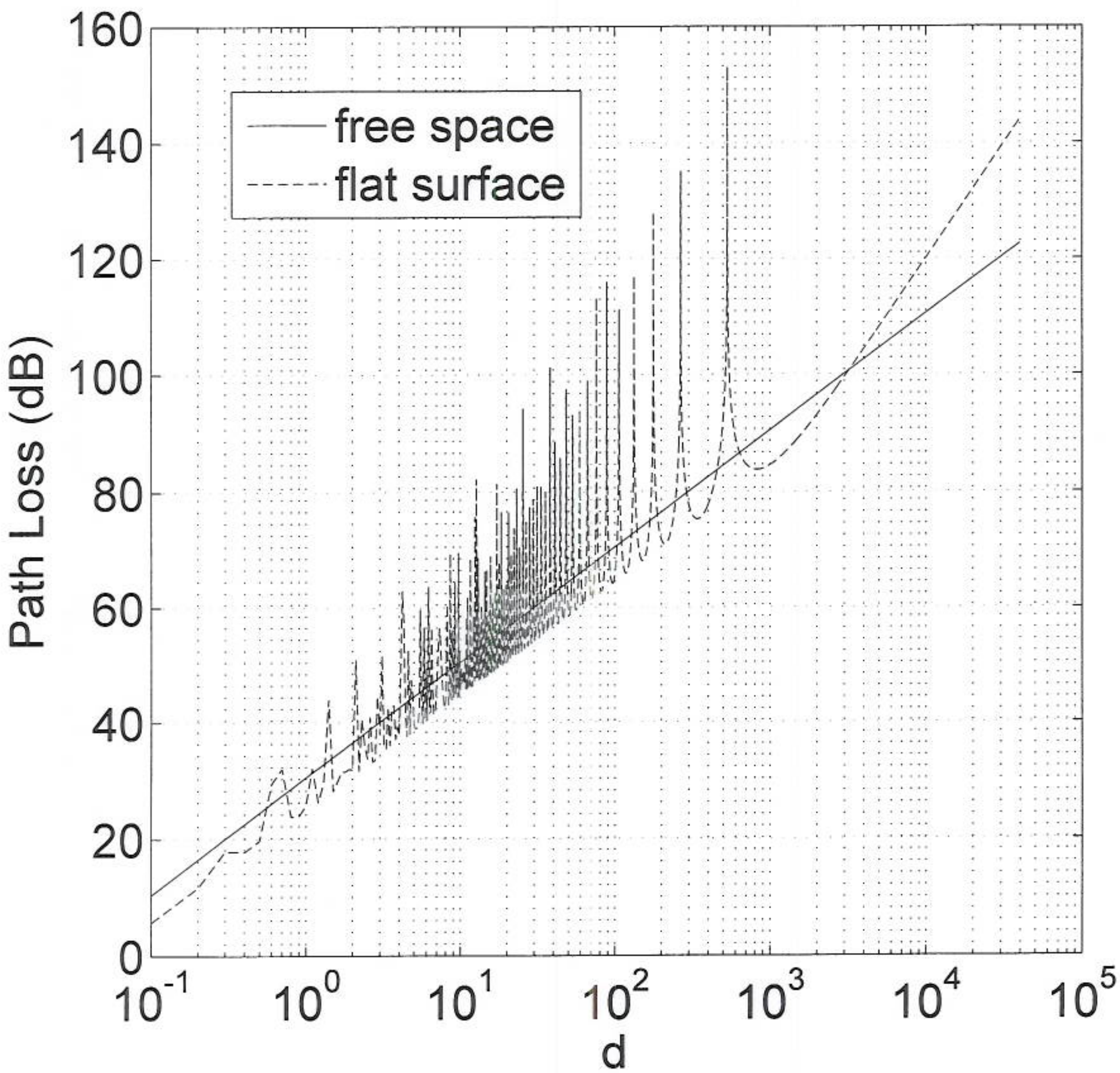
ECE 6604 Homework 1 Solutions



1 a) free-space: $L_p = 20\log_{10} f_c + 20\log_{10} d - 147.55$

flat surface: $L_p = 20\log_{10} f_c + 20\log_{10} d - 153.57$

$$- 20\log_{10}\left(\sin\left(\frac{2\pi h_b h_m f_c}{c d}\right)\right)$$




$f_c = 800$ MHz

$h_b = 10$

$h_m = 10$

1b)
$$L_p = 25\text{ dB} + 10\log_{10} d^{2.8}$$
$$= 25 + 28\log_{10} d \quad (\text{dB})$$

Also $\Omega_{t\,(\text{dBm})} = S_{Rx\,(\text{dBm})} + L_{p\,(\text{dB})}$

For $d = 10{,}000$ m, $S_{Rx\,(\text{dBm})} = -95$

$L_p = 137$ dB

$\therefore \quad \Omega_t = -95 + 137 = 42$ dBm

If $\beta = 3.1$, then

$$L_p = 25 + 31\log_{10} d \quad (\text{dB})$$

For $d = 10{,}000$m

$L_p = 149$

Hence,

$\Delta\Omega_{t\,(\text{dB})} = 149 - 137 = 12$ dB.

