ECE 6604 Fall 2019

Assignment #1 Solutions

1.4 $\Omega_p(d) = \frac{k}{d^{\beta}}$; $\beta = 3.5$

Since $\Omega_p(d=1\text{m}) = \frac{k}{1^{\beta}} = 1\text{mW} \Rightarrow k = 1\text{ mW}$

a) Assume the MS can be located anywhere in a cell, particularly in the corner of a cell. With $N=7$, the nearest co-channel BS is at distance

$$d = \sqrt{13}R$$

$$\Omega_p(d) = \frac{1\text{mW}}{d^{3.5}} = 10^{-10}\text{ mW} \Leftrightarrow d = 10^{10/3.5} = 2154\text{m}$$

$$R = d/\sqrt{13} = 597.5\text{ m}$$

b) For $N=4$, $d = \sqrt{\frac{27}{4}}R \Rightarrow R = \sqrt{\frac{4}{27}}d = 829\text{ m}$

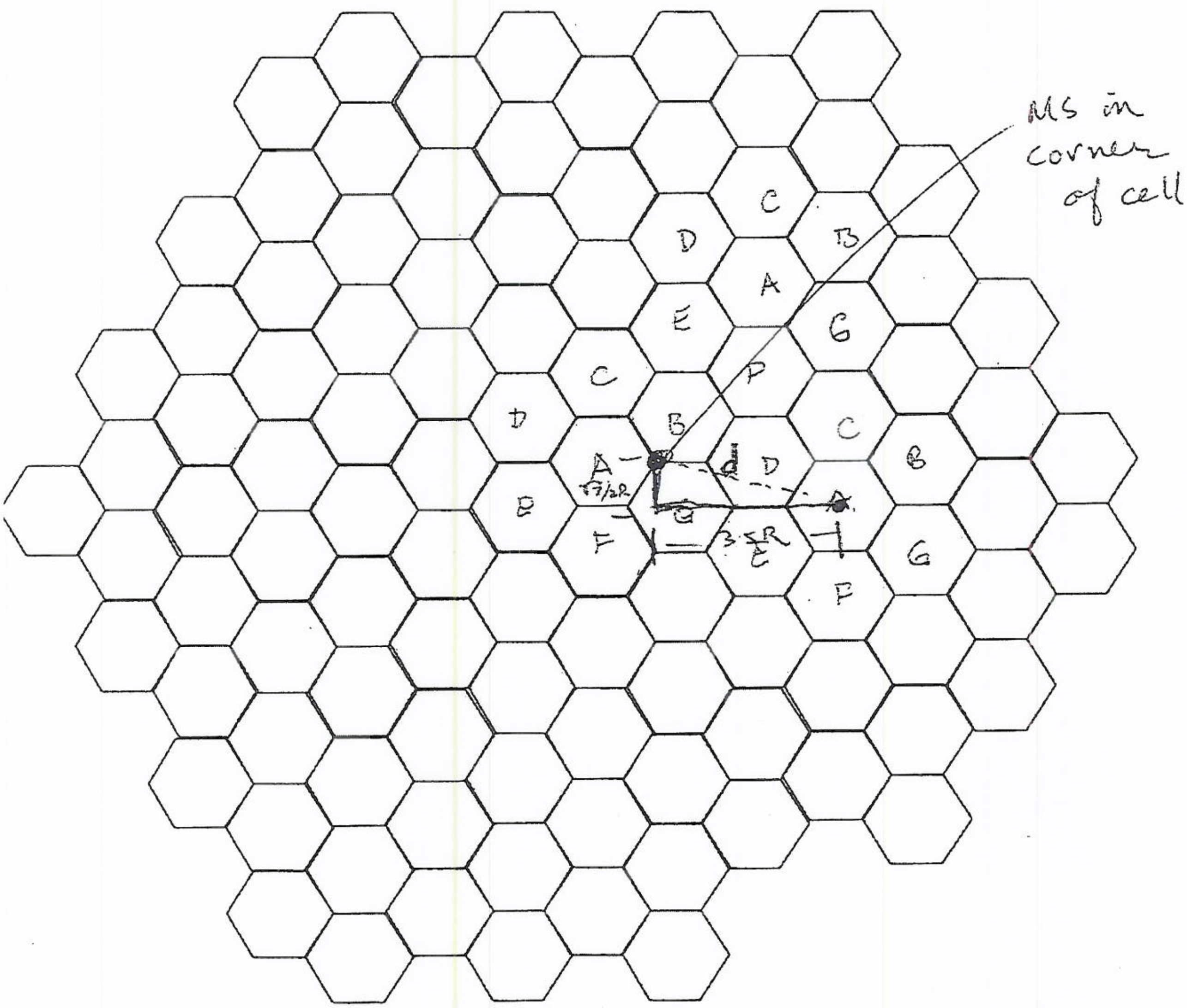
Note that the MS is located on the edge of a cell in this case.

