EE6604

Personal & Mobile Communications

Lecture 17

CPM and Orthogonal Multipulse

Continuous Phase Modulation (CPM)

- **The CPM bandpass signal is**

$$s(t) = \mathrm{Re}\left\{ A e^{j\phi(t)} e^{j2\pi f_c t} \right\} = A\cos\left(2\pi f_c t + \phi(t)\right) \quad (1)$$

where the excess phase is

$$\phi(t) = 2\pi \int_0^t \sum_{k=0}^{\infty} h_k x_k h_f(\tau - kT) d\tau$$

- $x_n \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$ **are the** M**-ary data symbols**
- $\{h_k\}$ **is the sequence of modulation indices. When** $h_k = h$ **the modulation index is fixed for all symbols. With multi-**h **CPM, the sequence** $\{h_k\}$ **is chosen in a cyclic fashion from set** $\{\hat{h}_1, \hat{h}_2, \ldots, \hat{h}_H\}$ **of** H **modulation indices. That is,** $h_{i+H} = h_i$**.**
- $h_f(t)$ **is the frequency shaping pulse of duration** LT**.**

Frequency Shaping Pulses

pulse type	$h_f(t)$
L-rectangular (LREC)	$\frac{1}{2LT} u_{LT}(t)$
L-raised cosine (LRC)	$\frac{1}{2LT} \left[1 - \cos\left(\frac{2\pi t}{T}\right)\right] u_{LT}(t)$
L-half sinusoid (LHS)	$\frac{\pi}{4LT} \sin(\pi t/T) u_{LT}(t)$
L-triangular (LTR)	$\frac{1}{LT} \left(1 - \frac{\lvert t-LT/2 \rvert}{LT/2}\right)$

Excess Phase and Tilted Phase

- **Assume single h modulation. During the time interval $nT \leq t \leq (n+1)T$, the excess phase $\phi(t)$ is**

$$\phi(t) = 2\pi h \sum_{k=0}^{n} x_k \beta(t - kT).$$

where the phase shaping pulse is

$$\beta(t) = \begin{cases} 0 & , \ t < 0 \\ \int_0^t h_f(\tau) d\tau & , \ 0 \leq t \leq LT \\ 1/2 & , \ t \geq LT \end{cases}$$

- **For the case of full response CPM $(L = 1)$, during the time interval $nT \leq t \leq (n+1)T$ the excess phase is**

$$\phi(t) = \pi h \sum_{k=0}^{n-1} x_k + 2\pi h x_n \beta(t - nT)$$

- **During the time interval $nT \leq t \leq (n+1)T$, the CPM tilted phase is**

$$\psi(t) = 2\pi h \sum_{k=0}^{n} x_k \beta(t - kT) + \pi h(M-1)t/T$$

- **Note that $s(t)$ can be generated by replacing $\phi(t)$ with $\psi(t)$ and f_c by $f_c - h(M-1)t/2T$ in (1).**

Continuous Phase Frequency Shift Keying (CPFSK)

