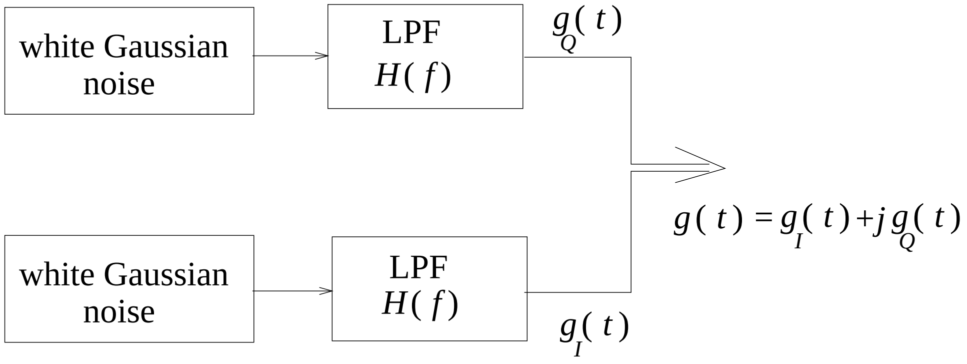
EE6604
Personal & Mobile Communications

Lecture 11

Fading Simulators

Filtered White Noise

- **Since the complex faded envelope can be modelled as a complex Gaussian random process, one approach for generating the complex faded envelope is to filter a white noise process with appropriately chosen low pass filters**



- **If the Gaussian noise sources are uncorrelated and have power spectral densities of $\Omega_p/2$ watts/Hz, and the low-pass filters have transfer function $H(f)$, then**

$$\begin{aligned} S_{g_I g_I}(f) &= S_{g_Q g_Q}(f) = \frac{\Omega_p}{2}|H(f)|^2 \\ S_{g_I g_Q}(f) &= 0 \end{aligned}$$

- **Two approaches: IIR filtering method and IFFT filtering method**

IIR Filtering Method

- **implement the filters in the time domain as finite impulse response (FIR) or infinite impulse response (IIR) filters. There are two main challenges with this approach.**
  - **the normalized Doppler frequency, $\hat{f}_m = f_m T_s$, where T_s is the simulation step size, is very small.**
    - **This can be overcome with an infinite impulse response (IIR) filter designed at a lower sampling frequency followed by an interpolator to increase the sampling frequency.**
  - **The second main challenge is that the square-root of the target Doppler spectrum for 2-D isotropic scattering and an isotropic antenna is irrational and, therefore, none of the straightforward filter design methods can be applied.**
    - **One possibility is to use the MATLAB function `iirlpnorm` to design the required filter.**

IIR Filtering Method

- **Here we consider an IIR filter of order $2K$ that is synthesized as the the cascade of K Direct-Form II second-order (two poles and two zeroes) sections (biquads) having the form**

