ECE 6604 Homework #4 Solutions

15. Average delay: $\mu_\tau = \dfrac{\sum_k \tau_k \psi_g(\tau_k)}{\sum_k \psi_g(\tau_k)} = \sum_k \tau_k \psi_g(\tau_k) \Longrightarrow \begin{cases} \mu_{\tau,\text{typical}} = 0.9024\ \mu s \\ \mu_{\tau,\text{bad}} = 2.6174\ \mu s \end{cases}$

rms delay spread: $\sigma_\tau = \sqrt{\dfrac{\sum_k (\tau_k - \mu_k)^2 \psi_g(\tau_k)}{\sum_k \psi_g(\tau_k)}} = \sqrt{\sum_k (\tau_k-\mu_k)^2 \psi_g(\tau_k)} \Longrightarrow \begin{cases} \sigma_{\tau,\text{typical}} = 1.0396\ \mu s \\ \sigma_{\tau,\text{bad}} = 2.5110\ \mu s \end{cases}$

power delay profile: $W_x = \tau_3 - \tau_1$ with $\sum_{\tau=0}^{\tau_1} \psi_g(\tau) = \sum_{\tau=\tau_3}^{\infty} \psi_g(\tau)$ & $\sum_{\tau=\tau_1}^{\tau_3} \psi_g(\tau) = x \sum_{\tau=0}^{\infty} \psi_g(\tau)$

For typical channel, $\sum_{\tau=0}^{0.3} \psi_g(\tau) = 0.207 \approx \sum_{\tau=1.1}^{\infty} \psi_g(\tau) = 0.246$

$\sum_{\tau=0.3}^{1.1} \psi_g(\tau) = 0.547 \approx 0.5 \sum_{\tau=0}^{\infty} \psi_g = 0.5$

$\Longrightarrow W_{50} \approx 1.1 - 0.3 = 0.8\ \mu s$

For bad channel, $\sum_{\tau=0}^{0.7} \psi_g(\tau) = 0.263 \approx \sum_{\tau=5}^{\infty} \psi_g(\tau) = 0.202$

$\sum_{\tau=0.7}^{5.0} \psi_g(\tau) = 0.535 \approx 0.5 \Longrightarrow W_{50} \approx 5 - 0.7 = 4.3\ \mu s$

Bad channels tend to have larger μ_τ, σ_τ, and W_x

18 (a) $\langle g_I(t) g_Q(t) \rangle = 2\sum_{n=1}^{M} \sin\beta_n \cos\beta_n + \sin\alpha\cos\alpha$

$\langle g_I^2(t) \rangle = M + \cos^2\alpha + \sum_{n=1}^{M} \cos 2\beta_n \quad ; \quad \langle g_Q^2(t) \rangle = M + \sin^2\alpha - \sum_{n=1}^{M}\cos 2\beta_n$

For $\alpha = 0$, $\beta_n = \frac{\pi n}{M+1}$, we have

$$\langle g_I(t) g_Q(t) \rangle = 2\sum_{n=1}^{M} \sin\beta_n\cos\beta_n = \sum_{n=1}^{M} \sin 2\beta_n = \sum_{n=1}^{M} \sin\frac{2\pi n}{M+1}$$

$$= \frac{1}{2j}\left[\sum_{n=1}^{M} e^{j\frac{2\pi n}{M+1}} - \sum_{n=1}^{M} e^{-j\frac{2\pi n}{M+1}}\right] = \frac{1}{2j}\left[\frac{e^{j\frac{2\pi}{M+1}} - 1}{1 - e^{j\frac{2\pi}{M+1}}} - \frac{e^{-j\frac{2\pi}{M+1}} - 1}{1 - e^{-j\frac{2\pi}{M+1}}}\right] = \frac{-1+1}{2j} = 0$$

and $\sum_{n=1}^{M} \cos 2\beta_n = \frac{1}{2}\left[\sum_{n=1}^{M} e^{j\frac{2\pi n}{M+1}} + \sum_{n=1}^{M} e^{-j\frac{2\pi n}{M+1}}\right] = \frac{1}{2}(-1-1) = -1$

$\Longrightarrow \langle g_I^2(t) \rangle = M \quad , \quad \langle g_Q^2(t) \rangle = M+1$

(b) In this case

$$\langle g_I(t) g_Q(t) \rangle = \sum_{n=1}^{M}{}' \sin\frac{2\pi n}{M}$$

$$= \frac{1}{2j}\left[\sum_{n=1}^{M}{}' e^{j\frac{2\pi n}{M}} - \sum_{n=1}^{M}{}' e^{-j\frac{2\pi n}{M}}\right] = \frac{1}{2j}\left[\frac{e^{j\frac{2\pi}{M}} - e^{j\frac{2\pi}{M}}}{1 - e^{j\frac{2\pi}{M}}} - \frac{e^{-j\frac{2\pi}{M}} - e^{-j\frac{2\pi}{M}}}{1 - e^{-j\frac{2\pi}{M}}}\right] = \frac{0+0}{2j} = 0$$

$$\sum_{n=1}^{M}{}' \cos 2\beta_n = \frac{1}{2}\left[\sum_{n=1}^{M}{}' e^{j\frac{2\pi n}{M}} + \sum_{n=1}^{M}{}' e^{-j\frac{2\pi n}{M}}\right] = \frac{1}{2}\left[0+0\right] = 0$$

