EE6604

Personal & Mobile Communications

Lecture 12

Fading Simulators,
Baud-Spaced Channel Models

Method of Exact Doppler Spreads - Deterministic

- **Patzold proposed a deterministic simulation model, called the "Method of Exact Doppler Spreads (MEDS)".**
- **The method is derived by using the integral representation for the zero-order Bessel function of the first kind**

$$J_0(x) = \frac{2}{\pi}\int_0^{\pi/2} \cos(x \sin\theta)d\theta$$

and replacing the integral by a series expansion as follows:

$$J_0(x) = \lim_{N_I \to \infty} \frac{2}{\pi} \sum_{n=1}^{N_I} \cos(x \sin\alpha_n)\Delta_\alpha$$

where $\alpha_n \triangleq \pi(2n-1)/(4N_I)$ **and** $\Delta_\alpha \triangleq \pi/(2N_I)$**.**

- **Hence,**

$$\phi_{g_I g_I}(\tau) = \frac{1}{2}J_0(2\pi f_m \tau) = \lim_{N_I \to \infty} \frac{1}{2N_I} \sum_{n=1}^{N_I} \cos(2\pi f_{n,i}\tau)$$

where

$$f_{n,I} = f_m \sin\left[\frac{\pi}{2N_I}\left(n - \frac{1}{2}\right)\right] \quad .$$

- **With finite** N_I**, the normalized autocorrelation is**

$$\phi^n_{g_I g_I}(\tau) = \frac{1}{N_I}\sum_{n=1}^{N_I} \cos(2\pi f_{n,I}\tau)$$

Method of Exact Doppler Spreads - Deterministic

- **The complex fading envelope is $g(t) = g_I(t) + jg_Q(t)$, where**

$$g_{I/Q}(t) = \sqrt{\frac{1}{N_{I/Q}}} \sum_{n=1}^{N_{I/Q}} \cos(2\pi f_{I/Q,n} t + \phi_{I/Q,n}) \quad .$$

- **The deterministic processes $g_I(t)$ and $g_Q(t)$ are uncorrelated if and only if $f_{I,n} \neq f_{Q,m}$ for all $n = 1, \ldots, N_I$ and $m = 1, \ldots, N_q Q$.**
  - **This can be achieved by setting $N_Q = N_i I + 1$.**
- **If $g_I(t)$ and $g_Q(t)$ are uncorrelated, the phases $\phi_{I/Q,n}$ have no influence on the statistical properties of $g(t)$.**
  - **Hence, the $\phi_{I/Q,n}$ can be chosen as arbitrary realizations of uniform random variables on $[-\pi, \pi)$.**

Zheng and Xiao's Model - Statistical

- **The complex fading envelope is $g(t) = g_I(t) + jg_Q(t)$, where**

$$
\begin{aligned}
g_I(t) &= \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos[2\pi f_m t \cos(\theta_n) + \phi_{I,n}] \\
g_Q(t) &= \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos[2\pi f_m t \sin(\theta_n) + \phi_{Q,n}]
\end{aligned}
$$

where the angles are

$$
\theta_n = \frac{2\pi n - \pi + \theta}{4N}, \qquad n = 1, 2, \ldots, N
$$

where θ, $\phi_{I,n}$, and $\phi_{Q,n}$ are all uniform on $[-\pi, \pi)$, and all values are mutually independent.

- **The statistical correlation functions of the quadrature components $\phi_{g_I g_I}(\tau)$, $\phi_{g_I g_Q}(\tau)$, $\phi_{gg}(\tau)$ match the desired functions.**

- **The autocorrelation function of the squared envelope is**

$$
\begin{aligned}
\phi_{|g|^2|g|^2}(\tau) &= 4 + \frac{J_0(4\pi f_m \tau)}{N} \\
&\quad + \frac{4}{N^2} \sum_{n=1}^{N} \sum_{\substack{m=1 \\ n \neq m}}^{N} \mathrm{E}\{\cos[2\pi f_m \tau \cos\theta_n] \cos[2\pi f_m \tau \cos\theta_m]\}
\end{aligned}
$$