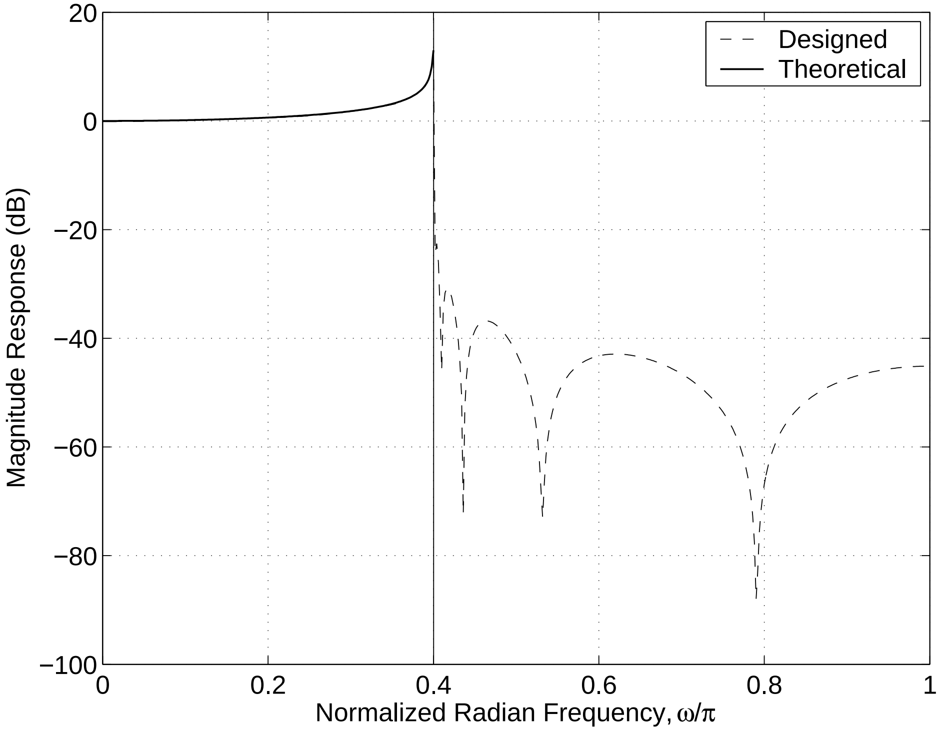
  $$H(z) = A \prod_{k=1}^{K} \frac{1 + a_k z^{-1} + b_k z^{-2}}{1 + c_k z^{-1} + d_k z^{-2}} \quad .$$

  For example, for $f_m T_s = 0.4$, $K = 5$, and an ellipsoidal accuracy of 0.01, we obtain the coefficients tabulated below

Coefficients for $K = 5$ biquad stage elliptical filter, $f_m T_s = 0.4$, $K = 5$

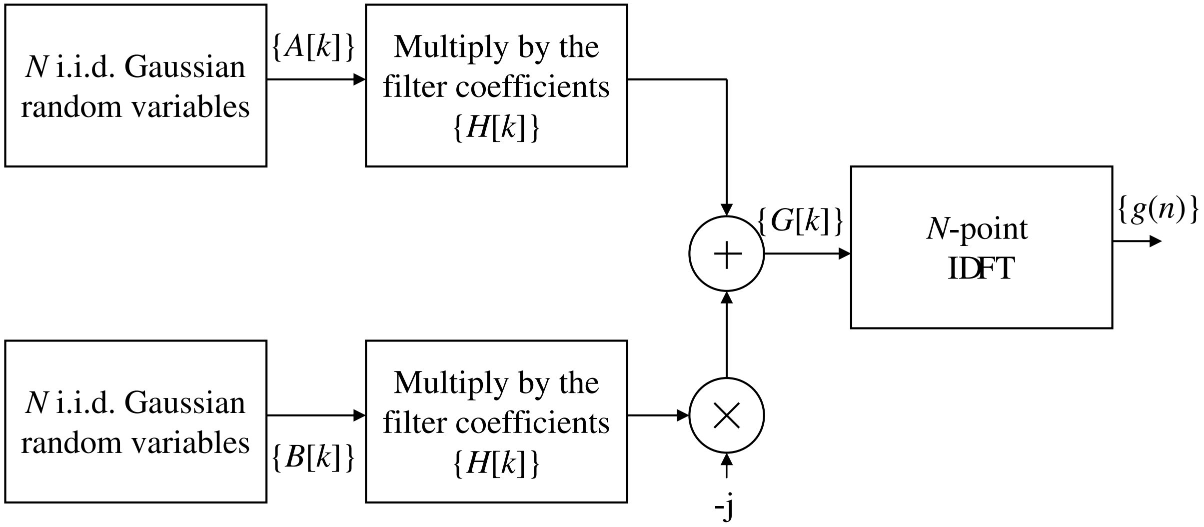
Stage	*Filter Coefficients*			
k	a_k	b_k	c_k	d_k
1	1.5806655278853	0.99720549234156	-0.64808639835819	0.88900798545419
2	0.19859624284546	0.99283177405702	-0.62521063559242	0.97280125737779
3	-0.60387555371625	0.9999939585621	-0.62031415619505	0.99996628706514
4	-0.56105447536557	0.9997677910713	-0.79222029531477	0.2514924845181
5	-0.39828788982331	0.99957862369507	-0.71405064745976	0.64701702807931
A	0.020939537466725			

IIR Filtering Method



Magnitude response of the designed shaping filter, $f_m T_s = 0.4$, $K = 5$.