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2 a) There are only two BSs (F1 & F2) interfering with the reference sector in the first tier.

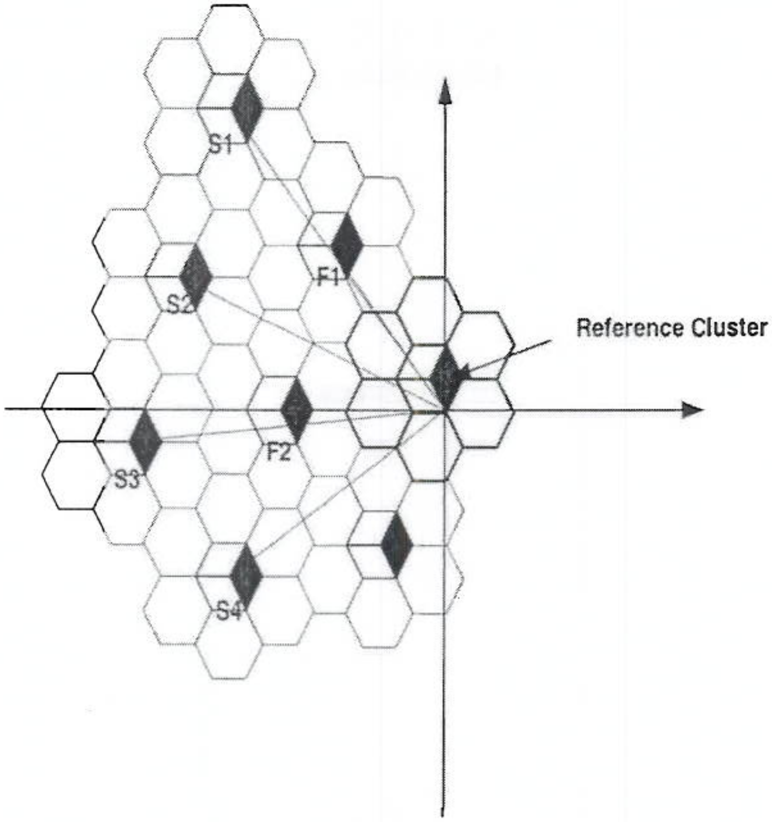


Figure 1: 7-cell hexagonal reuse cluster

Assuming $R = 1$, the coordinates of F1 and F2 are $(-3.5,\ 5\frac{\sqrt{3}}{2})$ and $(-5,\ 0)$, respectively. The co-channel interference ratio is then:

$$\begin{aligned}\Lambda &= \frac{R^{-1}}{d_{\mathrm{F1}}^{-4} + d_{\mathrm{F2}}^{-4}} \\ &= 378.9439 = 25.8 \text{ dB}\end{aligned}$$

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b) There are four additional BSs interfering with the reference sector in the second tier. Their coordinates with respect to the corner of the reference sector are as follows:

$$\begin{aligned}
S1 &= (-6.5,\ 9\frac{\sqrt{3}}{2}) \\
S2 &= (-8,\ 4\frac{\sqrt{3}}{2}) \\
S3 &= (-9.5,\ \frac{\sqrt{3}}{2}) \\
S4 &= (-6.5,\ 5\frac{\sqrt{3}}{2})
\end{aligned}$$

The co-channel interference ratio for the first two tiers is:

$$\begin{aligned}
\lambda &= \frac{R^{-1}}{d_{\mathrm{F1}}^{-4} + d_{\mathrm{F2}}^{-4} + d_{\mathrm{S1}}^{-4} + d_{\mathrm{S2}}^{-4} + d_{\mathrm{S3}}^{-4} + d_{\mathrm{S4}}^{-4}} \\
&= 303.4355 = 24.8 \text{ dB}
\end{aligned}$$

b) When $\beta = 4$, there is only 1 dB difference between the two results, which suggests that the interference contributions from the second tier co-channel BSs are minimal. In this case, it is a valid approximation to consider only the first tier co-channel BSs when calculating co-channel interference ratio. With $\beta = 3$, $\Lambda = 18.6$ dB for first tier and $\Lambda = 17.1$ dB for first two tiers. The interference contributions from the second tier BSs increase as $\beta$ gets smaller.

