

Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Term Test
Spring 2009

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

- 1) Consider the reverse link of a cellular system that uses a 7-cell reuse cluster with omnidirectional base station antennas. Ignore envelope fading and shadowing, and assume the simple path loss model

$$\mu_{\Omega_p \text{ (dBm)}}(d) = \mu_{\Omega_p \text{ (dBm)}}(d_0) - 10\beta \log_{10}(d/d_0) \text{ (dBm)},$$

where $\beta = 3.5$. Also assume that all mobile stations are transmitting at the same power level.

- a) **5 marks:** Show graphically the worst case co-channel interference geometry for the reverse channel. Use the hex grid on the next page.
- b) **5 marks:** Calculate the worst case carrier-to-interference ratio, Λ in terms of the co-channel reuse factor D/R .

The worst case interfering Mobile station locations are all at distance $\sqrt{13}R$ from the target Base station (located in the center cell). The serving mobile station is at distance R .
Hence,

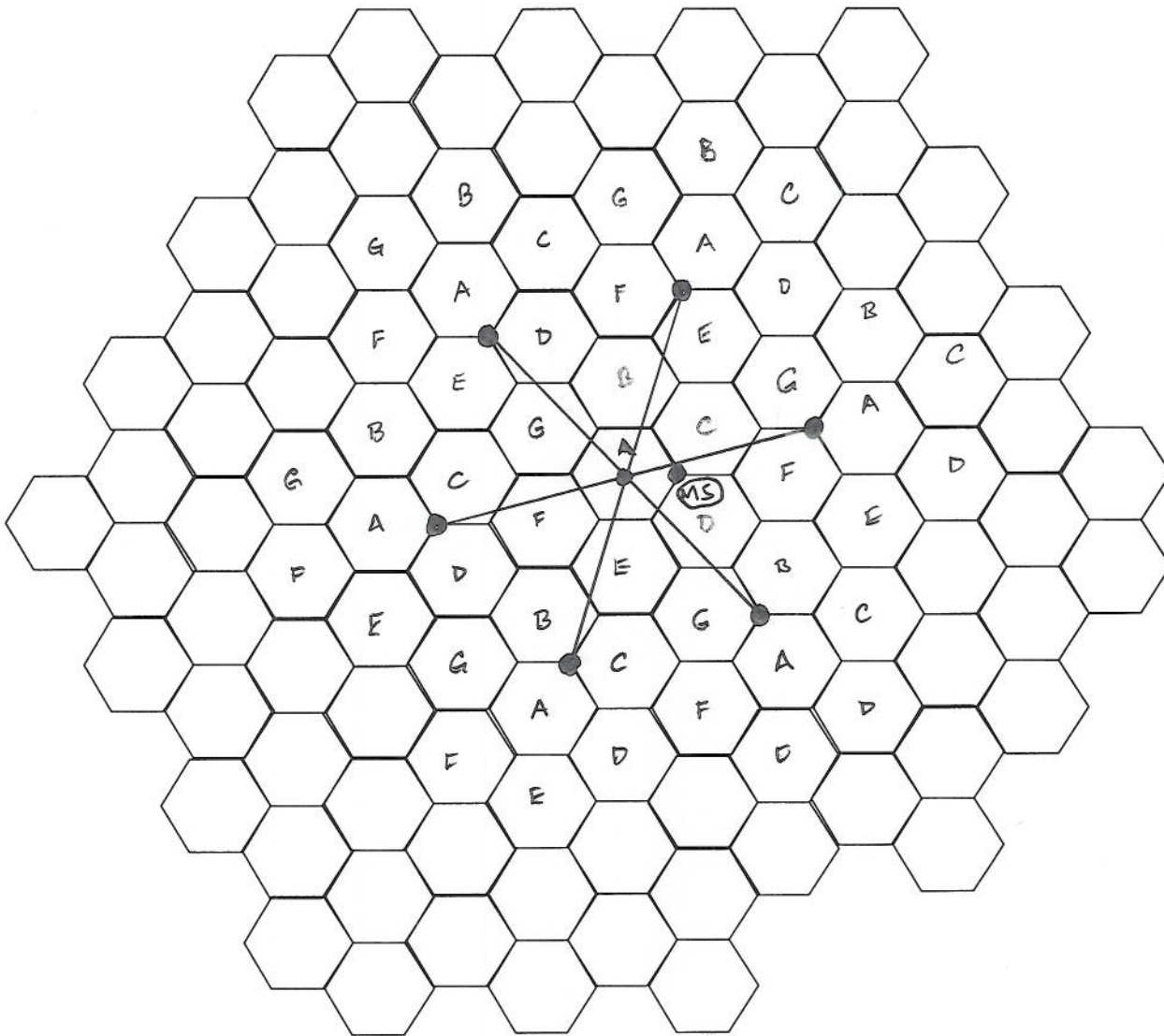
$$\Lambda = \frac{C}{I} = \frac{R^{-\beta}}{6 \times (\sqrt{13}R)^{-\beta}}$$

$$= \frac{1}{6 \times (\sqrt{13})^{-\beta}} = 11.71 \text{ dB for } \beta = 3.5.$$