IFFT Filtering Method



IDFT-based fading simulator.

- To implement 2-D isotropic scattering, the filter $H[k]$ can be specified as follows:

$$H[k] = \begin{cases} 0 & , \quad k = 0 \\ \sqrt{\frac{1}{2\pi f_m \sqrt{1-(k/(N\hat{f}_m))^2}}} & , \quad k = 1, 2, \ldots, k_m - 1 \\ \sqrt{k_m \left[\frac{\pi}{2} - \arctan\left(\frac{k_m-1}{\sqrt{2k_m-1}}\right)\right]} & , \quad k = k_m \\ 0 & , \quad k = k_m + 1, \ldots, N - k_m - 1 \\ \sqrt{k_m \left[\frac{\pi}{2} - \arctan\left(\frac{k_m-1}{\sqrt{2k_m-1}}\right)\right]} & , \quad k = N - k_m \\ \sqrt{\frac{1}{2\pi f_m \sqrt{1-(N-k/(N\hat{f}_m))^2}}} & , \quad N - k_m + 1, \ldots, N - 1 \end{cases}$$

- One problem with the IFFT method is that the faded envelope is discontinuous from one block of N samples to the next.

Sum of Sinusoids (SoS) Methods - Clarke's Model

- **With** N **equal strength (**$C_n = \sqrt{1/N}$**) arriving plane waves**

$$
\begin{aligned}
g(t) &= g_I(t) + jg_Q(t) \\
&= \sqrt{1/N}\sum_{n=1}^{N}\cos(2\pi f_m t\cos\theta_n + \hat{\phi}_n) + j\sqrt{1/N}\sum_{n=1}^{N}\sin(2\pi f_m t\cos\theta_n + \hat{\phi}_n)\ . \qquad (1)
\end{aligned}
$$

- **The normalization** $C_n = \sqrt{1/N}$ **makes** $\Omega_p = 1$**.**
- **The phases** $\hat{\phi}_n$ **are independent and uniform on** $[-\pi, \pi)$**.**
- **With 2-D isotropic scattering, the** θ_n **are also independent and uniform on** $[-\pi, \pi)$**, and are independent of the** $\hat{\phi}_n$**.**
- **Types of SoS simulators**
  - **deterministic -** $\{\theta_n\}$ **and** $\{\hat{\phi}_n\}$ **are fixed for all simulation runs.**
  - **statistical - either** $\{\theta_n\}$ **or** $\{\hat{\phi}_n\}$**, or both, are random for each simulation run.**
  - **ergodic statistical - either** $\{\theta_n\}$ **or** $\{\hat{\phi}_n\}$**, or both, are random, but only a single simulation run is required.**

Clarke's Model - Ensemble Averages

- **The statistical properties of Clarke's model in for** *finite* N **are**

$$\begin{aligned}\phi_{g_I g_I}(\tau) &= \phi_{g_Q g_Q}(\tau) = \frac{1}{2}J_0(2\pi f_m \tau) \\ \phi_{g_I g_Q}(\tau) &= \phi_{g_Q g_I}(\tau) = 0 \\ \phi_{gg}(\tau) &= \frac{1}{2}J_0(2\pi f_m \tau) \\ \phi_{|g|^2|g|^2}(\tau) &= \mathrm{E}[|g|^2(t)|g|^2(t+\tau)] \\ &= 1 + \frac{N-1}{N}J_0^2(2\pi f_m \tau)\end{aligned}$$

