ECE6604 Personal & Mobile Communications

Assignment #2

Date Assigned: January 28, 2010

Date Due: February 9, 2010

1) Suppose that $r(t)$ is a wide-sense stationary (WSS) band-pass random process, such that

$$r(t) = g_I(t)\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t$$

a) Show that the auto- and cross-correlations of $g_I(t)$ and $g_Q(t)$ must satisfy the following conditions:

$$\begin{aligned} \phi_{g_I g_I}(\tau) &= \phi_{g_Q g_Q}(\tau) \\ \phi_{g_I g_Q}(\tau) &= -\phi_{g_Q g_I}(\tau) \end{aligned}$$

b) Under the conditions in part a) show that the autocorrelation of $r(t)$ is

$$\begin{aligned} \mathrm{E}[r(t)r(t+\tau)] &= \phi_{g_I g_I}(\tau)\cos 2\pi f_c \tau \\ &\quad -\phi_{g_I g_Q}(\tau)\sin 2\pi f_c \tau \ . \end{aligned}$$

2) A wireless channel is characterized by the time-variant impulse response

$$\begin{aligned} g(t,\tau) &= u_T(\tau)\cos\left(\Omega t + \phi_0\right), \quad 0 \le \tau \le T \\ u_t(\tau) &= \begin{cases} 1, & 0 \le \tau \le T \\ 0, & \text{elsewhere} \end{cases} \end{aligned}$$

where $T = 10$ μs, $\Omega = 10\pi$, and $\phi_0 \in [-\pi, +\pi]$ is a constant.

a) Determine the channel time-variant transfer function.

b) Given the complex channel input signal

$$\tilde{s}(t) = \begin{cases} 1, & 0 \le t \le T_s \\ 0, & \text{otherwise} \end{cases} ,$$

where $0 \le T_s \le T$, determine the complex channel output signal, $\tilde{r}(t)$.

3) What is the maximum Doppler shift for the GSM mobile cellular system on the "downlink" from the base station to the mobile unit (935 to 960 MHz RF band)? What is it on the "uplink" direction, or mobile to base (890 to 915 MHz RF band)? Assume a high-speed train travelling at a speed of $v = 250$ km/h.

4) Suppose that the pdf of arriving plane waves is described by equation (2.41) in the text. Sometimes this is called the Parsons pdf.

$$p(\theta) = \begin{cases} \frac{\pi}{4|\theta_m|} \cos\left(\frac{\pi}{2} \cdot \frac{\theta}{\theta_m}\right) & , \quad |\theta| \leq |\theta_m| \leq \frac{\pi}{2} \\ 0 & , \quad \text{elsewhere} \end{cases} .$$

Find the Doppler power spectrum $S_{gg}(f)$.

5) A flat Rayleigh fading signal at 5.7 GHz is received by a vehicle travelling at 80 km/hr.

1. Determine the number of positive-going zero crossings about the rms value that occur over a 5 s interval.
2. Determine the average duration of a fade below the rms level.
3. Determine the average duration of a fade at a level of 20 dB below the rms value.