Geometry for
Problem 1.4a)

$$d = \sqrt{(3.5R)^2 + (\sqrt{3}/2\,R)^2}$$

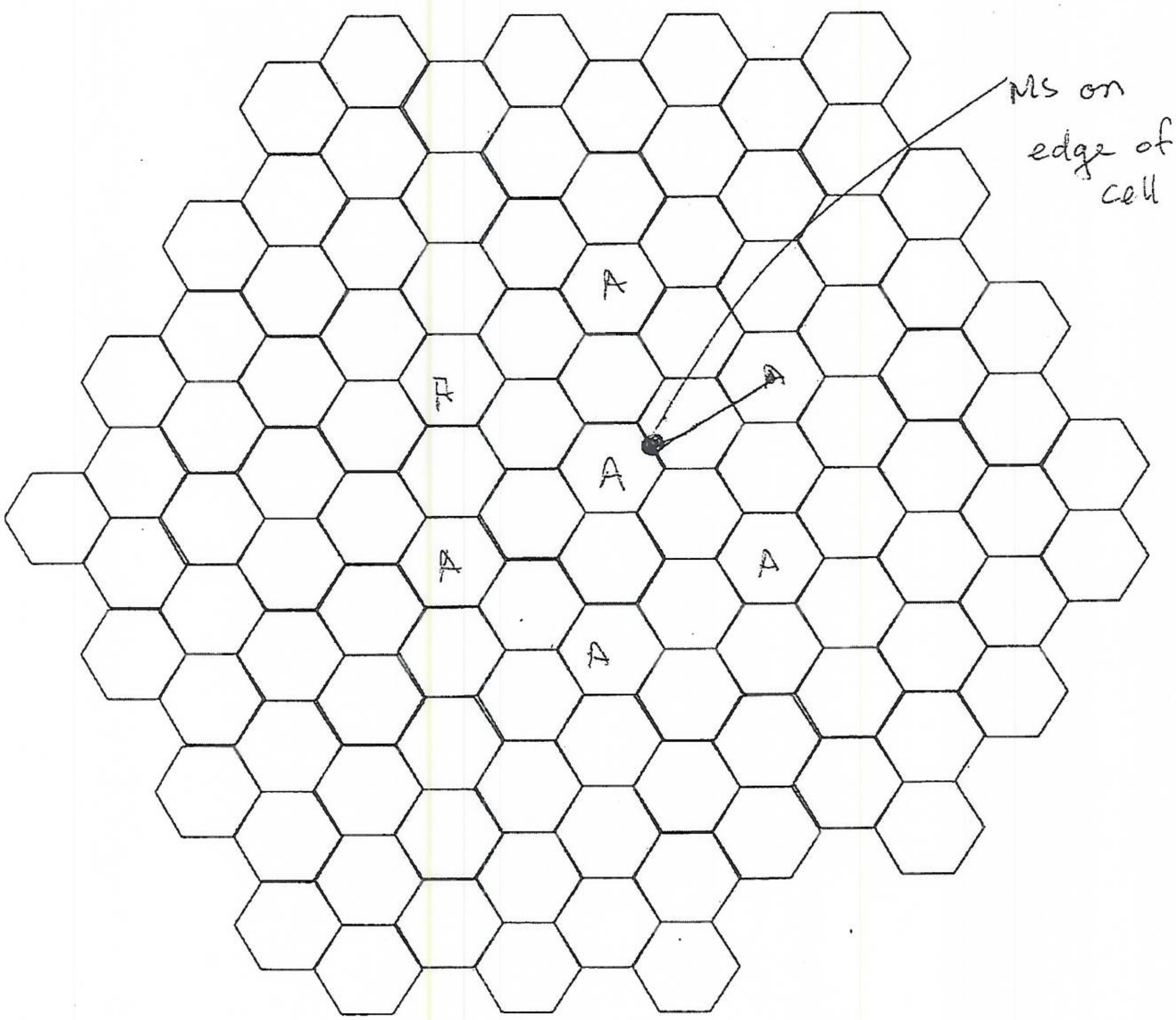
$$= \sqrt{\frac{49}{4} + \frac{3}{4}}\;R$$