- **For finite** N**, the ensemble averaged auto- and cross-correlation of the quadrature components match those of the 2-D isotropic scattering reference model.**
- **The squared envelope autocorrelation reaches the desired form** $1 + J_0^2(2\pi f_m \tau)$ **asymptotically as** $N \rightarrow \infty$**.**

Clarke's Model - Time Averages

- **In simulations, time averaging is often used in place of ensemble averaging. The corresponding time average correlation functions $\hat{\phi}(\cdot)$ (all time averaged quantities are distinguished from the statistical averages with a '$\hat{}$') are random and depend on the specific realization of the random parameters in a given simulation trial.**

- **The variances of the time average correlation functions, defined as**

$$\mathrm{Var}[\hat{\phi}(\cdot)] = \mathrm{E}\Big[\Big|\hat{\phi}(\cdot) \;-\; \lim_{N\,\to\,\infty} \phi(\cdot)\Big|^2\Big] \;,$$

characterizes the closeness of a simulation trial with finite N and the ideal case with $N \to \infty$.

- **These variances can be derived as follows:**

$$\begin{aligned}
\mathrm{Var}[\hat{\phi}_{g_I g_I}(\tau)] &= \mathrm{Var}[\hat{\phi}_{g_Q g_Q}(\tau)] \\
&= \frac{1 + J_0(4\pi f_m \tau) - 2J_0^2(2\pi f_m \tau)}{8N} \\
\mathrm{Var}[\hat{\phi}_{g_I g_Q}(\tau)] &= \mathrm{Var}[\hat{\phi}_{g_Q g_I}(\tau)] \\
&= \frac{1 - J_0(4\pi f_m \tau)}{8N} \\
\mathrm{Var}[\hat{\phi}_{gg}(\tau)] &= \frac{1 - J_0^2(2\pi f_m \tau)}{4N}
\end{aligned}$$

Jakes' Deterministic Method

- **To approximate an isotropic scattering channel, it is assumed that the N arriving plane waves uniformly distributed in angle of incidence:**

$$\theta_n = 2\pi n/N \ , \quad n = 1,\ 2, \ldots,\ N$$

- **By choosing $N/2$ to be an odd integer, the sum in (1) can be rearranged into the form**

$$\begin{aligned} g(t) \ &= \ \sqrt{\frac{1}{N}} \sum_{n=1}^{N/2-1} \left[e^{-j(2\pi f_m t \cos\theta_n + \hat{\phi}_{-n})} + e^{j(2\pi f_m t \cos\theta_n + \hat{\phi}_n)} \right] \\ &\qquad + e^{-j(2\pi f_m t + \hat{\phi}_{-N})} + e^{j(2\pi f_m t + \hat{\phi}_N)} \end{aligned} \tag{2}$$

- **The Doppler shifts progress from $-2\pi f_m \cos(2\pi/N)$ to $+2\pi f_m \cos(2\pi/N)$ as n progresses from 1 to $N/2 - 1$ in the first sum, while in the second sum they progress from $+2\pi f_m \cos(2\pi/N)$ to $-2\pi f_m \cos(2\pi/N)$.**

- **Jakes uses nonoverlapping frequencies to write $g(t)$ as**

$$\begin{aligned} g(t) \ &= \ \sqrt{2}\sqrt{\frac{1}{N}} \sum_{n=1}^{M} \left[e^{-j(\hat{\phi}_{-n} + 2\pi f_m t \cos\theta_n)} + e^{j(\hat{\phi}_n + 2\pi f_m t \cos\theta_n)} \right] \\ &\qquad + e^{-j(\hat{\phi}_{-N} + 2\pi f_m t)} + e^{j(\hat{\phi}_N + 2\pi f_m t)} \end{aligned} \tag{3}$$

  where

$$M = \frac{1}{2}\left(\frac{N}{2} - 1\right)$$

  and the factor $\sqrt{2}$ is included so that the total power remains unchanged.