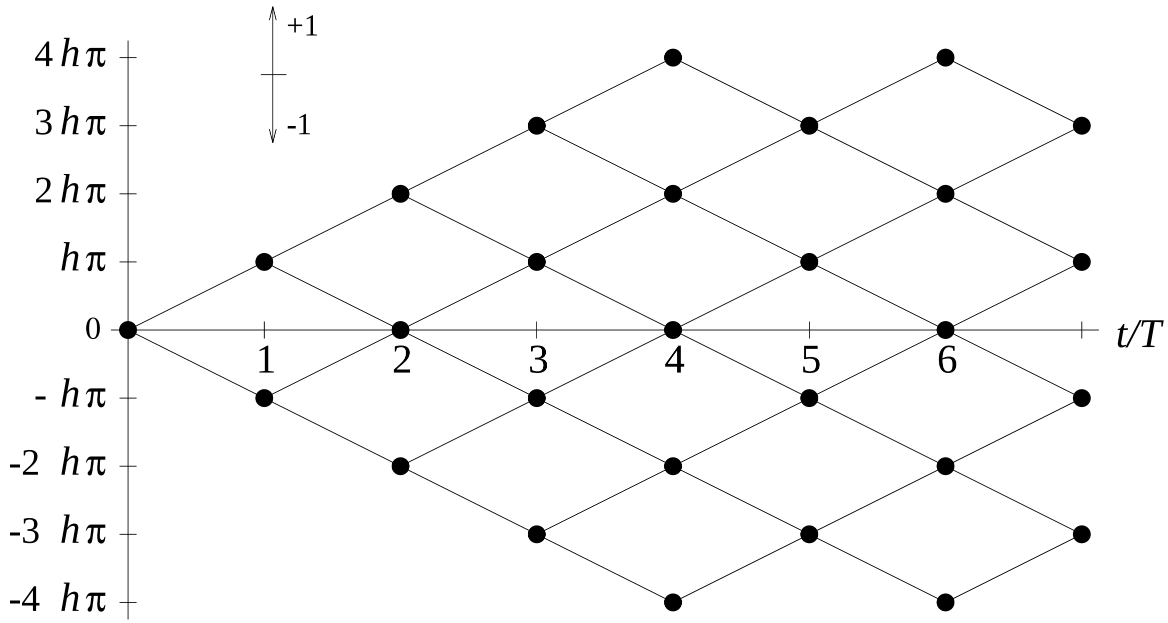
- **Continuous phase frequency shift keying (CPFSK) is a special type of CPM that uses the shaping function**

$$h_f(t) = \frac{1}{2T} u_T(t) = \frac{1}{2T} \left(u(t) - u(t - T) \right)$$

As a result

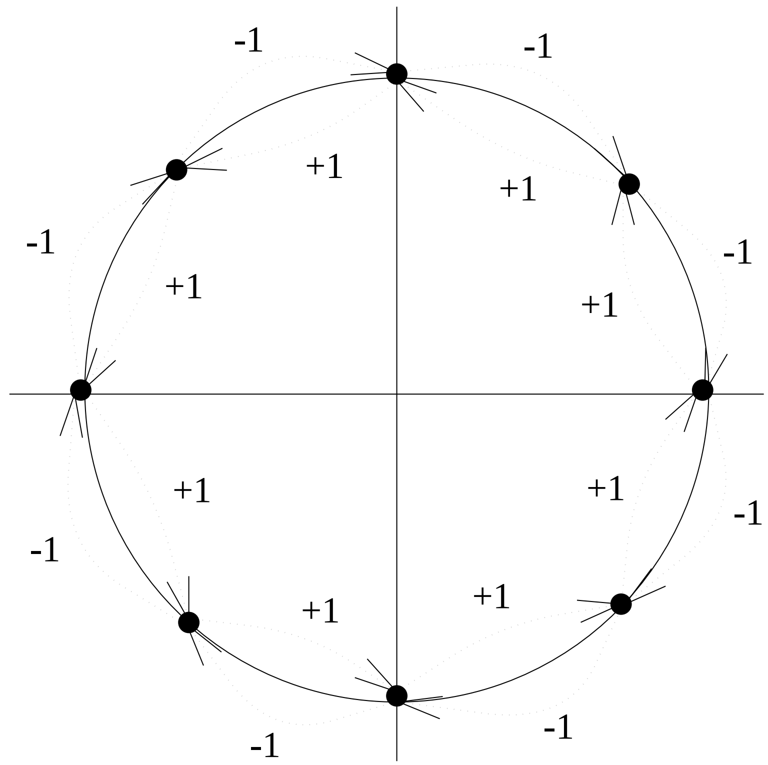
$$\beta(t) = \begin{cases} 0 & , \ t < 0 \\ t/2T & , \ 0 \leq t \leq T \\ 1/2 & , \ t \geq T \end{cases}$$

- **Since the shaping function is rectangular, the CPFSK excess phase trajectories are linear.**



Phase tree of binary CPFSK.

Phase-state Diagrams



Phase-state diagram of CPM with $h = 1/4$.