$$= \sqrt{13}\,R$$



Geometry for
Problem 1.4 b)

$$d = 3\frac{\sqrt{3}}{2}R$$
$$= \sqrt{27/4}\,R$$



1.5/ See the attached plot showing the locations of the downlink co-channel base stations w.r.t. the worst case mobile station location.

a) There are two base stations at distances $\sqrt{19}R$ and $\sqrt{28}R$

$$\frac{C}{I} = \frac{1}{(\sqrt{19})^{-4} + (\sqrt{28})^{-4}} = 23.93 \text{ dB}$$

b) There are 6 base stations at distances $\{\sqrt{19}R, \sqrt{28}R, \sqrt{67}R, \sqrt{79}R, \sqrt{79}R, \sqrt{97}R\}$

$$\frac{C}{I} = \frac{1}{(\sqrt{19})^{-4} + (\sqrt{28})^{-4} + (\sqrt{67})^{-4} + (\sqrt{79})^{-4} + (\sqrt{79})^{-4} + (\sqrt{97})^{-4}}$$

$$= 23.28 \text{ dB}$$

c) With $\beta = 4$ second tier degrades C/I by 0.65 dB

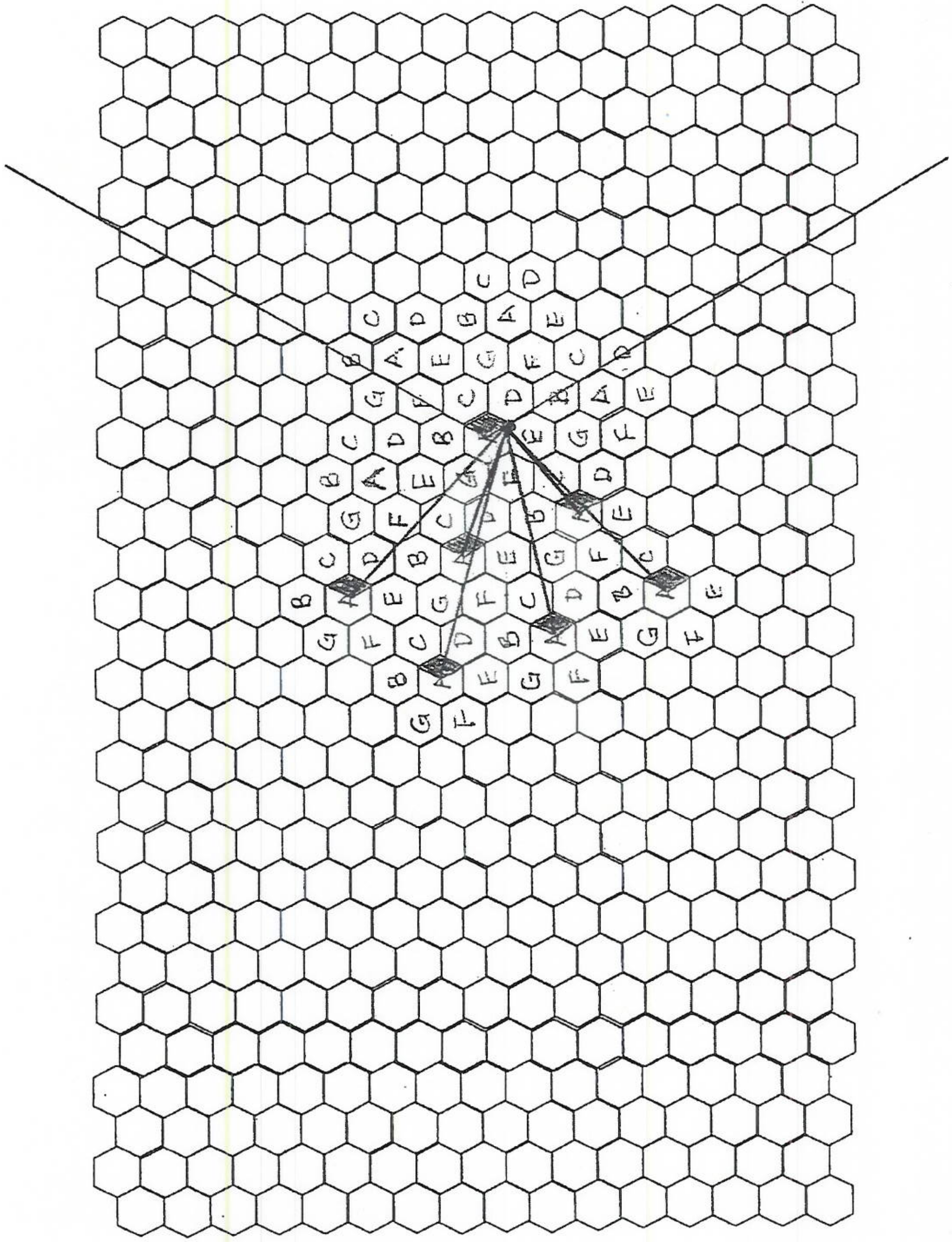
For $\beta = 3$ and first tier $C/I = 17.25$ dB

For $\beta = 3$ and first and second tier

$C/I = 16.10$ dB

For $\beta = 3$, the C/I is worse than $\beta = 4$.

For $\beta = 3$, the second tier degrades C/I by 1.15 dB