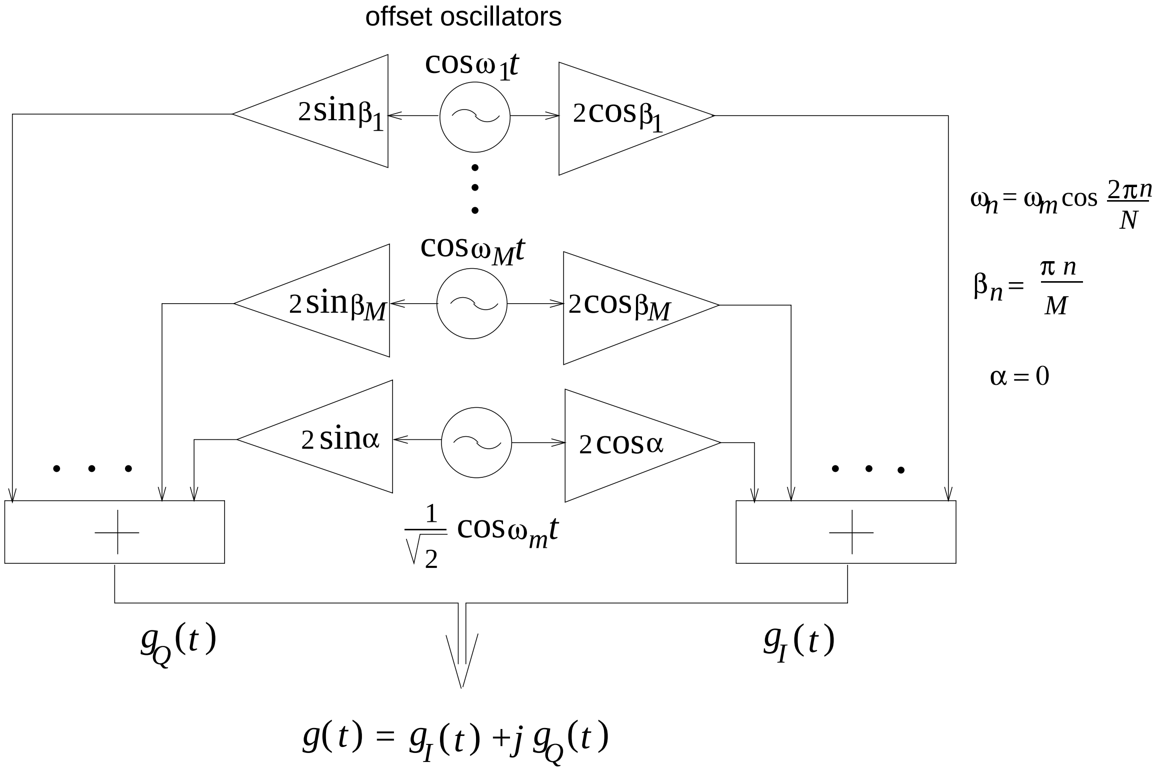
- **Note that (2) and (3) are not equal. In (2) all phases are independent. However, (3) implies that $-\hat{\phi}_i = \hat{\phi}_{N/2-1-i}$ and, therefore, correlation is introduced into the phases**

- **Jakes' further imposes the constraint $\hat{\phi}_n = -\hat{\phi}_{-n}$ to give**

$$\begin{aligned} g(t) &= \sqrt{2}\left\{\left[2\sum_{n=1}^{M}\cos\beta_n\cos 2\pi f_n t + \sqrt{2}\cos\alpha\cos 2\pi f_m t\right]\right. \\ &\qquad \left. +j\left[2\sum_{n=1}^{M}\sin\beta_n\cos 2\pi f_n t + \sqrt{2}\sin\alpha\cos 2\pi f_m t\right]\right\} \end{aligned}$$

where

$$\alpha = \hat{\phi}_N = -\hat{\phi}_{-N} \qquad \beta_n = \hat{\phi}_n = -\hat{\phi}_{-n} \qquad M = \frac{1}{2}\left(\frac{N}{2} - 1\right)$$



Jakes' fading simulator that generates a faded envelope by summing waveforms from $M+1$ low frequency oscillators.

