

EE6604

Personal & Mobile Communications

Lecture 15

Shadowing, Co-channel Interference, and Outage

Shadowing

- Shadows are very often modeled as being log-normally distributed.
- Let

$$\begin{aligned}\Omega_v &= E[\alpha(t)], & \mu_{\Omega_v} &= E[\Omega_v] \\ \Omega_p &= E[\alpha^2(t)], & \mu_{\Omega_p} &= E[\Omega_p]\end{aligned}$$

- Then distributions of Ω_v and Ω_p are

$$\begin{aligned}p_{\Omega_v}(x) &= \frac{2\xi}{x\sigma_{\Omega}\sqrt{2\pi}} \exp\left\{-\frac{\left(10\log_{10}x^2 - \mu_{\Omega_v} \text{ (dBm)}\right)^2}{2\sigma_{\Omega}^2}\right\} \\ p_{\Omega_p}(x) &= \frac{\xi}{x\sigma_{\Omega}\sqrt{2\pi}} \exp\left\{-\frac{\left(10\log_{10}x - \mu_{\Omega_p} \text{ (dBm)}\right)^2}{2\sigma_{\Omega}^2}\right\}\end{aligned}$$

where

$$\begin{aligned}\mu_{\Omega_v} \text{ (dBm)} &= 30 + 10E[\log_{10}\Omega_v^2] \\ \mu_{\Omega_p} \text{ (dBm)} &= 30 + 10E[\log_{10}\Omega_p]\end{aligned}$$

and $\xi = \ln 10/10$.

Shadowing

- By using a transformation of random variables, $\Omega_v \text{ (dBm)} = 30 + 10\log_{10}\Omega_v^2$ and $\Omega_p \text{ (dBm)} = 30 + 10\log_{10}\Omega_p$ have the Gaussian densities

$$p_{\Omega_v \text{ (dBm)}}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{-\frac{(x - \mu_{\Omega_v \text{ (dBm)}})^2}{2\sigma_\Omega^2}\right\}$$

$$p_{\Omega_p \text{ (dBm)}}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{-\frac{(x - \mu_{\Omega_p \text{ (dBm)}})^2}{2\sigma_\Omega^2}\right\}.$$

- Note that the standard deviation σ_Ω of $\Omega_v \text{ (dBm)}$ and $\Omega_p \text{ (dBm)}$ are the same. However, for Rician fading channels the means differ by

$$\mu_{\Omega_p \text{ (dBm)}} = \mu_{\Omega_v \text{ (dBm)}} + 10 \cdot \log_{10}C(K)$$

where

$$C(K) = \frac{4e^{2K}(K+1)}{\pi {}_1F_1(3/2, 1; K)}$$

${}_1F_1(\cdot, \cdot; \cdot)$ is the confluent hypergeometric function (see Appendix 2A).

Shadow Simulation

- Shadowing can be modeled as a Markov process.
- In a discrete-time simulation, the local mean Ω_{k+1} (dBm) at step $k + 1$ is generated recursively as follows:

$$\Omega_{k+1} \text{ (dBm)} = \xi \Omega_k \text{ (dBm)} + (1 - \xi)v_k$$

- k is the step index.
 - $\{v_k\}$ is a sequence of independent zero-mean Gaussian random variables with variance $\tilde{\sigma}^2$.
 - ξ controls the shadow correlation
- The autocorrelation function of Ω_k (dBm) is

$$\phi_{\Omega_{\text{(dBm)}}\Omega_{\text{(dBm)}}}(n) = \frac{1 - \xi}{1 + \xi} \tilde{\sigma}^2 \xi^{|n|}$$

- The variance of log-normal shadowing is

$$\sigma_{\Omega}^2 = \phi_{\Omega(\text{dBm})\Omega(\text{dBm})}(0) = \frac{1 - \xi}{1 + \xi} \tilde{\sigma}^2$$

- Consequently, we can express the autocorrelation of Ω_k as

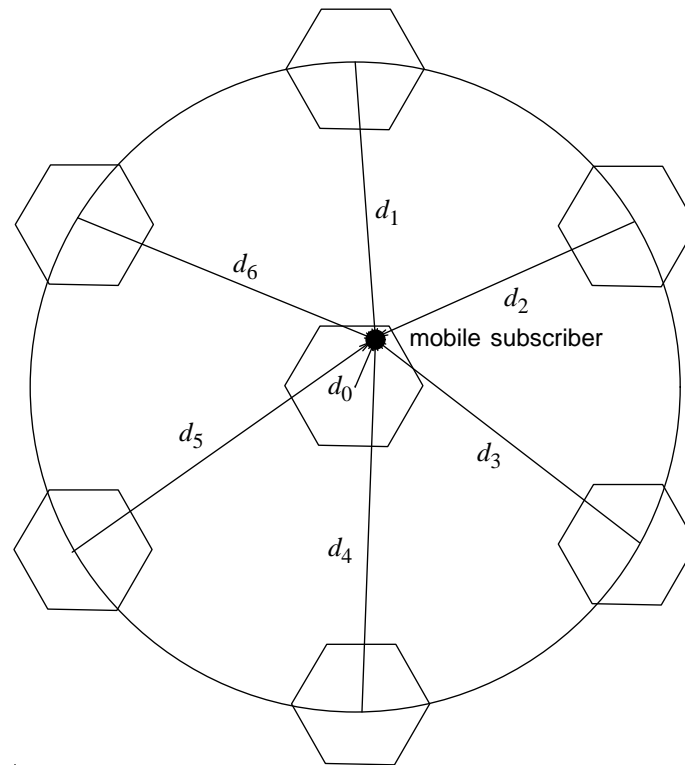
$$\phi_{\Omega(\text{dBm})\Omega(\text{dBm})}(n) = \sigma_{\Omega}^2 \xi^{|n|}$$