1.7 a) $F_{\Omega_p^o}(x) = P(\Omega_{p,1} \le x,\ \Omega_{p,2} \le x)$

$$= \Phi\left(\frac{x+83.57}{8}\right)\Phi\left(\frac{x+85.09}{8}\right)$$

$$f_{\Omega_p^o}(x) = \Phi\left(\frac{x+83.57}{8}\right)\frac{1}{\sqrt{2\pi}\,8}\, e^{-\frac{(x+85.09)^2}{128}} + \Phi\left(\frac{x+85.09}{8}\right)\frac{1}{\sqrt{2\pi}\,8}\, e^{-\frac{(x+83.57)^2}{128}}$$

$$\mu_{\Omega_{p_1}} = -80 - 36.8\log_{10}\left(\frac{2}{1.6}\right) = -83.57$$

$$\mu_{\Omega_{p_2}} = -80 - 36.8\log_{10}\left(\frac{2.2}{1.6}\right) = -85.09$$

b) $P(\text{outage}) = F_{\Omega_p^o}(-100)$

$$= \Phi\left(\frac{-100+83.57}{8}\right)\Phi\left(\frac{-100+85.09}{8}\right)$$

$$= \Phi(-2.05)\,\Phi(-1.86) = Q(2.05)\,Q(1.86)$$

$$= (1-0.97982)(1-0.96856)$$

$$= 6.34 \times 10^{-4}$$

2.1 $r(t) = g_I(t)\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t$

$$E[r(t)\, r(t+\tau)] = E[\{g_I(t)\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t\}\{g_I(t+\tau)\cos[2\pi f_c(t+\tau)] - g_Q(t+\tau)\sin[2\pi f_c(t+\tau)]\}]$$