Modified Hoeher Model - Statistical

- **Consider the complex faded envelope $g(t) = g_I(t) + jg_Q(t)$, where**

$$
\begin{aligned}
g_I(t) &= \sqrt{\frac{1}{N}} \sum_{n=1}^{N_I} \cos(2\pi f_{I,n} t + \phi_{I,n}) \\
g_Q(t) &= \sqrt{\frac{1}{N}} \sum_{m=1}^{N_Q} \cos(2\pi f_{Q,m} t + \phi_{Q,m})
\end{aligned}
$$

and

$$
f_{I/Q,n/m} = f_m \sin\left(\frac{\pi}{2} u_{I/Q,n/m}\right) \quad .
$$

- **The Doppler frequencies $f_{I/Q,n/m}$ for the I and Q components are determined by $u_{I/Q,n/m}$, where the $u_{I/Q,n/m}$ are uniform on $(0,1]$ and are mutually independent for all n and m. The random phases $\phi_{I/Q,n/m}$ are uniform on $[-\pi,\pi)$, are mutually independent for all n and m, and are also independent of the $u_{I/Q,n/m}$.**

- **For convenience, the number of sinusoids in the quadrature components are usually set equal, i.e., $N_I = N_Q = N$.**

- **(MEDS) and the modified Hoeher model are similar**
  - **The set of numbers $\{(n-1/2)/N_I, i = 1, \ldots, N_I\}$ are uniformly spaced on the interval $(0,1]$, while the $u_{I/Q,n/m}$ are uniformly distributed on the interval $(0,1]$.**
- **The statistical correlation functions for the quadrature components match the desired values.**
- **The squared envelope correlation is**

$$\phi_{|g|^2|g|^2}(\tau) = 4 + 4\frac{N-1}{N}J_0^2(2\pi f_m \tau) + \frac{1}{N}J_0(4\pi f_m \tau)$$

  which differs from the ideal value for finite N.

Multiple Faded Envelopes - Zheng & Xiao Model

- **The Zheng & Xiao statistical method can be easily extended to generate multiple faded envelopes.**
- **The** k**th complex envelope,** $g_k(t) = g_{I,k}(t) + jg_{Q,k}(t)$**, is generated as**

$$
\begin{aligned}
g_{I,k}(t) &= \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos[2\pi f_m t \cos(\theta_{n,k}) + \phi_{I,n,k}] \\
g_{Q,k}(t) &= \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos[2\pi f_m t \sin(\theta_{n,k}) + \phi_{Q,n,k}]
\end{aligned}
$$

where

$$
\theta_{n,k} = \frac{2\pi n - \pi + \theta_k}{4N}, \qquad n = 1, 2.., N
$$

and where θ_k**,** $\phi_{I,n,k}$**, and** $\phi_{Q,n,k}$ **are all uniform on** $[-\pi, \pi)$**, and all values are mutually independent.**

- **The** $g_k(t)$ **are all uncorrelated.**

Multiple Faded Envelopes - Li & Huang Model

- **Assume that P uncorrelated fading envelopes are required, each composed of N sinusoids.**
- **The kth faded envelope, $g_k(t) = g_{I,k}(t) + jg_{Q,k}(t)$, is generated as**

$$\begin{aligned} g_{I,k}(t) &= \sqrt{\frac{1}{N}} \sum_{n=0}^{N-1} \cos(2\pi f_m \cos\theta_{n,k} t + \phi_{n,k}) \\ g_{Q,k}(t) &= \sqrt{\frac{1}{N}} \sum_{n=0}^{N-1} \sin(2\pi f_m \sin\theta_{n,k} t + \phi'_{n,k}) \end{aligned}$$

- **The phases $\phi_{n,k}$ and $\phi'_{n,k}$ arbitrary realizations of independent random variables uniform on $[-\pi, \pi)$, and**

$$\theta_{n,k} = \frac{2\pi n}{N} + \frac{2\pi k}{PN} + \theta_{00}, \quad n = 0, \ldots, N, k = 0, \ldots, P-1$$

  where θ_{00} is an initial arrival angle chosen to satisfy $0 < \theta_{00} < 2\pi/PN$ and $\theta_{00} \neq \pi/PN$.