Minimum Shift Keying (MSK)

- **MSK is a special case of continuous phase frequency shift keying (CPFSK), where the modulation index $h = \frac{1}{2}$ is used.**
- **The phase shaping pulse is**

$$\beta(t) = \begin{cases} 0 & , \ t < 0 \\ t/2T & , \ 0 \leq t \leq T \\ 1/2 & , \ t \geq T \end{cases}$$

- **The MSK bandpass waveform is**

$$s(t) = A \cos\left(2\pi f_c t + + \frac{\pi}{2} \sum_{k=0}^{n-1} x_k + \frac{t - nT}{2T} \pi x_n\right) \quad , \quad nT \leq t \leq (n+1)T$$

- **The excess phase on the interval $nT \leq t \leq (n+1)T$ is**

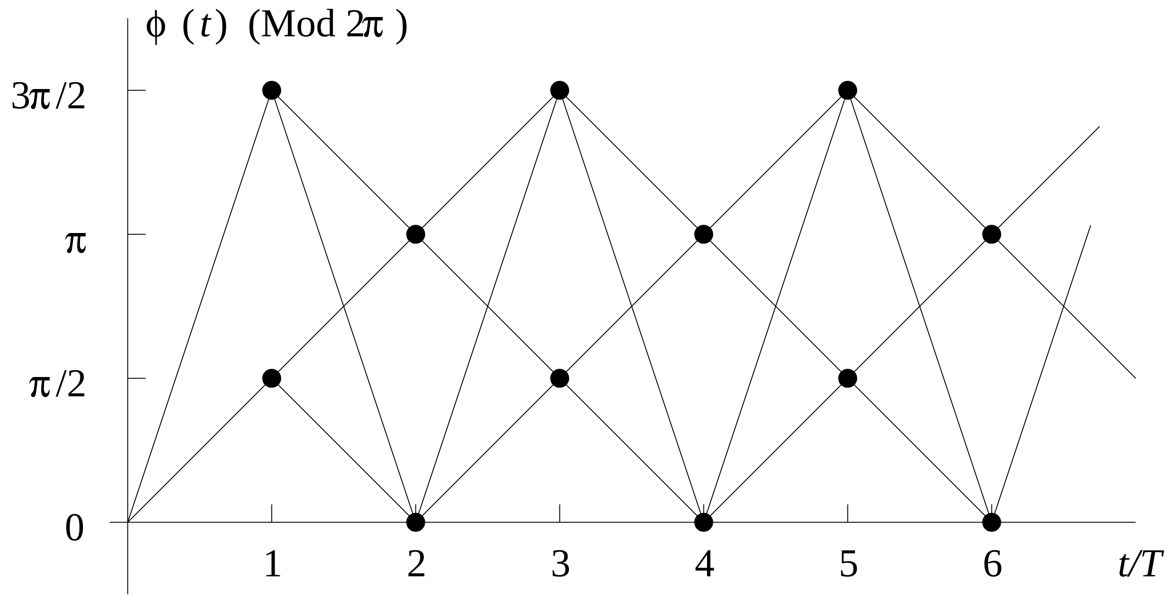
$$\phi(t) = \frac{\pi}{2} \sum_{k=0}^{n-1} x_k + \frac{t - nT}{2T} \pi x_n$$