$$= E[g_I(t)g_I(t+\tau)\{\tfrac{1}{2}\cos 2\pi f_c\tau + \tfrac{1}{2}\cos[2\pi f_c(2t+\tau)]\}]$$

$$+ E[g_Q(t)g_Q(t+\tau)\{\tfrac{1}{2}\cos 2\pi f_c\tau - \tfrac{1}{2}\cos[2\pi f_c(2t+\tau)]\}]$$

$$- E[g_Q(t)g_I(t+\tau)\{-\tfrac{1}{2}\sin 2\pi f_c\tau + \tfrac{1}{2}\sin[2\pi f_c(2t+\tau)]\}]$$

$$- E[g_I(t)g_Q(t+\tau)\{\tfrac{1}{2}\sin 2\pi f_c\tau + \tfrac{1}{2}\sin[2\pi f_c(2t+\tau)]\}]$$

$$= \tfrac{1}{2}\cos 2\pi f_c\tau\,\{E[g_I(t)g_I(t+\tau)] + E[g_Q(t)g_Q(t+\tau)]\}$$

$$- \tfrac{1}{2}\sin 2\pi f_c\tau\,\{E[g_I(t)g_Q(t+\tau)] - E[g_Q(t)g_I(t+\tau)]\}$$

$$+ \tfrac{1}{2}\cos 2\pi f_c(2t+\tau)\,\{E[g_I(t)g_I(t+\tau)] - E[g_Q(t)g_Q(t+\tau)]\}$$

$$- \tfrac{1}{2}\sin 2\pi f_c(2t+\tau)\,\{E[g_Q(t)g_I(t+\tau)] + E[g_I(t)g_Q(t+\tau)]\}$$

$$= \frac{1}{2}\cos(2\pi f_c\tau)\{\phi_{g_Ig_I}(\tau) + \phi_{g_Qg_Q}(\tau)\}$$

$$-\frac{1}{2}\sin(2\pi f_c\tau)\{\phi_{g_Ig_Q}(\tau) - \phi_{g_Qg_I}(\tau)\}$$

$$+\frac{1}{2}\cos(2\pi f_c(2t+\tau))\{\phi_{g_Ig_I}(\tau) - \phi_{g_Qg_Q}(\tau)\}$$

$$-\frac{1}{2}\sin(2\pi f_c(2t+\tau))\{\phi_{g_Qg_I}(\tau) + \phi_{g_Ig_Q}(\tau)\}$$

Since $r(t)$ is wide sense stationary, the last two terms must be zero.

Hence, $\phi_{g_Ig_I}(\tau) = \phi_{g_Qg_Q}(\tau)$

$$\phi_{g_Ig_Q}(\tau) = -\phi_{g_Qg_I}(\tau)$$

$$E[r(t)r(t+\tau)] = \phi_{g_Ig_I}(\tau)\cos(2\pi f_c\tau) - \phi_{g_Ig_Q}(\tau)\sin(2\pi f_c\tau)$$

$$= \phi_{rr}(\tau)$$

2.4 a) $T(f,t) = \mathcal{F}_{\tau}\{g(t,\tau)\}$

$$= \cos(\Omega t + \phi_0)\int_0^T \left(1-\frac{\tau}{T}\right)e^{-j\omega\tau}\,d\tau$$

$$= \left(\frac{1}{j\omega} - \frac{(1-e^{-j\omega T})}{\omega^2 T}\right)\cos(\Omega t + \phi_0)$$