$\rightarrow \langle g_I^2(t) \rangle = M+1 \qquad \langle g_Q^2(t) \rangle = M$

21

19,20 In order to generate a faded envelope with Rician distribution, we need to modify the Jake's fading simulator for Rayleigh fading in Fig. 2. 26. The new fading simulator with Rice factor K is as follows ($\alpha = 0$):

$$
\begin{aligned}
g_I(t) &= \frac{1}{\sqrt{M+1}}\Big[2\sum_{n=1}^{M}\cos\beta_n\cos(\omega_n t+\beta_n+\gamma_{nj})+\sqrt{(2)}\cos 2\pi f_m t\Big]+\sqrt{2K} \\
g_Q(t) &= \frac{1}{\sqrt{M}}\Big[2\sum_{n=1}^{M}\sin\beta_n\cos(\omega_n t+\beta_n+\gamma_{nj})\Big]+\sqrt{2K},
\end{aligned}
$$

where f_m is the Doppler frequency and

$$
\begin{aligned}
\omega_n &\triangleq 2\pi f_n = 2\pi f_m\cos(2\pi n/N) \\
\beta_n &= \frac{\pi n}{M}, \qquad n = 1,2,\cdots,M \\
\gamma_{nj} &= \frac{2\pi(j-1)n}{M}, \qquad n,j = 1,2,\cdots,M \\
N &= 2(2M+1).
\end{aligned}
$$

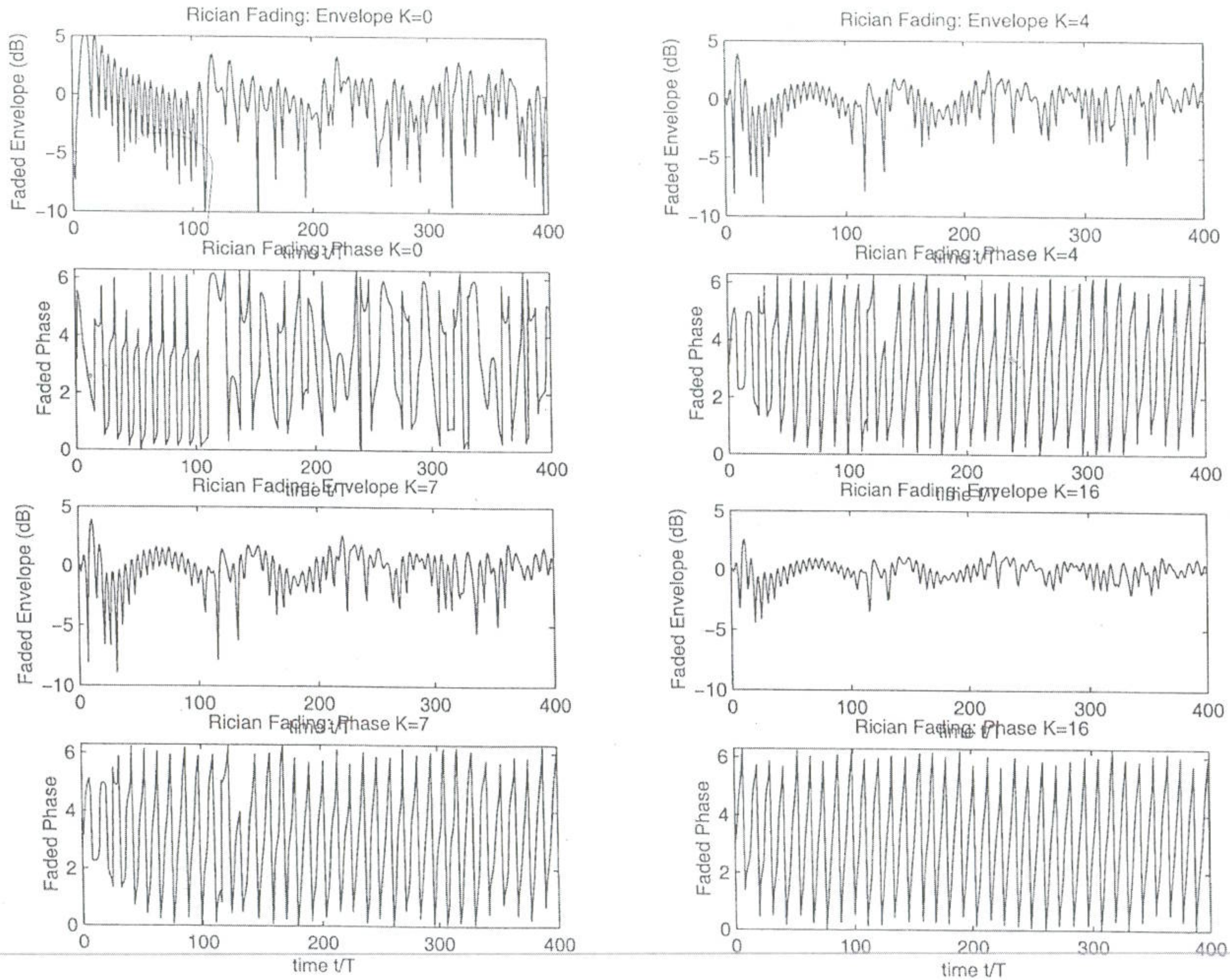
Assume that Aulin's model is used to generate the means $m_i(t) = s\cdot\cos(2\pi f_m\cos\theta_0)$ and $m_Q(t) = s\cdot\sin(2\pi f_m\cos\theta_0)$ of $g_I(t)$ and $g_Q(t)$, respectively. Then, the fading simulator for $\Omega_p = 1$ will be