- **The tilted phase on the interval $nT \leq t \leq (n+1)T$ is**

$$\psi(t) = \phi(t) + \frac{\pi t}{2T}$$

- **From this, we have**

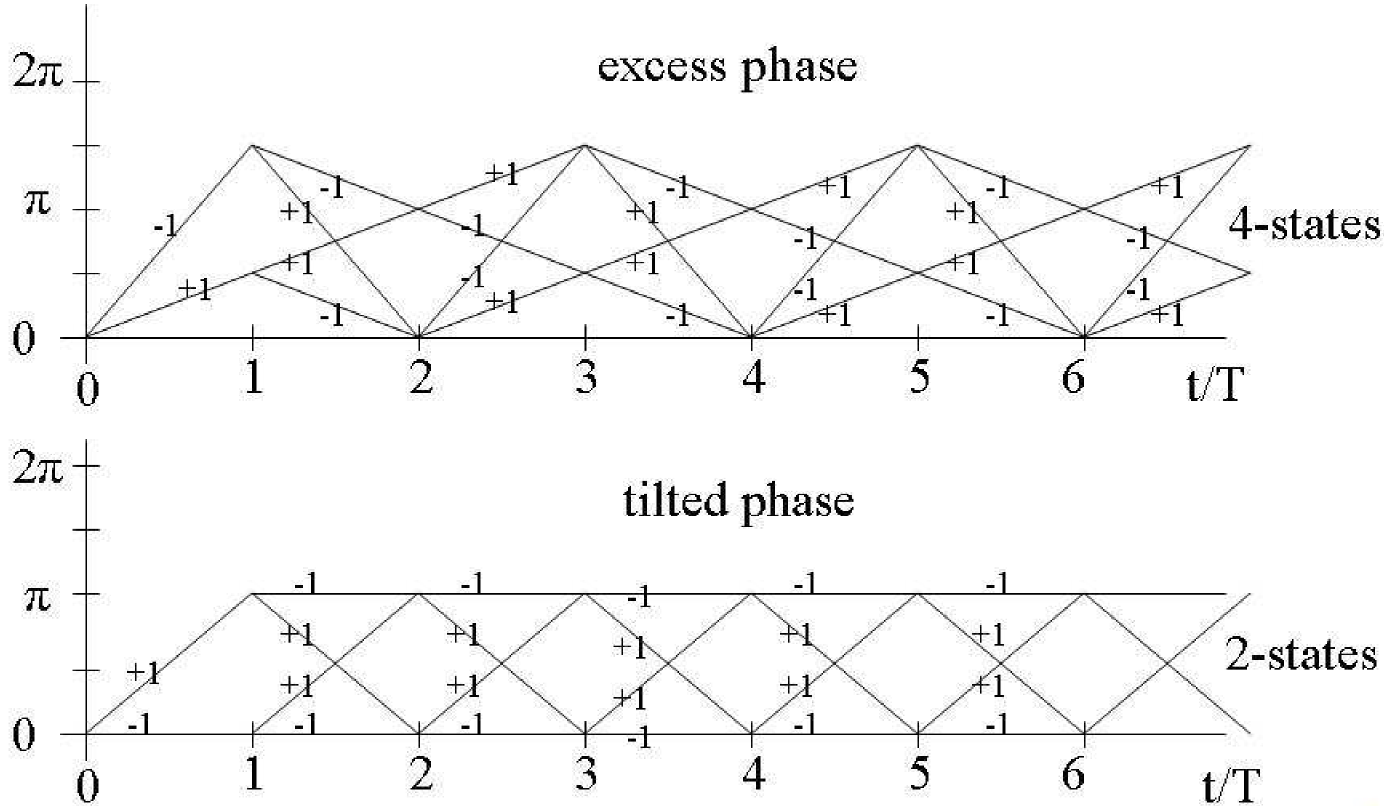
$$\psi((n+1)T) = \psi(nT) + \frac{\pi}{2}(1 + x_n)$$