- **Although the Li & Huang model generates uncorrelated faded envelopes, it does not satisfy the correlation functions of the reference model.**

Zajić & Stüber Deterministic Model

- **The** $k^{th}, 1 \leq k \leq P$ **complex faded envelope is defined as** $g_k(t) = g_{I,k}(t) + j g_{Q,k}(t)$**, where**

$$g_{I,k}(t) = \frac{2}{\sqrt{N}} \left[\sum_{n=0}^{M} a_n \cos(2\pi f_m t \cos \alpha_{nk} + \phi_{nk}) \right],$$

$$g_{Q,k}(t) = \frac{2}{\sqrt{N}} \left[\sum_{n=0}^{M} b_n \sin(2\pi f_m t \sin \alpha_{nk} + \phi_{nk}) \right].$$

- **The phases** ϕ_{nk} **are chosen to be independent random variables uniformly distributed on the interval** $[0, 2\pi)$

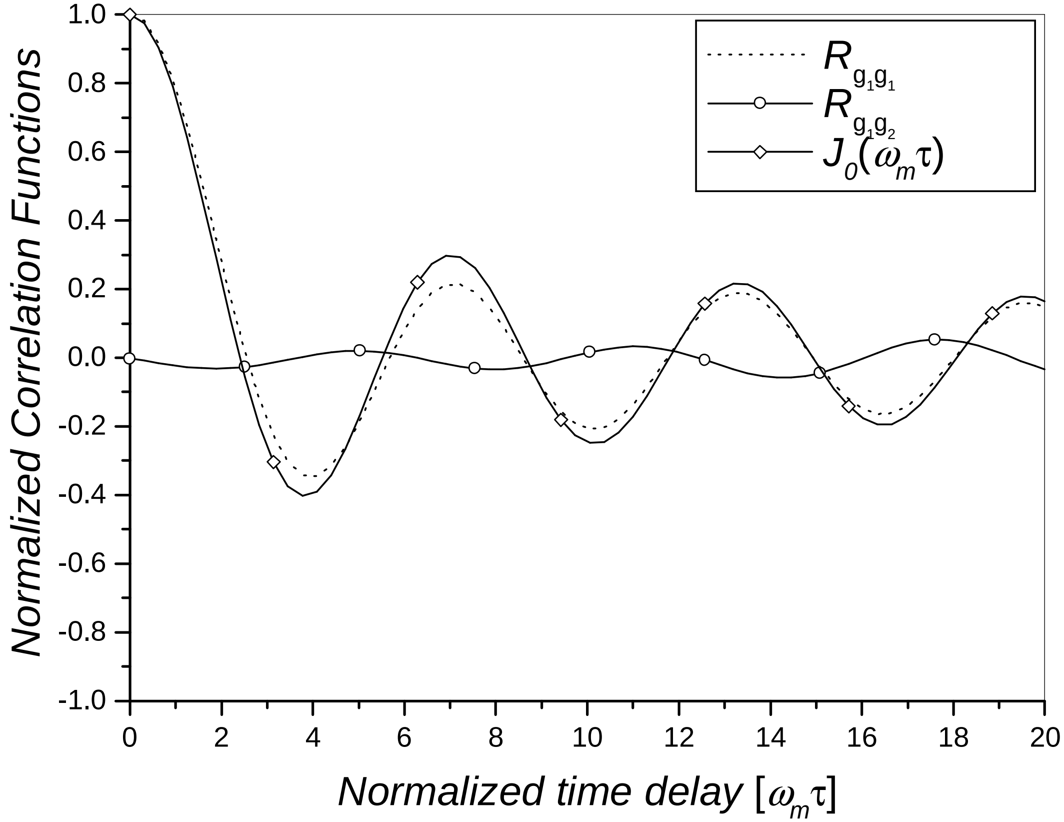
- **The** β_n **are defined as** $\beta_n = \pi n / M$ **for** $n = 0, \ldots, M$**, where** $N = 4M + 2$**.**

- **Parameters** a_n **and** b_n **are defined as follows:**

$$a_n = \begin{cases} 2\cos(\beta_n), & n = 1, ..., M \\ \sqrt{2}\cos(\beta_n), & n = 0 \end{cases},$$

$$b_n = \begin{cases} 2\sin(\beta_n), & n = 1, ..., M \\ \sqrt{2}\sin(\beta_n), & n = 0 \end{cases}.$$

- **The angles of arrival** α_{nk} **are defined as** $\alpha_{nk} = (2\pi n)/N + (2\pi k)/(PN) + \alpha_{00}$ **for** $n = 0, \ldots, M$**,** $k = 0, \ldots, P-1$**, where** $\alpha_{00} = (0.2\pi)/(PN)$**.**



Theoretical and simulated auto-correlation functions and the cross-correlation function of the first and the second complex envelope of the Zajić and Stüber "deterministic" model.