If you express in terms of D/R , we have

$$\frac{D}{R} = \sqrt{21} \text{ and } \sqrt{13}R = \left(\frac{D}{R}\right) \sqrt{\frac{13}{21}} R$$

$$\therefore \Lambda = \frac{C}{I} = \frac{1}{6 \times \left(\frac{D}{R} \sqrt{\frac{13}{21}}\right)^{-\beta}}$$



Note: For the UPLINK, mobile stations in co-channel cells generate co-channel interference. The worst case location for the interfering mobile stations is at the corner of their respective cells that is closest to the target base station (all at distance $\sqrt{13}R$). The desired mobile station is also located in a corner of its cell (at distance R).

2) The scattering function for a WSSUS scattering channel is given by the following

$$\psi_S(\tau, \nu) = \Omega_p \cdot \frac{2a}{a^2 + (2\pi\nu)^2} \cdot be^{-b\tau} u(\tau)$$

a) 5 marks: What is the spaced-time spaced-frequency correlation function?

b) 2 marks: What is the average delay?

b) 3 marks: What is the rms delay spread?

a) we have

$$\begin{aligned} \psi_g(\Delta t, \tau) &= \mathcal{F}^{-1} \{ \psi_S(\tau, \nu) \} \\ &= \Omega_p e^{-a|\Delta t|} \cdot be^{-b\tau} u(\tau) \end{aligned}$$

Also,

$$\begin{aligned} \phi_T(\Delta t : \Delta f) &= \mathcal{F} \{ \psi_g(\Delta t, \tau) \} \\ &= \Omega_p e^{-a|\Delta t|} \cdot \frac{b}{b + j2\pi f} \end{aligned}$$

b) $\psi_g(\tau) = \Omega_p be^{-b\tau} u(\tau)$

$$\begin{aligned} \mu_\tau &= \frac{\int_0^\infty \tau \psi_g(\tau) d\tau}{\int_0^\infty \psi_g(\tau) d\tau} = \frac{b \int_0^\infty \tau e^{-b\tau} d\tau}{\int_0^\infty e^{-b\tau} d\tau} \\ &= b \cdot \frac{1}{b^2} = \frac{1}{b} \end{aligned}$$

c)
$$\sigma_\tau^2 = \frac{\int_0^\infty (\tau - \mu_\tau)^2 \psi_g(\tau) d\tau}{\int_0^\infty \psi_g(\tau) d\tau}$$

cont'd

$$= \int_0^{\infty} \tau^2 b e^{-b\tau} d\tau - \int_0^{\infty} \frac{2\tau}{b} \cdot b e^{-b\tau} d\tau + \frac{1}{b^2} \int_0^{\infty} b e^{-b\tau} d\tau$$

$$= \frac{2}{b^2} - \frac{2}{b^2} + \frac{1}{b^2}$$

$$= \frac{1}{b^2}$$

$$\therefore \sigma_{\tau} = \frac{1}{b}$$

3) A vehicle experiences 2-D isotropic scattering and receives a Rayleigh faded 900 MHz signal while traveling at a constant velocity for 10 s. Assume that the local mean remains constant during travel, and the average duration of fades 10 dB below the rms envelope level is 1 ms.

a) 5 marks: How far does the vehicle travel during the 10 s interval?

b) 5 marks: How many fades is the envelope expected to undergo that are 10 dB below the rms envelope level during the 10 s interval?

$$a) \quad \bar{T} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} \Rightarrow f_m = \frac{v}{\lambda_c} = \frac{e^{\rho^2} - 1}{\bar{T} \rho \sqrt{2\pi}}$$

$$\text{For } 900 \text{ MHz, } \lambda_c = \frac{c}{f_c} = \frac{3 \times 10^8}{9 \times 10^8} = .33 \text{ m}$$

Then for $\rho = -10 \text{ dB}$

$$(\rho = 10^{-1/2} = .31623)$$

and $\bar{T} = 1 \text{ ms}$, the vehicle speed is 43.78 m/s. Hence, the vehicle travels 437.8 m in 10 seconds.

$$b) \quad L_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} \quad \begin{array}{l} \rho = .31623 \\ f_m = 132.68 \end{array}$$

$$= 95.16 \text{ crossings/second}$$

Expected number of envelope crossings in 10 s is $95.16 \times 10 = 951.6$ crossings.