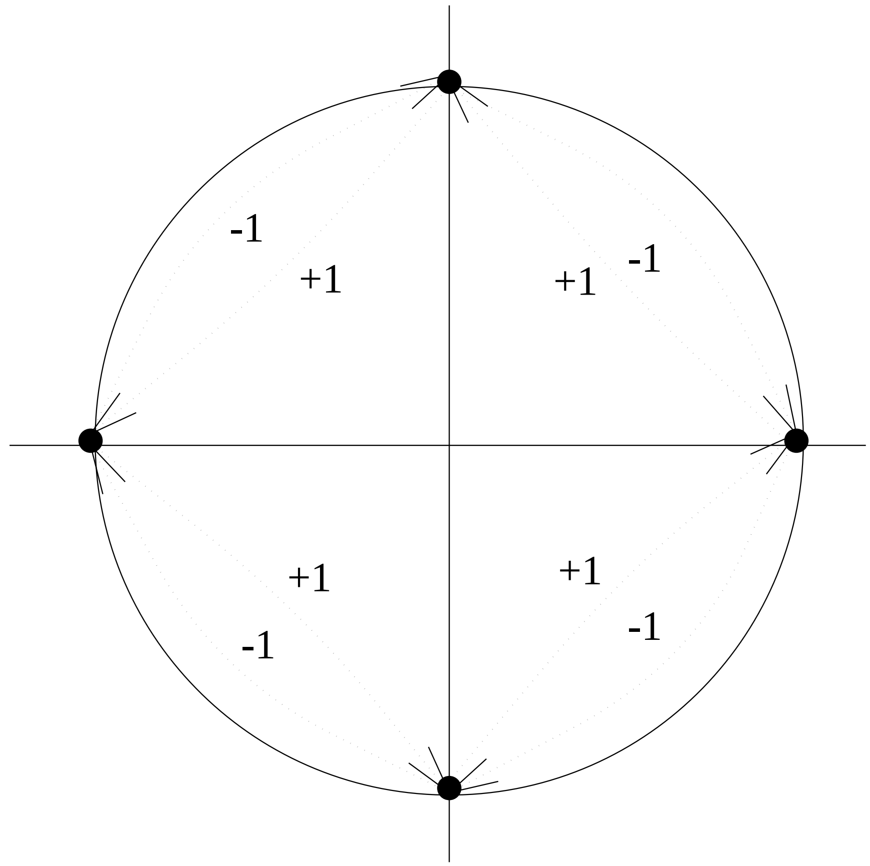
excess phase trellis diagram for MSK.

❖Excess Phase vs. Tilted Phase

- **Example: MSK (h=1/2)**



Excess Phase and Tilted Phase, example MSK ($h = 1/2$).



Phase state diagram for MSK signals.

- **Using various trig identities**

$$\begin{aligned} s(t) &= A\left[x_k^I \cos\left(\frac{\pi t}{2T}\right)\cos\left(2\pi f_c t\right)\right. \\ &\qquad \left. - x_k^Q \sin\left(\frac{\pi t}{2T}\right)\sin\left(2\pi f_c t\right)\right] \quad , \quad kT \le t \le (k+1)T \end{aligned}$$

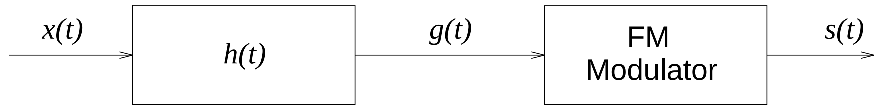
where

$$\begin{aligned} x_k^I &= -x_{k-1}^Q x_{2k-1} \\ x_k^Q &= x_k^I x_{2k} \end{aligned}$$

– $x_k^I, x_k^Q \in \{-1, +1\}$.

- **MSK is equivalent to OQASK with half sinusoid (HS) pulse shaping.**

Gaussian MSK (GMSK)



Gaussian Pre-modulation filtered MSK (GMSK).

- **With MSK the modulating signal is**

$$x(t) = \frac{1}{2T} \sum_{n=-\infty}^{\infty} x_n u_T(t - nT)$$

- **We filter $x(t)$ with a low-pass filter to remove high frequency content prior to modulation, i.e., $g(t) = x(t) * h(t)$.**
- **For GMSK, the low-pass filter transfer function is**

$$H(f) = \exp\left\{ - \left(\frac{f}{B} \right)^2 \frac{\ln 2}{2} \right\}$$

where B is the 3 dB bandwidth.