- If we use discrete-time simulation, then each simulation step correspond to T seconds.
- For a mobile station traveling at velocity v , the distance traveled in T seconds is vT meters.
- Let ξ_D be the shadow correlation between two points separated by a spatial distance of D meters.
- Then the time autocorrelation of the shadowing is

$$\phi_{\Omega(\text{dBm})\Omega(\text{dBm})}(k) \equiv \phi_{\Omega(\text{dBm})\Omega(\text{dBm})}(kT) = \sigma_{\Omega}^2 \xi_D^{(vT/D)|k|}$$

- Typically, $\xi_D = 0.1$ for $D = 30$ meters. However, this can vary greatly depending on local topography.

Co-channel interference on the forward channel



The mobile station is being served by the center base station.

- At a particular location, let $\mathbf{d} = (d_0, d_1, \dots, d_{N_I})$ be the vector of distances between a mobile station and the serving base station BS_0 and N_I co-channel base stations $\text{BS}_k, k = 1, \dots, N_I$.

- The received signal power power at distance d , $\Omega_p \text{ (dBm)}(d)$, is a Gaussian random variable that depends on the distance d through the path loss model, i.e.,

$$\mu_{\Omega_p \text{ (dBm)}}(d) = \text{E}[\Omega_p \text{ (dBm)}(d)] = \mu_{\Omega_p \text{ (dBm)}}(d_o) - 10\beta \log_{10}(d/d_o)$$

- Experiments have verified that co-channel interferers add noncoherently (power addition) rather than coherently (amplitude addition).
- The C/I a function of the vector \mathbf{d} is

$$\Lambda(\mathbf{d}) = \frac{\Omega_p(d_0)}{\sum_{k=1}^{N_I} \Omega_p(d_k)}$$

or in decibel units

$$\Lambda(\mathbf{d})_{\text{(dB)}} = \Omega_p \text{ (dBm)}(d_0) - 10 \log_{10} \left(\sum_{k=1}^{N_I} \Omega_p(d_k) \right)$$

- The outage probability given vector \mathbf{d} is

$$O(\mathbf{d}) = \text{P}_r \left(\Lambda(\mathbf{d})_{\text{(dB)}} < \Lambda_{\text{th(dB)}} \right)$$

- Although the $\Omega_p(d_k)$ are log-normal random variables, the sum $\sum_{k=1}^{N_I} \Omega_p(d_k)$ is *not* a log-normal random variable.

Multiple Log-normal Interferers

- Consider the sum of N_I log-normal random variables

$$I = \sum_{k=1}^{N_I} \Omega_k = \sum_{k=1}^{N_I} 10^{\Omega_k(\text{dBm})/10}$$

where the Ω_k (dBm) are Gaussian with mean μ_{Ω_k} (dBm) and variance $\sigma_{\Omega_k}^2$.

- The sum I can be approximated by another log-normal random variable with appropriately chosen parameters, i.e.,

$$I = \sum_{k=1}^{N_I} 10^{\Omega_k(\text{dBm})/10} \approx 10^{Z(\text{dBm})/10} = \hat{I}$$

where Z (dBm) is a Gaussian random variable with mean μ_Z (dBm) and variance σ_Z^2 .

Fenton-Wilkinson Method

- The mean μ_Z (dBm) and variance σ_Z^2 of $Z_{(\text{dBm})}$ are obtained by matching the first two moments of I and \hat{I} .

- Using natural logarithms

$$\Omega_k = 10^{\Omega_k (\text{dBm})/10} = e^{\xi \Omega_k (\text{dBm})} = e^{\hat{\Omega}_k}$$

where $\xi = (\ln 10)/10 = 0.23026$.

- Note that

$$\begin{aligned}\mu_{\hat{\Omega}_k} &= \xi \mu_{\Omega_k (\text{dBm})} \\ \sigma_{\hat{\Omega}_k}^2 &= \xi^2 \sigma_{\Omega_k}^2\end{aligned}$$

- The n th moment of the log-normal random variable Ω_k can be obtained from the moment generating function of the Gaussian random variable $\hat{\Omega}_k$ as

$$E[\Omega_k^n] = E[e^{n\hat{\Omega}_k}] = e^{n\mu_{\hat{\Omega}_k} + (1/2)n^2\sigma_{\hat{\Omega}_k}^2}$$

– Here we have assumed identical variances, $\sigma_{\hat{\Omega}_k}^2 = \sigma_{\hat{\Omega}}^2$.