3. The Erlang-B blocking formula is

$$B(\rho, m) = \frac{\rho^m}{m! \sum_{k=0}^{m} \frac{\rho^k}{k!}}$$

In general, direct evaluation of $B(\rho,m)$ is not possible for large $m$ and small $\rho$, due to overflows in calculating $m!$, $k!$ and $\rho^k$, respectively. First write $B(\rho,m)$ as:

$$B(\rho, m) = \frac{1}{\sum_{k=0}^{m} \rho^{k-m} \frac{m!}{k!}}$$

Now reverse the order of summation

$$B(\rho, m) = \frac{1}{\sum_{k=m}^{0} \rho^{k-m} \frac{m!}{k!}}$$

You should be able to show that this is equivalent to

$$B(\rho, m) = \frac{1}{\sum_{\ell=0}^{m} t_\ell}$$

where $t_0 = 1$, $t_{i+1} = (m-i)\rho^{-1} t_i$

The only remaining problem is that the terms $t_i$ continue to decrease and eventually we will get an underflow. So at some point you need to truncate the summation at $m' \leq m$. You can do this by examining how much the term $t_{m'}$ contributes to $B(\rho, m)$. If it is a small fraction, then the remaining terms $t_i$, $m'+1 \leq i \leq m$ can be neglected.

Now for the particular problem at hand the solutions follow.

**a)** The number of channels per cell in the three systems are $15, 100$ and $60$. The Erlang-B blocking probability is shown here.

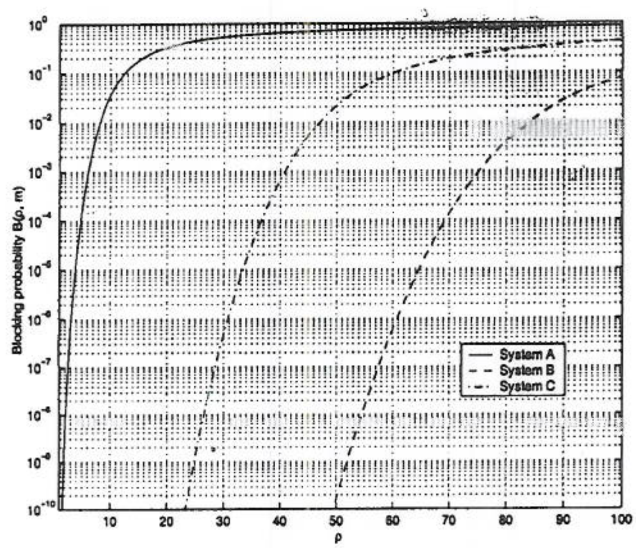


Figure 1: Erlang-B blocking probability vs. $\rho$

**b)** The traffic loading offered $\rho$ per cell are $\rho = 9.01, 87.97, 49.64$, for **System A, B, C,** respectively. Assume the cell number is $C$ and the total number of users is $n$. Since the traffic loading offered by each user is 0.1 Erlangs, the traffic loading offered per cell is $\rho = 0.1n/C$.

The number of users in System A, B and C are $36040, 43985, 49640$, respectively.

4 $O(r)=Q\left(\frac{\mu_{\Omega_p(dBm)}(r)-\Omega_{th(dBm)}}{\sigma_\Omega}\right)$ with $\mu_{\Omega_p}(r) = A\,r^{-\beta}$

$$\mu_{\Omega_p(dBm)}(r) = 10\log_{10} A - 10\beta\log_{10} r = \mu_{\Omega_p(dBm)}(R) + 10\beta\log_{10}\left(\frac{R}{r}\right)$$

$$\Longrightarrow O(r) = Q\left(\frac{\mu_{\Omega_p(dBm)}(R)-\Omega_{th(dBm)}+10\beta\log_{10}\left(\frac{R}{r}\right)}{\sigma_\Omega}\right) = Q\left(X+\frac{2}{Y}\ln\left(\frac{R}{r}\right)\right)$$

where $X=\frac{\mu_{\Omega_p(dBm)}(R)-\Omega_{th(dBm)}}{\sigma_\Omega}$ and $Y=\frac{2\sigma_\Omega}{\beta\xi}$ , $\xi=\frac{10}{\ln 10}$