- **A rectangular pulse** $\text{rect}(t/T) = u_T(t + T/2)$ **transmitted through this filter yields the frequency shaping pulse**

$$h_f(t) = \frac{1}{2T}\sqrt{\frac{2\pi}{\ln 2}}(BT)\int_{t/T-1/2}^{t/T+1/2} \exp\left\{-\frac{2\pi^2(BT)^2x^2}{\ln 2}\right\} dx$$

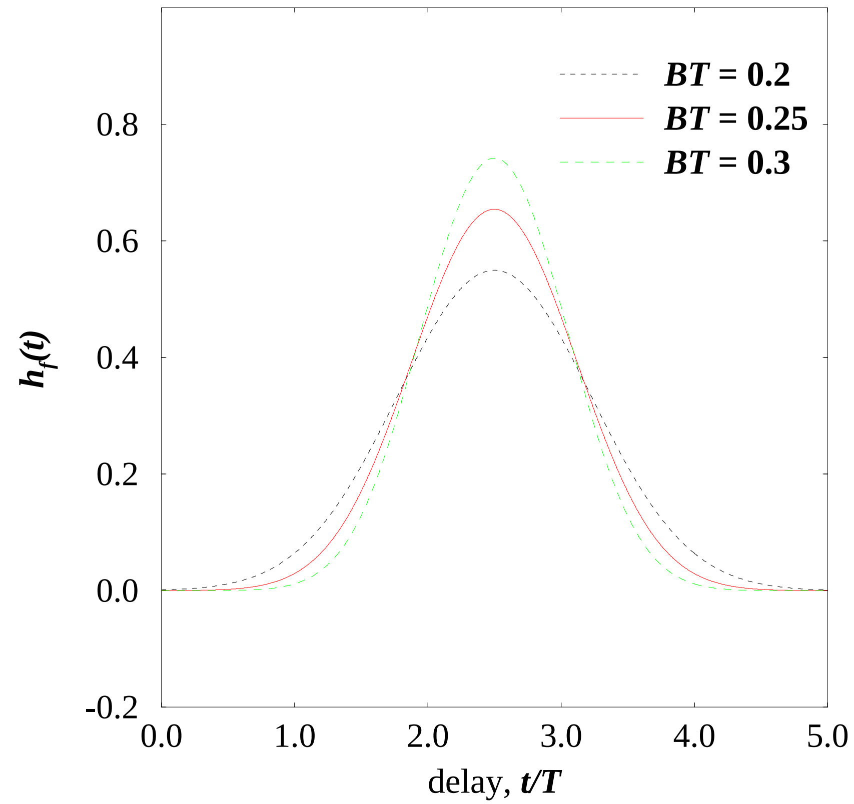
$$= \frac{1}{2T}\left[Q\left(\frac{t/T-1/2}{\sigma}\right) - Q\left(\frac{t/T+1/2}{\sigma}\right)\right] \qquad \text{Correction!}$$

where

$$Q(\alpha) = \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2} dx$$

$$\sigma^2 = \frac{\ln 2}{4\pi^2(BT)^2} \quad .$$

- **the total pulse area is** $\int_{-\infty}^{\infty} h_f(t)dt = 1/2$ **and, therefore, the total contribution to the excess phase for each data symbol is** $\pm\pi/2$.



GMSK frequency shaping pulse for various normalized filter bandwidths BT.

- **The excess phase change over the interval from $-T/2$ to $T/2$ is**

$$\phi(T/2) - \phi(-T/2) = x_0\beta_0(T) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} x_n\beta_n(T)$$