- Suppose that $\hat{\Omega}_1, \dots, \hat{\Omega}_{N_I}$ are independent with means $\mu_{\hat{\Omega}_1}, \dots, \mu_{\hat{\Omega}_{N_I}}$ and identical variances $\sigma_{\hat{\Omega}}^2$.
- The appropriate moments of the log-normal approximation are obtained by equating the means on both sides of

$$\mu_I = \mathbb{E}[I] = \sum_{k=1}^{N_I} \mathbb{E}[e^{\hat{\Omega}_k}] \approx \mathbb{E}[e^{\hat{Z}}] = \mathbb{E}[\hat{I}] = \mu_{\hat{I}}$$

where $\hat{Z} = \xi Z_{(\text{dBm})}$.

- This gives

$$\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right) e^{(1/2)\sigma_{\hat{\Omega}}^2} = e^{\mu_{\hat{Z}} + (1/2)\sigma_{\hat{Z}}^2} \quad (1)$$

- Also equate the variances on both sides of

$$\sigma_I^2 = \mathbb{E}[I^2] - \mu_I^2 \approx \mathbb{E}[\hat{I}^2] - \mu_{\hat{I}}^2 = \sigma_{\hat{I}}^2$$

- This gives

$$\left(\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}} \right) e^{\sigma_{\hat{\Omega}}^2} (e^{\sigma_{\hat{\Omega}}^2} - 1) = e^{2\mu_{\hat{Z}}} e^{\sigma_{\hat{Z}}^2} (e^{\sigma_{\hat{Z}}^2} - 1) \quad (2)$$

- To obtain $\mu_{\hat{Z}}$ and $\sigma_{\hat{Z}}^2$
 1. Square the Eq. (1) and divide by Eq. (2) to obtain $\sigma_{\hat{Z}}^2$.
 2. Obtain $\mu_{\hat{Z}}$ from Eq. (1)
- The above procedure yields

$$\mu_{\hat{Z}} = \frac{\sigma_{\hat{\Omega}}^2 - \sigma_{\hat{Z}}^2}{2} + \ln \left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right)$$

$$\sigma_{\hat{Z}}^2 = \ln \left((e^{\sigma_{\hat{\Omega}}^2} - 1) \frac{\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}}{\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right)^2} + 1 \right)$$

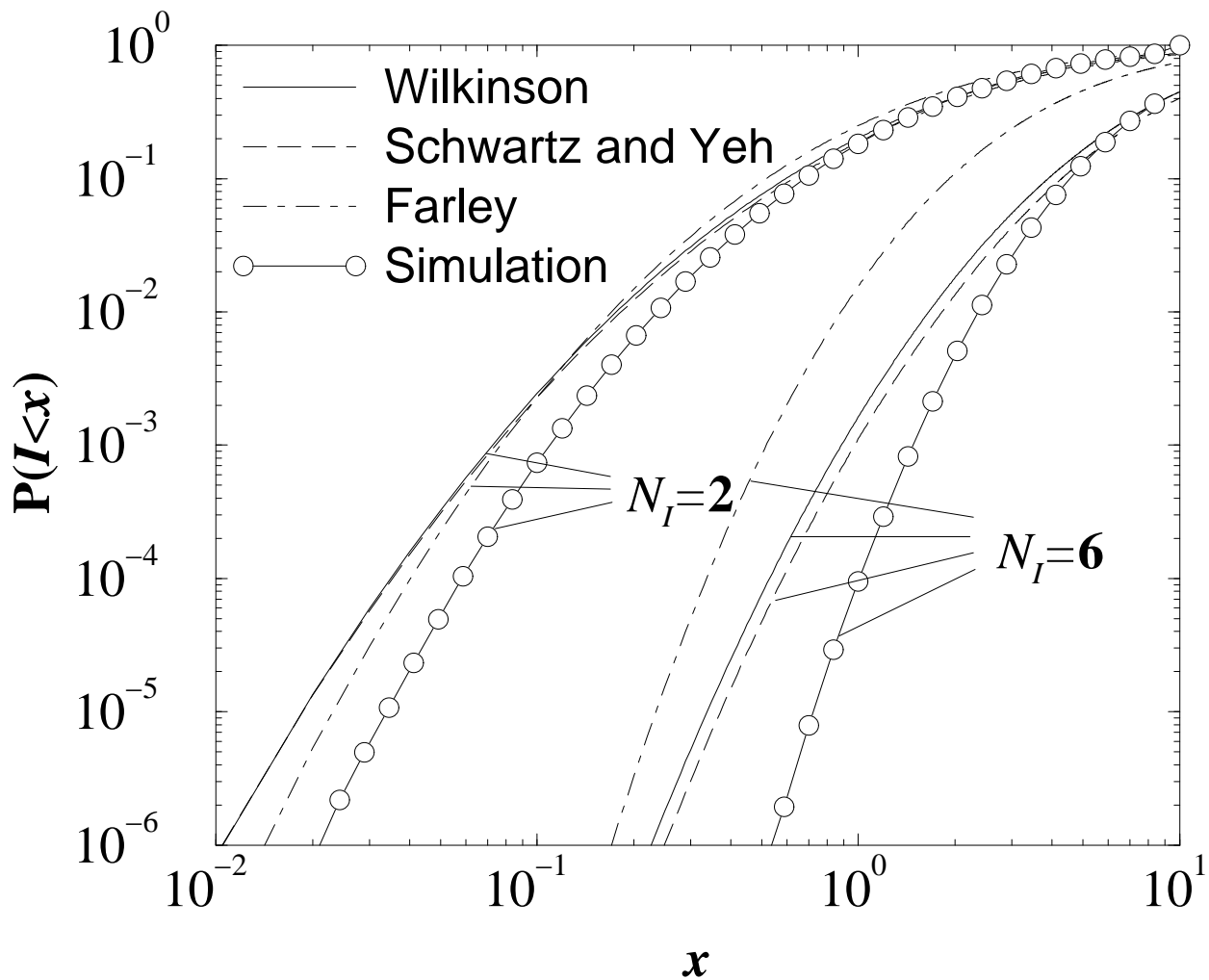
- Fenton's method breaks down in the prediction of the first and second moments for $\sigma_\Omega > 4$ dB.
 - Schwartz and Yeh's method yields the exact first and second moments.
- However, Fenton's method accurately predicts the tails of the complementary distribution function *cdfc* $F_I^c(x) = \text{P}_r(I \geq x)$ and the *cdf* $F_I(x) = 1 - F_I^c(x) = \text{P}_r(I < x)$.
 - We are interested in the accuracy of the approximations