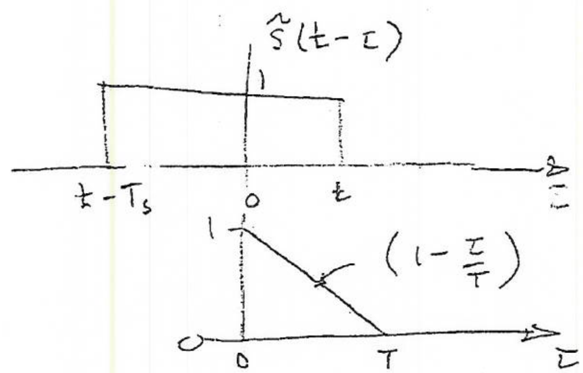
$$= \left(\frac{\cos\omega T - 1 - j[\omega T + \sin\omega T]}{\omega^2 T}\right)\cos(\Omega t + \phi_0)$$

b) $\hat{r}(t) = g(t,\tau) * \tilde{s}(t)$

$$= \int_0^t g(t,\tau)\,\tilde{s}(t-\tau)\,d\tau$$

$$= \int_0^t \left(1-\frac{\tau}{T}\right)\cos(\Omega t + \phi_0)\,\tilde{s}(t-\tau)\,d\tau$$

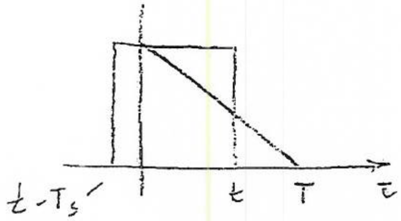
It is easier to use graphical convolution to visualize the integral



Case 1 : $T_s < T$

i) For $t \le 0$, $\tilde{r}(t) = 0$

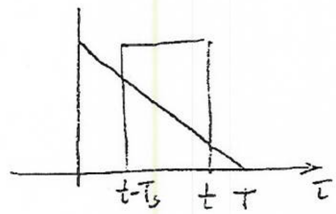
ii) For $0 \le t \le T_s$



$$\tilde{r}(t) = \cos(\Omega t + \phi_0)\int_0^t \left(1 - \frac{\tau}{T}\right)d\tau$$

$$= \cos(\Omega t + \phi_0)\left(t - \frac{t^2}{2T}\right)$$

iii) For $T_s \le t \le T$



$$\tilde{r}(t) = \cos(\Omega t + \phi_0)\int_{t-T_s}^{t} \left(1 - \frac{\tau}{T}\right)d\tau$$

$$= \cos(\Omega t + \phi_0)\left(1 - \frac{t}{T} + \frac{T_s}{2T}\right)T_s$$

iv) For $T \le t \le T + T_s$

$$\tilde{r}(t) = \cos(\Omega t + \phi_0)\int_{t-T_s}^{T} \left(1 - \frac{\tau}{T}\right)d\tau$$

$$= \cos(\Omega t + \phi_0)\left(\frac{T}{2} - t + T_s + \frac{(t-T_s)^2}{2T}\right)$$

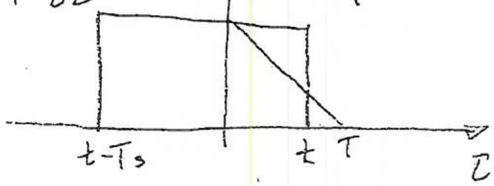
v) For $t > T + T_s$ $\tilde{r}(t) = 0$

$$\therefore \quad \tilde{r}(t) = \begin{cases} \cos(\Omega t + \phi_0)\left(t - \frac{t^2}{2T}\right), & 0 \le t \le T_s \\ \cos(\Omega t + \phi_0)\left(1 - \frac{t}{T} + \frac{T_s}{2T}\right)T_s, & T_s \le t \le T \\ \cos(\Omega t + \phi_0)\left(\frac{T}{2} - t + T_s + \frac{(t-T_s)^2}{2T}\right), & T \le t \le T + T_s \\ 0, & \text{elsewhere} \end{cases}$$