Zajić & Stüber Statistical Model

- **The** $k^{th}, 1 \leq k \leq P$ **complex faded envelope is defined as** $g_k(t) = g_{I,k}(t) + j g_{Q,k}(t)$**, where**

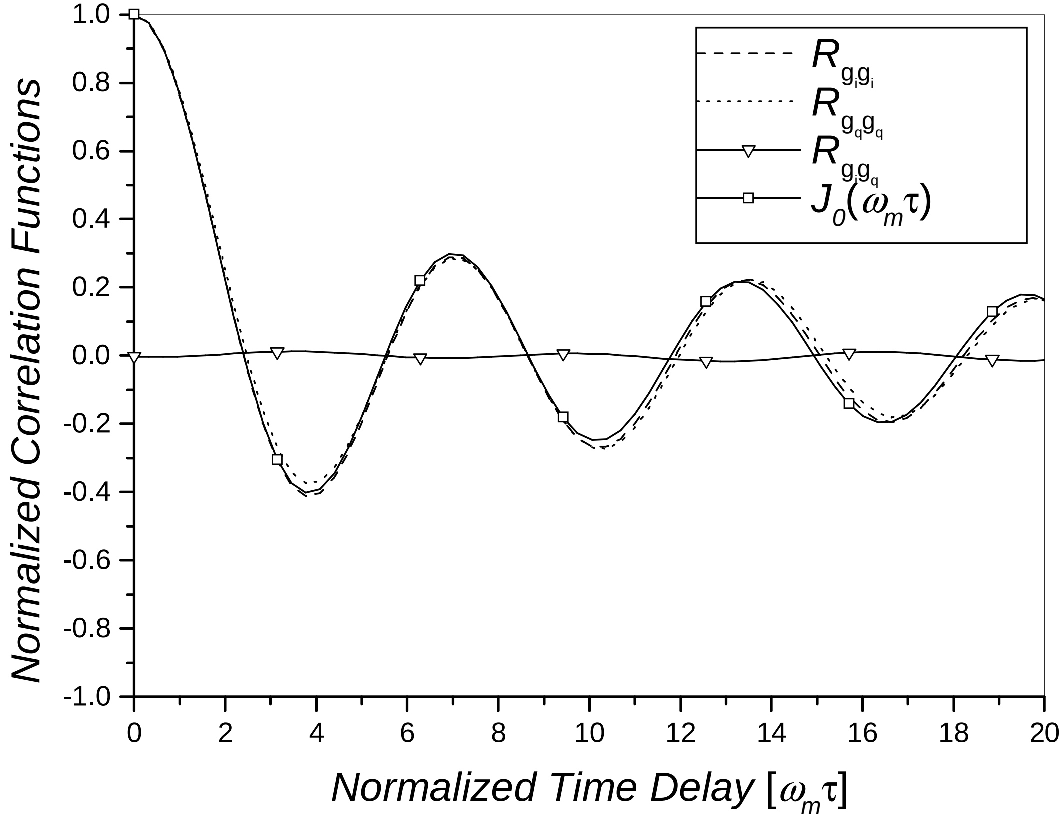
$$g_{I,k}(t) = \frac{2}{\sqrt{N}} \sum_{n=1}^{M} 2\cos(\beta_{nk}) \cos(\omega_m t \cos \alpha_{nk} + \phi_{nk}), \tag{1}$$

$$g_{Q,k}(t) = \frac{2}{\sqrt{N}} \sum_{n=1}^{M} 2\sin(\beta_{nk}) \sin(\omega_m t \sin \alpha_{nk} + \phi_{nk}). \tag{2}$$