$$F_I(x) \approx Q\left(\frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}}\right)$$

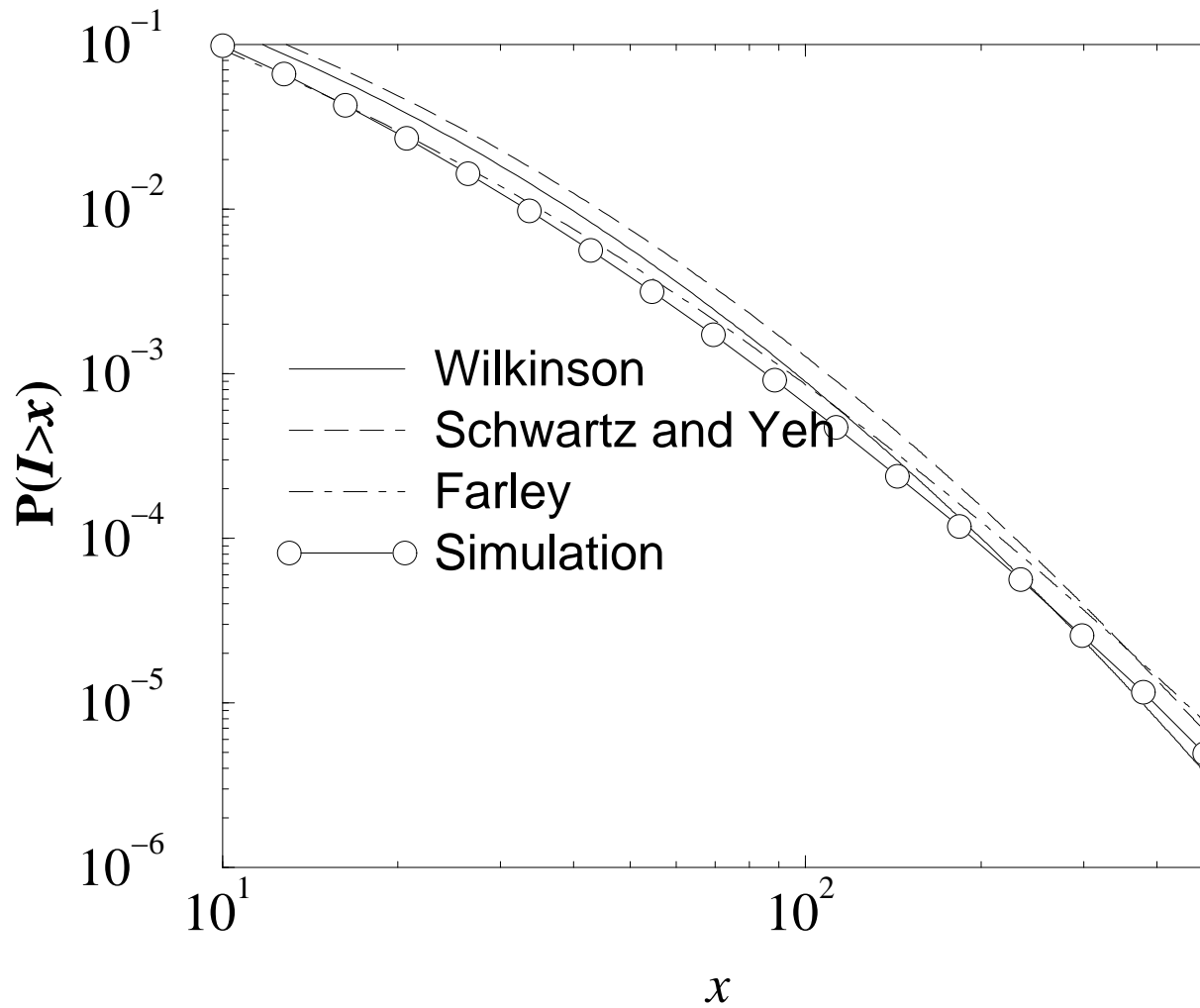
$$F_I^c(x) \approx 1 - Q\left(\frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}}\right)$$

when x is large and small, respectively.