where

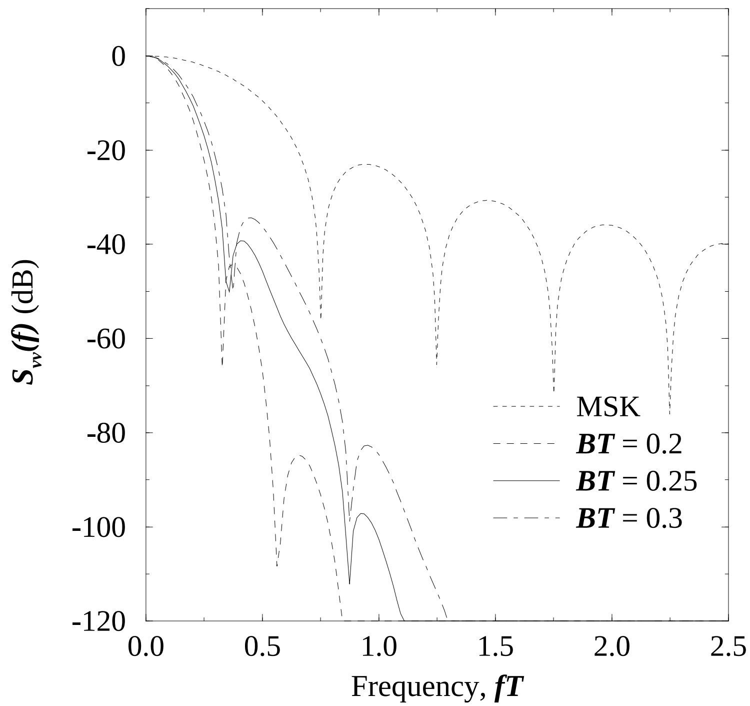
$$\beta_n(T) = \int_{-T/2-nT}^{T/2-nT} h_f(\nu)\ d\nu \quad .$$

and

$$h_f(t) = \frac{1}{2T}\left[Q\left(\frac{t/T - 1/2}{\sigma}\right) - Q\left(\frac{t/T + 1/2}{\sigma}\right)\right]$$

- **The first term, $x_0\beta_0(T)$ is the desired term, and the second term, $\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} x_n\beta_n(T)$, is the intersymbol interference (ISI) introduced by the Gaussian low-pass filter.**

- **Conclusion: GMSK trades off power efficiency for a greatly improved bandwdith efficiency.**



Power spectral density of GMSK with various normalized filter bandwidths BT.

Orthogonal Modulation with Binary Orthogonal Codes

- **Construct a Hadamard matrix using the recursion**

$$\mathbf{H}_M = \begin{bmatrix} \mathbf{H}_{M/2} & \mathbf{H}_{M/2} \\ \mathbf{H}_{M/2} & -\mathbf{H}_{M/2} \end{bmatrix} .$$

  where $\mathbf{H}_1 = [1]$.

- **For example,**

$$\mathbf{H}_8 = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \end{bmatrix} .$$

- **Notice that the rows of the Hadamard matrix are mutually orthogonal.**

Orthogonal Modulation with Binary Orthogonal Codes

- **A set of M equal energy orthogonal waveforms can be constructed according to**

$$\tilde{s}_m(t) = A \sum_{k=1}^{M} h_{m_k} h_c(t - kT_c) \ , \quad m = 0, \ldots, M-1$$

  - h_{m_k} **is the kth co-ordinate in the mth row of the Hadamard matrix**
  - $T = MT_c$ **is the symbol duration**
  - $h_c(t)$ **is a "chip" shaping pulse**

- **This type of orthogonal modulation is used with CDMA cellular, e.g., IS-95 reverse link.**

- **The same waveforms can be used for synchronous CDMA multi-access. One waveform is assigned to each user, e.g., IS-95 forward link.**

Orthogonal Multipulse Modulation

- **A more bandwidth efficient scheme can be obtained by using the rows of the Hadamard matrix $\mathbf{H}_N$ to define N orthogonal amplitude shaping pulses**

$$h_i(t) = A \sum_{k=0}^{N-1} h_{ik} h_c(t - kT_c) \ , \quad i = 0, \ldots, N-1$$

- **A block of N serial data symbols, each of duration T_c, is first converted into a block of N parallel data symbols.**
- **The block of N information symbols is transmitted in parallel by using the N orthogonal amplitude shaping pulses.**
- **The transmitted complex envelope is**

$$\tilde{s}(t) = \sum_n b(t - nT, \mathbf{x}_n)$$

where

$$b(t, \mathbf{x}_n) = \sum_{k=0}^{N-1} x_{n_k} h_k(t)$$

$T = NT_c$**, and** $\mathbf{x}_n = (x_{n_0}, x_{n_1}, \ldots, x_{n_{N-1}})$ **is the block of** N **data symbols transmitted at epoch** n**.**