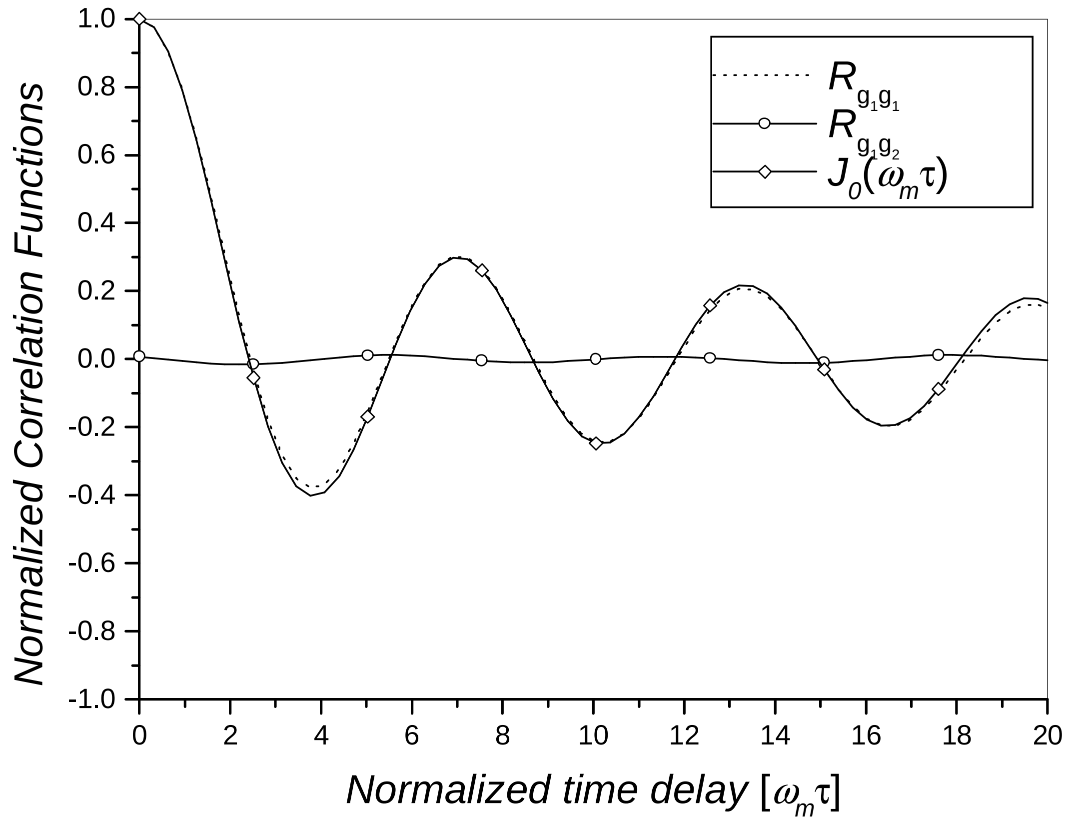
- **It is assumed that** P **independent complex envelopes are desired** $(k = 0, \ldots, P-1)$**, each having** $M = N/4$ **sinusoidal terms in the I and Q components.**

- **The parameters** ϕ_{nk}**,** β_{nk}**, and** θ **are independent random variables uniformly distributed on the interval** $[-\pi, \pi)$**.**

- **The angles of arrival are chosen as follows:** $\alpha_{nk} = (2\pi n)/N + (2\pi k)/(PN) + (\theta - \pi)/N$**, for** $n = 1, \ldots, M$**,** $k = 0, \ldots, P-1$**. The angles of arrival in the** k^{th} **complex faded envelope are obtained by rotating the angles of arrivals in the** $(k-1)^{th}$ **complex envelope by** $(2\pi)/(PN)$**.**



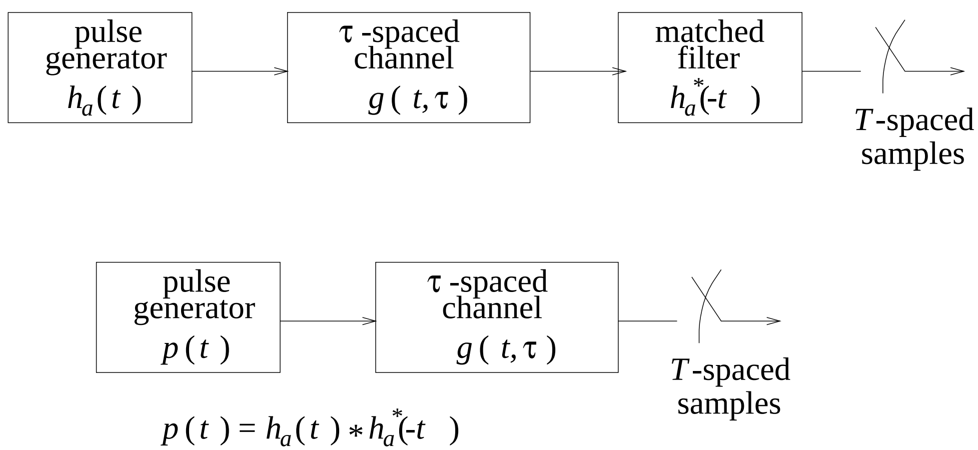
Theoretical and simulated ($N_{stat} = 30$) auto-correlation functions and the cross-correlation function of the in-phase and the quadrature component of the Zajić and Stüber "statistical" model.