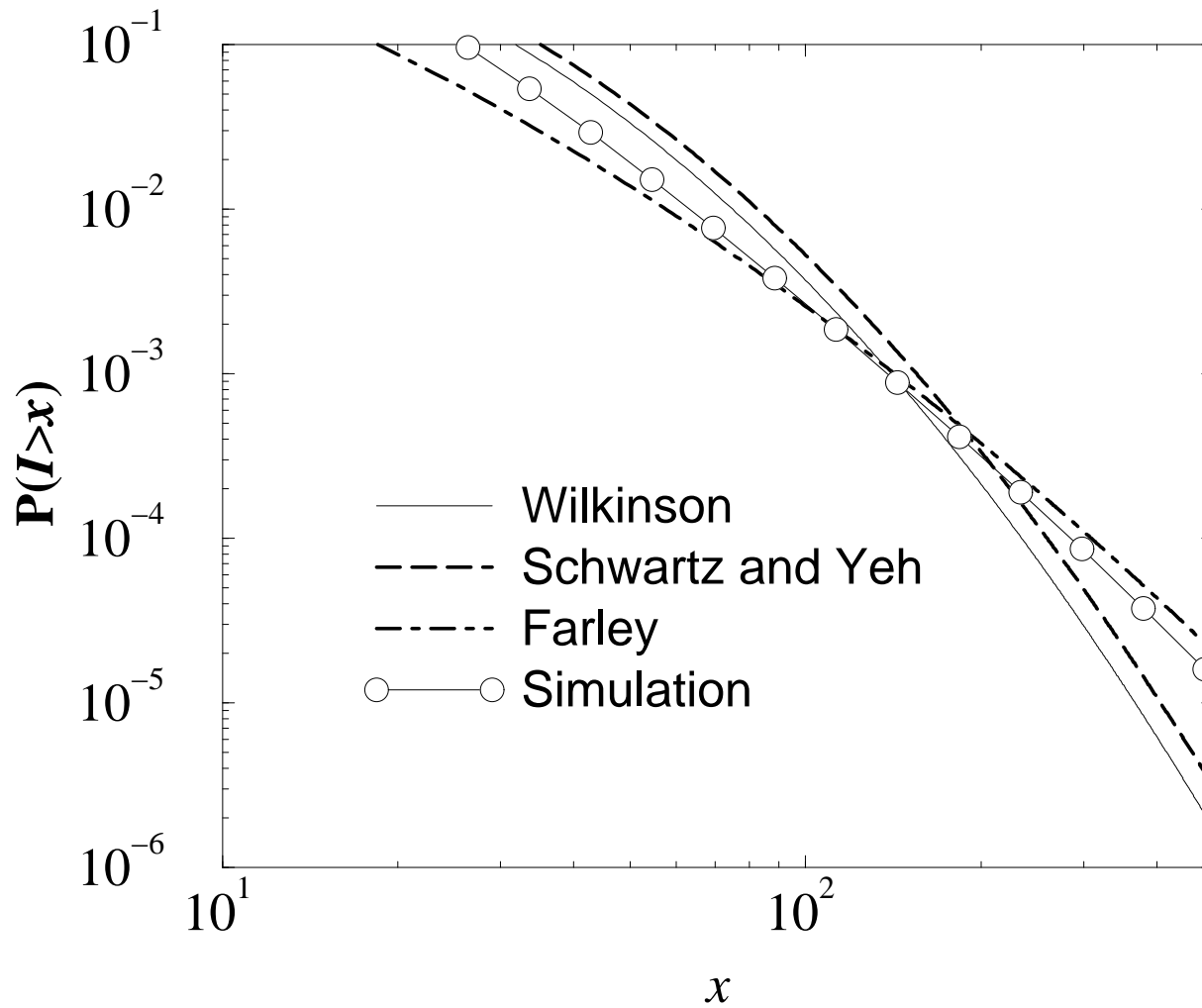
- The cdf and cdfc are more important for outage predictions.