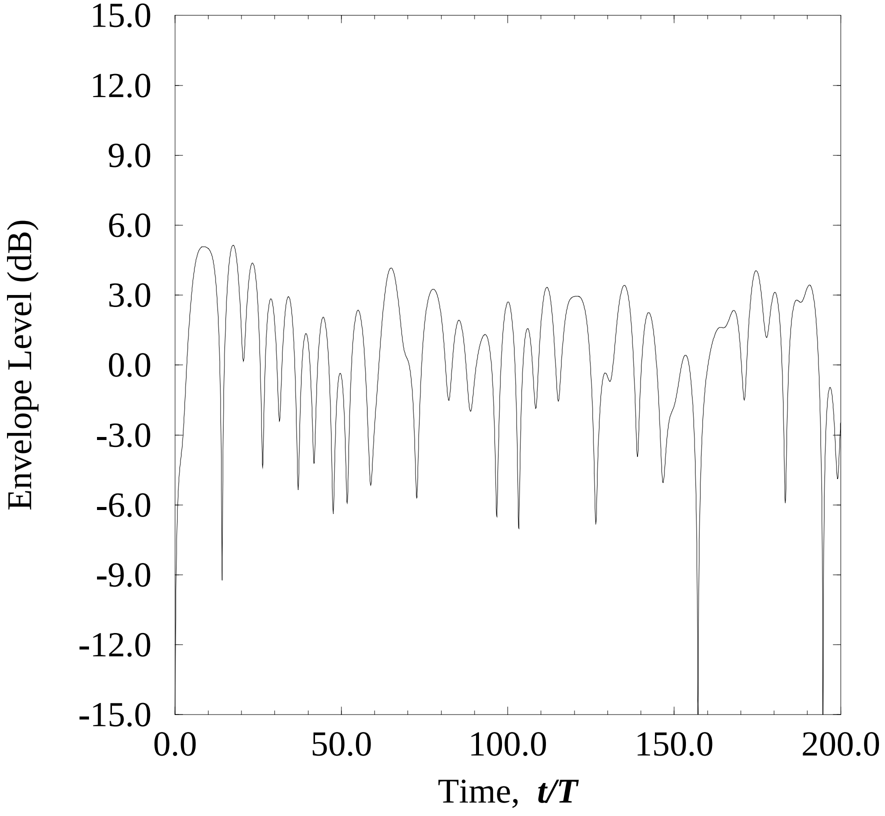
- **Time averages:**

$$\begin{aligned} < g_I^2(t) > &= 2\sum_{n=1}^{M} \cos^2\beta_n + \cos^2\alpha \\ &= M + \cos^2\alpha + \sum_{n=1}^{M} \cos 2\beta_n \end{aligned}$$

$$\begin{aligned} < g_Q^2(t) > &= 2\sum_{n=1}^{M} \sin^2\beta_n + \sin^2\alpha \\ &= M + \sin^2\alpha - \sum_{n=1}^{M} \cos 2\beta_n \end{aligned}$$

$$< g_I(t)g_Q(t) >= 2\sum_{n=1}^{M} \sin\beta_n\cos\beta_n + \sin\alpha\cos\alpha \quad .$$

- **Choose the β_n and α so that $g_I(t)$ and $g_Q(t)$ have zero-mean, equal variance, and zero cross-correlation.**
- **The choices $\alpha = 0$ and $\beta_n = \pi n/M$ will yield $< g_Q^2(t) >= M$, $< g_I^2(t) >= M+1$, and $< g_I(t)g_Q(t) >= 0$.**
- **The envelope power $< g_I^2(t) > + < g_Q^2(t) >$ can be scaled to any desired value.**



Typical faded envelope generated with 8 oscillators.