Theoretical and simulated ($N_{stat} = 30$) auto-correlation functions and the cross-correlation function of the first and the second fader in the Zajić and Stüber "statistical" model.

Baud-Spaced Channel Models

- **Wideband channel models usually have a delay profile where the delays, τ_i, not related to the modulated symbol duration T. However, time domain simulation models typically have a simulation step size, T_s, that is related to the symbol duration, i.e., $T_s = T/2^k$ for some integer k. Typically, $k = 4, 8, 16$.**
- **We need a T_s-spaced channel model.**
- **Consider the method below showing the generation of a T-spaced channel model.**



Method for generating correlated tap coefficients in a T-spaced channel model.

- **The pulse generator produces pulses having a shape that is determined by the combination of the transmitter and receiver filter, e.g., a raised cosine pulse.**
- **After passing the pulse through the τ-spaced channel, T-spaced samples are extracted. The T-spaced samples are just a linear combination of the taps in the τ-spaced model.**
- **Suppose that a vector of M, T-spaced, tap coefficients is generated in this manner**

$$\mathbf{g}_T(t) = (g_{1T}(t),\ g_{2T}(t),\ \ldots,\ g_{MT}(t))^T$$

- **Then $\mathbf{g}_T(t) = \mathbf{A}\mathbf{g}(t)$, where**

$$\mathbf{g}(t) = (g_1(t),\ g_2(t),\ \ldots,\ g_\ell(t))^T$$

and $\mathbf{A}$ is an $M \times \ell$ real matrix, and $\mathbf{x}^T$ denotes the transpose of $\mathbf{x}$.

- **The value of ℓ is given, and the value of M depends on the length of the channel $g(t,\tau)$ and the pulse $p(t)$.**
- **The entries of the matrix $\mathbf{A}$ are determined by the overall pulse response of the transmitter and receiver filters, the relative power and delays of the rays in the τ-spaced model, and the T-spaced sampler timing phase.**
- **The matrix $\mathbf{A}$ only needs to be generated whenever the relative delays of the rays in the τ-spaced channel and/or the sampler timing phase changes.**

- **The covariance matrix of the T-spaced tap gain vector $\mathbf{g}_T(t)$ is**

$$\begin{aligned}\Phi_{\mathbf{g}_T}(\tau) &= \frac{1}{2}\mathrm{E}\left[\mathbf{g}_T(t+\tau)\mathbf{g}_T^H(t)\right] \\ &= \frac{1}{2}\mathrm{E}\left[\mathbf{A}\mathbf{g}(t+\tau)\mathbf{g}^H(t)\mathbf{A}^T\right] \\ &= \mathbf{A}\Phi_{\mathbf{g}}(\tau)\mathbf{A}^T \quad .\end{aligned}$$

- **For a WSSUS channel with 2-D isotropic scattering**

$$\Phi_{\mathbf{g}_T}(\tau) = \frac{1}{2}\mathbf{A}\boldsymbol{\Omega}\mathbf{A}^T J_0(2\pi f_m \tau)$$

where

$$\boldsymbol{\Omega} \triangleq \mathbf{diag}[\Omega_0,\ \Omega_1, \ldots,\ \Omega_\ell]$$

is an $\ell \times \ell$ diagonal matrix, and $\Omega_k = \mathrm{E}[|g_k(t)|^2]$ is the envelope power that is associated with the kth tap in the τ-spaced model.

Example

- **Suppose that**

$$p(t) = \mathrm{Sa}(\pi t/T) \cdot \frac{\cos(\pi\beta t/T)}{1 - 4\beta^2 t^2/T^2}$$

  where $\beta = 0.35$**.**

- **The** τ**-spaced waveform channel is characterized by two equal strength taps (**$|g_1(t)|^2 = |g_2(t)|^2$**) with a differential delay of** $\tau = |\tau_1(t) - \tau_0(t)|$**.**

- **We wish to generate a 2-tap** T**-spaced channel model with taps** $g_{0T}(t)$ **and** $g_{1T}(t)$**, under the condition that** $\tau = T/4$**.**