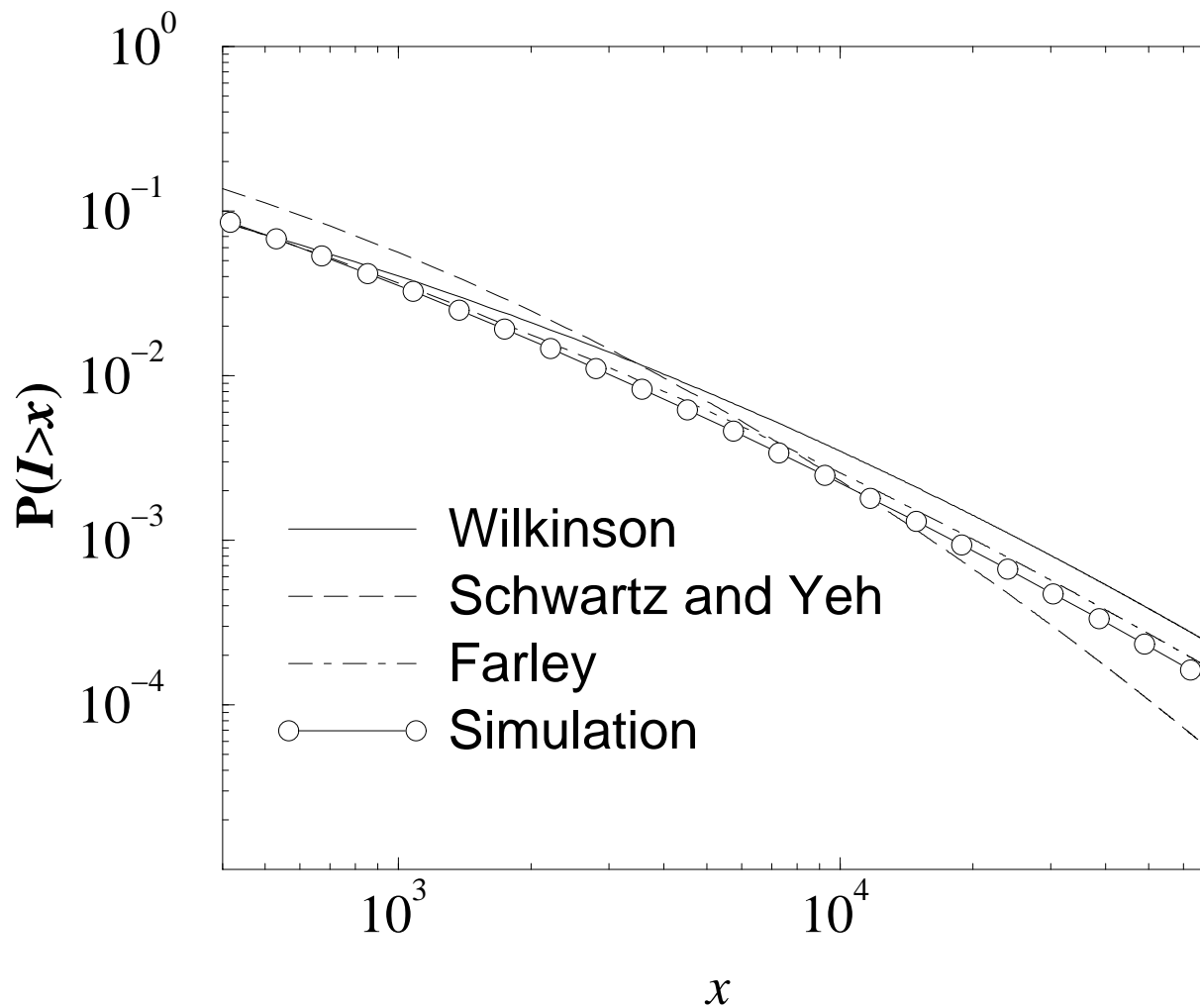
Comparison of the cdf for the sum of two and six log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.



Comparison of the cdfc for the sum of two log-normal random variables with various approximations; $\sigma_{\Omega} = 6$ dB.



Comparison of the cdf for the sum of six log-normal random variables with various approximations; $\sigma_{\Omega} = 6$ dB.



Comparison of the cdf for the sum of six log-normal random variables with various approximations; $\sigma_{\Omega} = 12$ dB.

Outage with Multiple Interferers

1. First obtain the mean and variance

$$\begin{aligned}\mu_Z &= \mu_{\hat{Z}}/\xi \\ \sigma_Z^2 &= \sigma_{\hat{Z}}^2/\xi^2 \quad \zeta = 0.23026\end{aligned}$$

2. Treat the average CIR as Gaussian distributed with mean and variance

$$\begin{aligned}\mu_{\Lambda(\mathbf{d})} &= \mu_{\Omega(d_0)} - \mu_Z \\ \sigma_{\Lambda(\mathbf{d})}^2 &= \sigma_{\Omega}^2 + \sigma_Z^2 .\end{aligned}$$

3. Compute the outage for a given location, described by \mathbf{d}

$$O(\mathbf{d}) = Q\left(\frac{\mu_{\Omega(d_0)} - \mu_Z - \Lambda_{\text{th(dB)}}}{\sqrt{\sigma_{\Omega}^2 + \sigma_Z^2}}\right)$$