$$
\begin{aligned}
g_I(t) &= \frac{1}{\sqrt{2(K+1)(M+1)}}\Big[2\sum_{n=1}^{M}\cos\beta_n\cos(\omega_n t+\beta_n+\gamma_{nj})+\sqrt{(2)}\cos 2\pi f_m t\Big] \\
&\quad +\frac{\sqrt{K}}{\sqrt{K+1}}\cos(2\pi f_m\cos(\theta_0)t) \\
g_Q(t) &= \frac{1}{\sqrt{2(K+1)M}}\Big[2\sum_{n=1}^{M}\sin\beta_n\cos(\omega_n t+\beta_n+\gamma_{nj})\Big]+\frac{\sqrt{K}}{\sqrt{K+1}}\sin(2\pi f_m\cos(\theta_0)t)(2)
\end{aligned}
$$

The Matlab program to implement the above equation is listed here for $K = 0$ case. The envelope and phase simulation for $K = 0, 4, 7, 16$ is shown in

```
clear all;
K=0;
fmt = 0.1; % f_m * T
tt = 0:1:399; % t/T
t = tt;


M = 16;
N = (2*M+1)*2;
alpha = 0;
theta_0=pi/6;
env = 2;
wm = 2*pi*fmt; % Doppler frequency
```

```
gamma = 2 * pi * (1:M) * (env-1)/M;
beta = pi * (1:M) / M;

i=1;
for t0 = tt,
    cos_wn(i,:) = cos(wm * t0* cos(2*pi*(1:M) / N) + gamma + beta);
    xre(i) = sum( 2*cos(beta) .* cos_wn(i,:));
    xim(i) = sum( 2*sin(beta) .* cos_wn(i,:));
    i=i+1;
end;

Xre = xre + sqrt(2) * cos(alpha) * cos(wm*t);
Xim = xim + sqrt(2) * sin(alpha) * cos(wm*t);
Xre = Xre / sqrt(2*(K+1)*(M+1));
Xim = Xim / sqrt(2*(K+1)*M);
Xre = Xre + sqrt(K/(K+1)) * cos(wm*t * cos(theta_0));
Xim = Xim + sqrt(K/(K+1)) * sin(wm*t * cos(theta_0));
z = sqrt(Xre .* Xre + Xim .* Xim);
phase = atan2(Xim, Xre) + pi;
zdb = 10*log10(z);
subplot(2, 1, 1), plot(zdb), title('Rician Fading: Envelope K=0'),
xlabel('time t/T'), ylabel('Faded Envelope (dB)'),
axis([0 400 -10 5]);

subplot(2, 1, 2), plot(phase),
title('Rician Fading: Phase K=0'),
xlabel('time t/T'), ylabel('Faded Phase'),
axis([0 400 0 2*pi]);
```


Rician Fading: Envelope K=0
Faded Envelope (dB)
time t/T
Rician Fading: Phase K=0
Faded Phase
time t/T
Rician Fading: Envelope K=7
Faded Envelope (dB)
time t/T
Rician Fading: Phase K=7
Faded Phase
time t/T
Rician Fading: Envelope K=4
Faded Envelope (dB)
time t/T
Rician Fading: Phase K=4
Faded Phase
time t/T
Rician Fading: Envelope K=16
Faded Envelope (dB)
time t/T
Rician Fading: Phase K=16
Faded Phase
time t/T

4/

5/

21. a) In order to have 20 wavelength in block size of 200, $f_m T$=0.1. The Figure shows the sample variance of the Ω_p estimate as a function of block size N.

b) As we can see from the Figure the sample variance decreases as the averaging window length increases for any Rice factor K. Also as the Rice factor K increases the sample variance of the Ω_p decreases. This is an important observation for resource management algorithms, such as handoff algorithms. Because these algorithms use local mean of the received signal to estimate the link quality to better utilize the system resources and the performance of the algorithms increase as the variance of Ω_p decrease.