Case 2: $T_s > T$

i) For $t < 0$, $\tilde{r}(t) = 0$

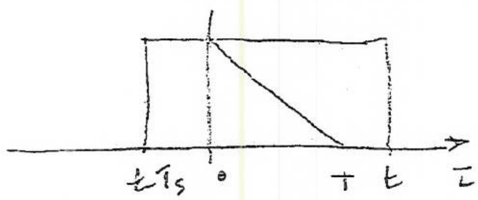
ii) For $0 \le t \le T$



$$\tilde{r}(t) = \cos(\Omega t + \phi_0)\int_0^t \left(1 - \frac{\tau}{T}\right) d\tau$$

$$= \cos(\Omega t + \phi_0)\left(t - \frac{t^2}{2T}\right)$$

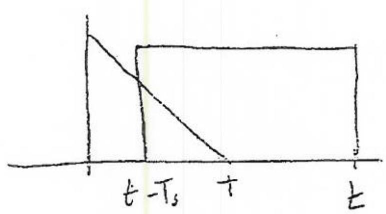
iii) For $T \le t \le T_s$



$$\tilde{r}(t) = \cos(\Omega t + \phi_0)\int_0^T \left(1 - \frac{\tau}{T}\right) d\tau$$

$$= \cos(\Omega t + \phi_0)\left(\frac{T}{2}\right)$$

iv) For $T_s \le t \le T_s + T$



$$\tilde{r}(t) = \cos(\Omega t + \phi_0)\int_{t-T_s}^T \left(1 - \frac{\tau}{T}\right) d\tau$$

$$= \cos(\Omega t + \phi_0)\left(\frac{T}{2} - t + T_s + \frac{(t-T_s)^2}{2T}\right)$$

v) For $t > T + T_s$ $\tilde{r}(t) = 0$

$$\therefore \quad \tilde{r}(t) = \begin{cases} \cos(\Omega t + \phi_0)\left(t - \frac{t^2}{2T}\right), & 0 \le t \le T \\ \frac{T}{2}\cos(\Omega t + \phi_0), & T \le t \le T_s \\ \cos(\Omega t + \phi_0)\left(\frac{T}{2} - t + T_s + \frac{(t-T_s)^2}{2T}\right), & T_s \le t \le T_s + T \\ 0, & \text{elsewhere} \end{cases}$$

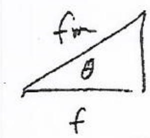
c) If $T \gtrsim 0.2T_s$, the channel will exhibit frequency selective fading in that an equalizer is required.

2.7 From (2.42) in text

$$S_{gg}(f) = \frac{\Omega_p/2}{\sqrt{f_m^2 - f^2}} \left(G(\theta)p(\theta) + G(-\theta)p(-\theta) \right)$$

$\theta = \cos^{-1}(f/f_m)$

In all cases $p(\theta) = \frac{1}{2\pi}$



$\sin\theta = \frac{\sqrt{f_m^2 - f^2}}{f_m}$

a) $G(\theta) = \frac{3}{2}\sin^2(\theta) = G(-\theta)$

$$S_{gg}(f) = \frac{\Omega_p/8\pi}{\sqrt{f_m^2 - f^2}} \cdot 3\sin^2\theta$$

$$= \frac{3\Omega_p}{8\pi f_m}\sqrt{1 - (f/f_m)^2}$$

b) $G(\theta) = \frac{3}{2}\cos^2\theta = G(-\theta)$ But $\cos\theta = f/f_m$

$$S_{gg}(f) = \frac{\Omega_p/8\pi}{\sqrt{f_m^2 - f^2}} \cdot 3\cos^2\theta$$

$$= \frac{3\Omega_p}{8\pi f_m} \cdot \frac{(f/f_m)^2}{\sqrt{1 - (f/f_m)^2}}$$

c) $S_{gg}(f) = \frac{\Omega_p/4}{\sqrt{f_m^2 - f^2}} \cdot G_0$ $\quad |\frac{\pi}{2} - \theta| < \beta/2$

$\Rightarrow -f_m\cos\left(\frac{\pi-\beta}{2}\right) \le f \le f_m\cos\left(\frac{\pi-\beta}{2}\right)$

d) $S_{gg}(f) = \frac{\Omega_p/4}{\sqrt{f_m^2 - f^2}} \cdot G_0$ $\quad |\theta| < \beta/2$

$\Rightarrow f_m\cos\frac{\beta}{2} \le f \le f_m$