4. Average over all locations \mathbf{d} by Monte Carlo integration

$$O = \int_{R^N} O(\mathbf{d})p_{\mathbf{d}}(\mathbf{d})d\mathbf{d}$$

Single Co-channel Interferer

- For a single co-channel interferer

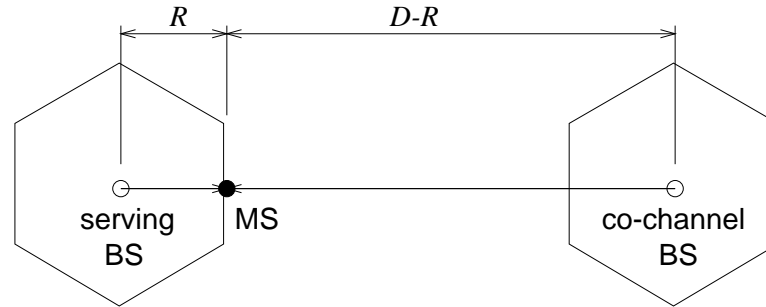
$$p_{\Lambda(\mathbf{d})_{(\text{dB})}}(x) = \frac{1}{\sqrt{4\pi}\sigma_{\Omega}} \exp \left\{ -\frac{(x - \mu_{\Lambda(\mathbf{d})_{(\text{dB})}})^2}{4\sigma_{\Omega}^2} \right\}$$

where

$$\mu_{\Lambda(\mathbf{d})_{(\text{dB})}} = \mu_{\Omega(d_0)_{(\text{dB})}} - \mu_{\Omega(d_1)_{(\text{dB})}}$$

- The outage for a given \mathbf{d} is

$$\begin{aligned} O(\mathbf{d}) &= \Pr(\Lambda(\mathbf{d})_{(\text{dB})} < \Lambda_{\text{th}(\text{dB})}) \\ &= \int_{-\infty}^{\Lambda_{\text{th}(\text{dB})}} \frac{1}{\sqrt{4\pi}\sigma_{\Omega}} \exp \left\{ -\frac{(x - \mu_{\Lambda(\mathbf{d})_{(\text{dB})}})^2}{4\sigma_{\Omega}^2} \right\} dx \\ &= Q \left(\frac{\mu_{\Lambda(\mathbf{d})_{(\text{dB})}} - \Lambda_{\text{th}(\text{dB})}}{2\sigma_{\Omega}} \right) \end{aligned}$$



Worst case interference from a single co-channel base-station.

- In this case $\mathbf{d} = (R, D - R)$.
- The worst case outage due to a single co-channel interferer is

$$O(R) = Q \left(\frac{\mu_{\Omega(R)}(\text{dB}) - \mu_{\Omega(D-R)}(\text{dB}) - \Lambda_{\text{th}}(\text{dB})}{2\sigma_{\Omega}} \right)$$

- Using a simple inverse- β path loss characteristic

$$\mu_{\Omega(\text{dB})} = \Omega_{(\text{dB})}(d_o) - 10\beta \log_{10}(d/d_o)$$

gives

$$O(R) = Q \left(\frac{10 \log_{10} \left(\frac{D}{R} - 1 \right)^\beta - \Lambda_{\text{th}} (\text{dB})}{2\sigma_\Omega} \right)$$

- The minimum CIR margin on the cell fringe is

$$M_\Lambda = 10 \log_{10} \left(\frac{D}{R} - 1 \right)^\beta - \Lambda_{\text{th}} (\text{dB})$$

- For an ideal hexagonal layout $\frac{D}{R} = \sqrt{3N}$, so that

$$N = \frac{1}{3} \left[10^{\frac{M_\Lambda + \Lambda_{\text{th}} (\text{dB})}{10\beta}} + 1 \right]^2$$

- A small cluster size is achieved by making the margin M_Λ and receiver threshold Λ_{th} small.

Rician/Multiple Rayleigh Interferers

- Sometimes propagation conditions exist such that the received signals experience fading, but not shadowing. In this section, we calculate the outage probability for the case of fading only.
 - The received signal may consist of a direct line of sight (LoS) component, or perhaps a specular component, accompanied by a diffuse component. The envelope of the received desired signal experiences Rician fading.
 - The interfering signals are often assumed to be Rayleigh faded, because a direct LoS is unlikely to exist due to the larger physical distances between the co-channel interferers and the receiver.
- Let the instantaneous power in the desired signal and the N_I interfering signals be denoted by s_0 and s_k , $k = 1, \dots, N_I$, respectively. Note that $s_i = \alpha_i^2$, where α_i^2 is the squared-envelope.
- The carrier-to-interference ratio is defined as $\lambda = s_0 / \sum_{k=1}^{N_I} s_k$, and for a specified receiver threshold λ_{th} , the outage probability is

$$O_I = \text{P}(\lambda < \lambda_{\text{th}}) \ .$$

Single Interferer

- For the case of a single interferer, the outage probability reduces to the simple closed form

$$O_I = \frac{\lambda_{\text{th}}}{\lambda_{\text{th}} + A_1} \exp \left\{ -\frac{K A_1}{\lambda_{\text{th}} + A_1} \right\} ,$$

where K is the Rice factor of the desired signal, $A_1 = \Omega_0 / (K + 1)\Omega_1$, and $\Omega_k = \text{E}[s_k]$.