Auto- and Cross-correlations

- **The normalized autocorrelation function is**

$$\phi_{gg}^{n}(\tau) = \frac{\mathrm{E}[g^*(t)g(t+\tau)]}{\mathrm{E}[|g(t)|^2]}$$

- **With 2-D isotropic scattering**

$$\begin{aligned} \phi_{g_I g_I}(\tau) &= \phi_{g_Q g_Q}(\tau) = \frac{\Omega_p}{2} J_0\left(2\pi f_m \tau\right) \\ \phi_{g_I g_Q}(\tau) &= \phi_{g_Q g_I}(\tau) = 0 \end{aligned}$$

- **Therefore,**

$$\begin{aligned} \phi_{gg}^{n}(\tau) &= \frac{\mathrm{E}[g^*(t)g(t+\tau)]}{\mathrm{E}[|g(t)|^2]} \\ &= J_0\left(2\pi f_m \tau\right) \end{aligned}$$

Auto- and Cross-correlations

- **For Clarke's model with angles θ_n that are independent and uniform on $[-\pi, \pi)$, the normalized autocorrelation function is**

$$\phi_{gg}^{n}(\tau) = \frac{\mathrm{E}[g^*(t)g(t+\tau)]}{\mathrm{E}[|g(t)|^2]} = J_0(2\pi f_m \tau) \quad .$$

- **Clark's model with even N and the restriction $\theta_n = \frac{2\pi n}{N}$, yields the normalized ensemble averaged autocorrelation function**

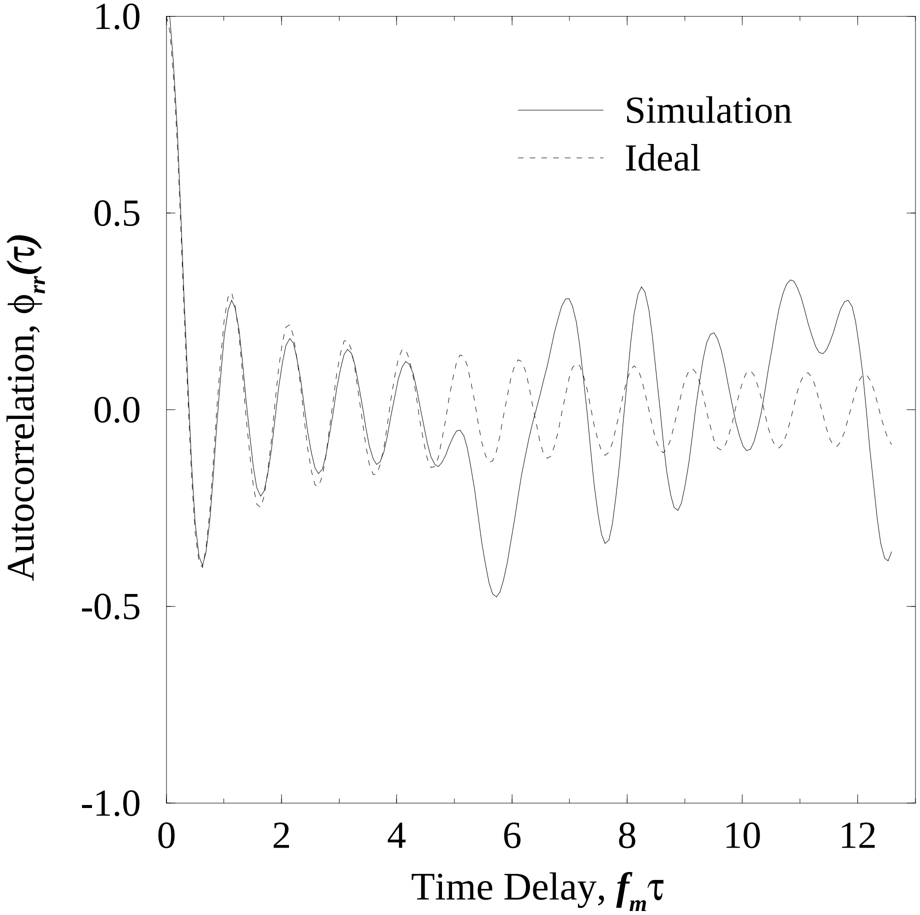
$$\phi_{gg}^{n}(\tau) = \frac{1}{N}\sum_{n=1}^{N} \cos\left(2\pi f_m \tau \cos\frac{2\pi n}{N}\right) \quad .$$