- **Let**

$$\begin{aligned} \mathbf{g}(t) &= (g_0(t), g_1(t))^T \\ \mathbf{g}_T(t) &= (g_{0T}(t), g_{1T}(t))^T \end{aligned}$$

  and

$$\mathbf{g}_T(t) = \mathbf{A}\mathbf{g}(t) \quad .$$

- **The matrix A depends on the timing phase of the** T**-spaced samples taken at the output of the pulse generator. In a practical system, the sampler timing phase is determined by the synchronization process in the receiver.**

Example

- **Suppose that the sampler timing phase is chosen so that the T-spaced taps $g_{0T}(t)$ and $g_{1T}(t)$ have equal magnitude. This makes sense since $|g_0(t)|^2 = |g_1(t)|^2$ for the τ-spaced channel in this example.**
- **The entries of matrix A can be obtained by writing (refer to following slide)**

$$\begin{aligned} g_{0T}(t) &= g_0(t)p(\tau/2 - T/2) + g_1(t)p(-\tau/2 - T/2) \\ g_{1T}(t) &= g_0(t)p(\tau/2 + T/2) + g_1(t)p(-\tau/2 + T/2) \end{aligned}$$

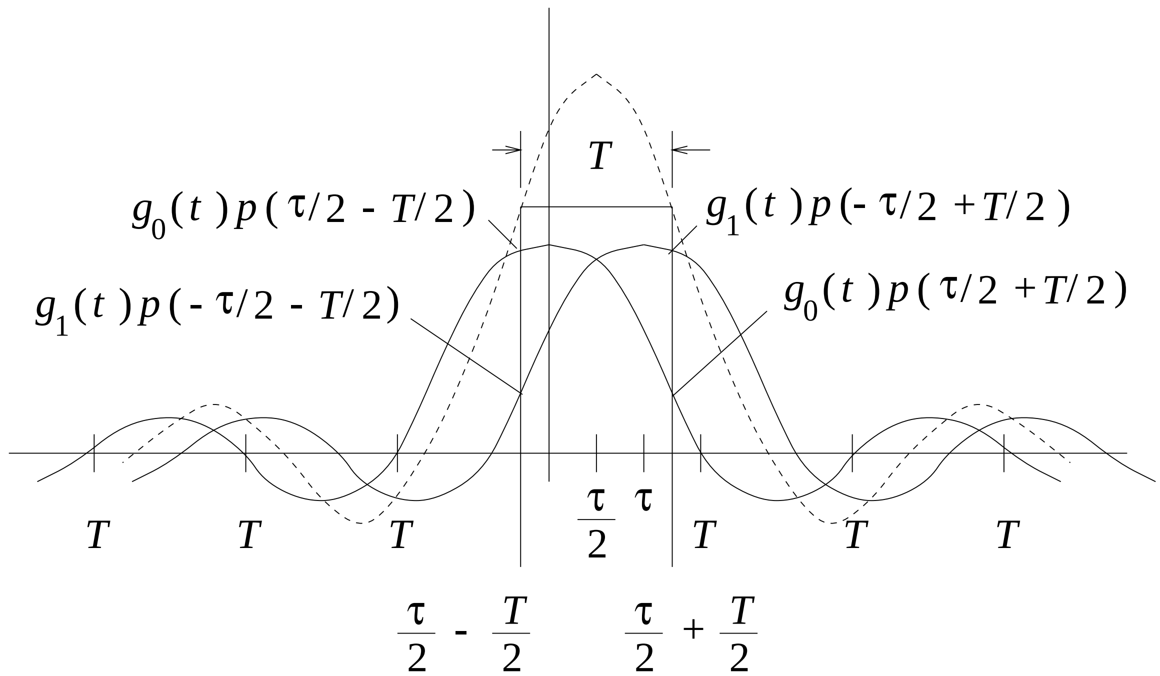
Hence,

$$\mathbf{A} = \begin{bmatrix} p(\tau/2 - T/2) & p(-\tau/2 - T/2) \\ p(\tau/2 + T/2) & p(-\tau/2 + T/2) \end{bmatrix}$$

For $\tau = T/4$ **and** $\beta = 0.35$

$$\mathbf{A} = \begin{bmatrix} p(-3T/8) & p(-5T/8) \\ p(5T/8) & p(3T/8) \end{bmatrix} = \begin{bmatrix} 0.7717 & 0.4498 \\ 0.4498 & 0.7717 \end{bmatrix}$$

Example



Generation T-spaced taps from a τ-spaced model.