- Note that A_1 can be interpreted as the ratio of the average desired signal power to the total interfering power.
- If the desired signal is Rayleigh faded, then the outage probability can be obtained by setting $K = 0$.

Multiple Interferers

- For the case of multiple interferers, each with mean power Ω_k , the outage probability has the closed form

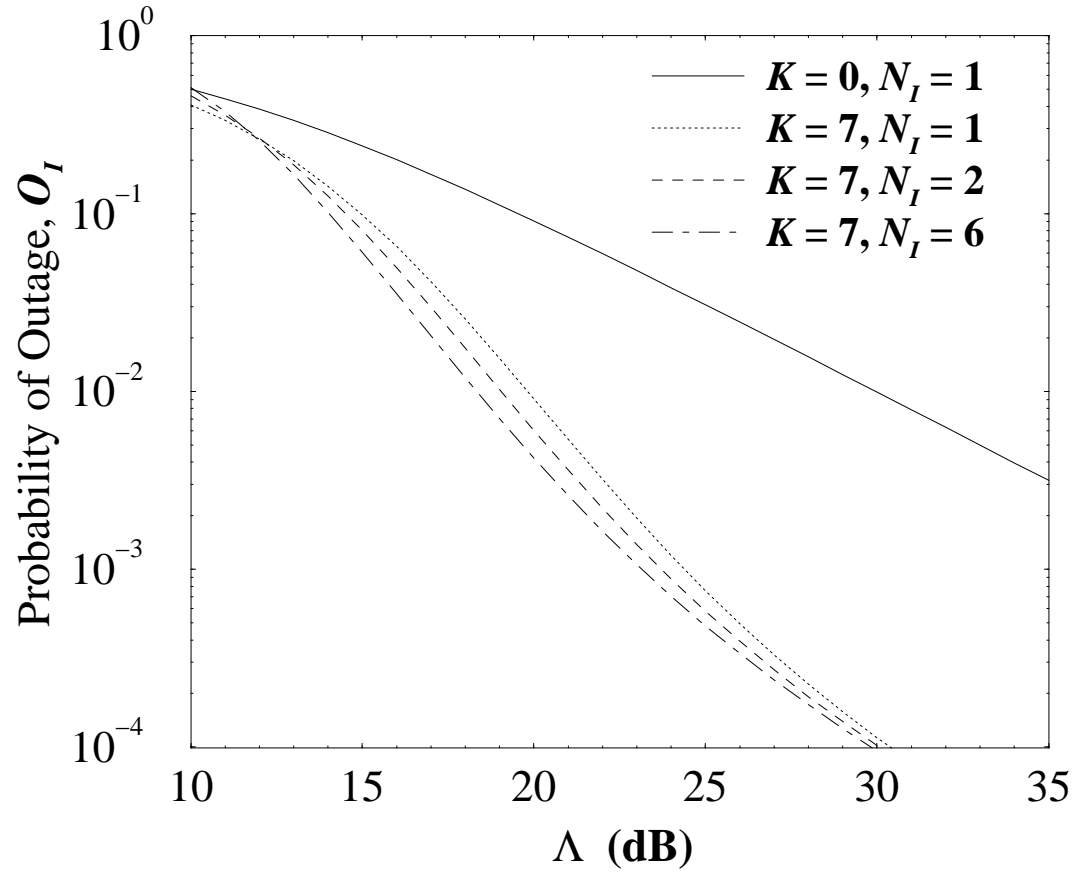
$$O_I = 1 - \sum_{k=1}^{N_I} \left[1 - \frac{\lambda_{\text{th}}}{\lambda_{\text{th}} + A_k} \exp \left\{ -\frac{K A_k}{\lambda_{\text{th}} + A_k} \right\} \right] \prod_{\substack{j=1 \\ j \neq k}}^{N_I} \frac{A_j}{A_j - A_k} ,$$

where $A_k = \Omega_0 / (K + 1) \Omega_k$. This expression is only valid if $\Omega_i \neq \Omega_j$ when $i \neq j$, i.e., the different interferers have different mean power.

- If all the interferers have the same mean power, then the outage probability can be derived as

$$O_I = \frac{\lambda_{\text{th}}}{\lambda_{\text{th}} + A_1} \exp \left\{ -\frac{K A_1}{\lambda_{\text{th}} + A_1} \right\} \\ \times \sum_{k=0}^{N_I-1} \left(\frac{A_1}{(\lambda_{\text{th}} + A_1)} \right)^k \sum_{m=0}^k \binom{k}{m} \frac{1}{m!} \left(\frac{K \lambda_{\text{th}}}{\lambda_{\text{th}} + A_1} \right)^m .$$

- If the desired signal is Rayleigh faded, then the probability of outage with multiple Rayleigh faded interferers can be obtained by setting $K = 0$.



Probability of outage with multiple interferers. The desired signal is Ricean faded with various Rice factors, while the interfering signals are Rayleigh faded and of equal power; $\lambda_{\text{th}} = 10.0$ dB.

$$\Lambda = \frac{\Omega_0}{N_I \Omega_1}$$