$$O = \frac{1}{\pi R^2}\int_0^R O(r)\,2\pi r\,dr = \frac{2}{R^2}\int_0^R Q\left(X+\frac{2}{Y}\ln\left(\frac{R}{r}\right)\right) r\,dr$$

$$= \frac{2}{R^2}\int_\infty^X Q(t)\left[R\,e^{-\frac{(t-X)Y}{2}}\right]\cdot\frac{d}{dt}\left[R\,e^{-\frac{(t-X)Y}{2}}\right]\cdot dt$$

$$= Y e^{XY}\int_X^\infty Q(t)\,e^{-tY}dt \qquad \left\langle \begin{array}{ll} u=Q(t) & dv=e^{-tY}dt \\ du=\frac{-1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}} & v=-\frac{1}{Y}e^{-tY} \end{array}\right\rangle$$

$$= Y e^{XY}\left\{\frac{-Q(t)}{Y}e^{-tY}\Big|_X^\infty - \int_X^\infty \frac{1}{Y\sqrt{2\pi}}e^{-\left(\frac{t^2}{2}+tY\right)}dt\right\}$$

$$= Q(X) - e^{XY}e^{\frac{Y^2}{2}}\int_X^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(t+Y)^2}dt$$

$$= Q(X) - e^{\left(XY+\frac{Y^2}{2}\right)}Q(X+Y)$$

5) For this problem, it helps to read the derivation of the Rice pdf on pages 690-1 of the text.

We have

$$m_I(t) = s \cdot \cos(2\pi f_m t \cos\theta_0 + \phi_0)$$

$$m_Q(t) = s \cdot \sin(2\pi f_m t \cos\theta_0 + \phi_0)$$

Hence $s = \sqrt{m_I^2(t) + m_Q^2(t)}$

Now define

$$\phi(t) = \mathrm{Tan}^{-1}\frac{m_Q(t)}{m_I(t)}$$

$$= \mathrm{Tan}^{-1}\frac{\sin(2\pi f_m t \cos\theta_0 + \phi_0)}{\cos(2\pi f_m t \cos\theta_0 + \phi_0)}$$

$$= 2\pi f_m t \cos\theta_0 + \phi_0$$

Because $\phi(t)$ is time varying, the envelope phase

$$\theta(t) = \mathrm{Tan}^{-1}\frac{g_Q(t)}{g_I(t)}$$

is a random process. The joint pdf of the envelope and envelope phase is

$$p_{R,\theta(t)}(r,\theta) = \frac{1}{2\pi b_0}\exp\left\{-\frac{r^2 + s^2 - 2rs\cos(\theta - \phi(t))}{2b_0}\right\}$$

The pdf of the envelope phase is the marginal pdf

$$P_{\Theta(t)}(\theta) = \int_0^\infty P_{R,\theta(t)}(r, \theta)\, dr$$

$$= \int_0^\infty \frac{r}{2\pi b_o} \exp\left\{-\frac{r^2+s^2-2rs\cos(\theta-\phi(t))}{2b_o}\right\} dr$$

To proceed further, we complete the square in the exponent

$$r^2+s^2-2rs\cos(\theta-\phi(t)) = (r - s\cos(\theta-\phi(t)))^2 + s^2\sin^2(\theta-\phi(t))$$

Hence,

$$P_{\theta(t)}(\theta) = \frac{1}{\pi} e^{-\frac{s^2}{2b_o}\sin^2(\theta-\phi(t))} \int_0^\infty \frac{r}{2b_o} \exp\left\{-\frac{(r-s\cos(\theta-\phi(t)))^2}{2b_o}\right\} dr$$

Now change variable of integration.

Let $x = \frac{r}{\sqrt{2b_o}}$ $\quad r = \sqrt{2b_o}\, x \quad dr = \sqrt{2b_o}\, dx$

$$P_{\theta(t)}(\theta) = \frac{1}{\pi} e^{-\frac{s^2}{2b_o}\sin^2(\theta-\phi(t))} \int_0^\infty x e^{-\left(x + \sqrt{\frac{s^2}{2b_o}}\cos(\theta-\phi(t))\right)^2} dx$$

$$\boxed{= \frac{1}{\pi} e^{-K\sin^2(\theta-\phi(t))} \int_0^\infty x e^{-(x+\sqrt{K}\cos(\theta-\phi(t)))^2} dx}$$

where $\phi(t) = 2\pi f_m t \cos\theta_o + \phi_o$