  - **Clark's model with $\theta_n = \frac{2\pi n}{N}$ yields an autocorrelation function that deviates from the desired values at large lags.**

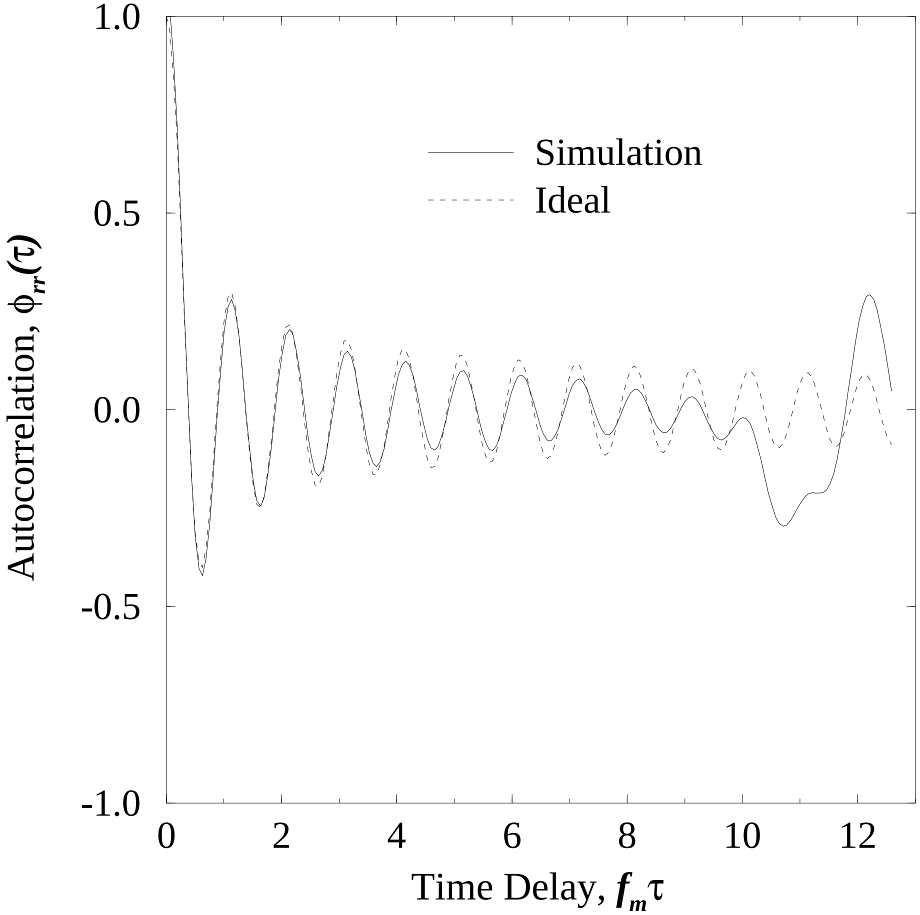
- **Finally, the normalized time averaged autocorrelation function for Jakes' method is**

$$\begin{aligned}\phi_{gg}^{n}(t, t+\tau) \;&=\; \frac{2}{N}\left(\cos 2\pi f_m \tau + \cos 2\pi f_m (2t+\tau)\right) \\ &\quad + \frac{4}{N}\sum_{n=1}^{M}\left(\cos 2\pi f_n \tau + \cos 2\pi f_n (2t+\tau)\right)\end{aligned}$$

  - **Jakes' simulator is not wide-sense stationary.**



Autocorrelation of inphase and quadrature components obtained with Clarke's method, using $\theta_n = \frac{2\pi n}{N}$ and $N = 8$ oscillators.



Autocorrelation of inphase and quadrature components obtained with Clarke's method, using $\theta_n = \frac{2\pi n}{N}$ and $N = 16$ oscillators.