EE6604

Personal & Mobile Communications

Lecture 22

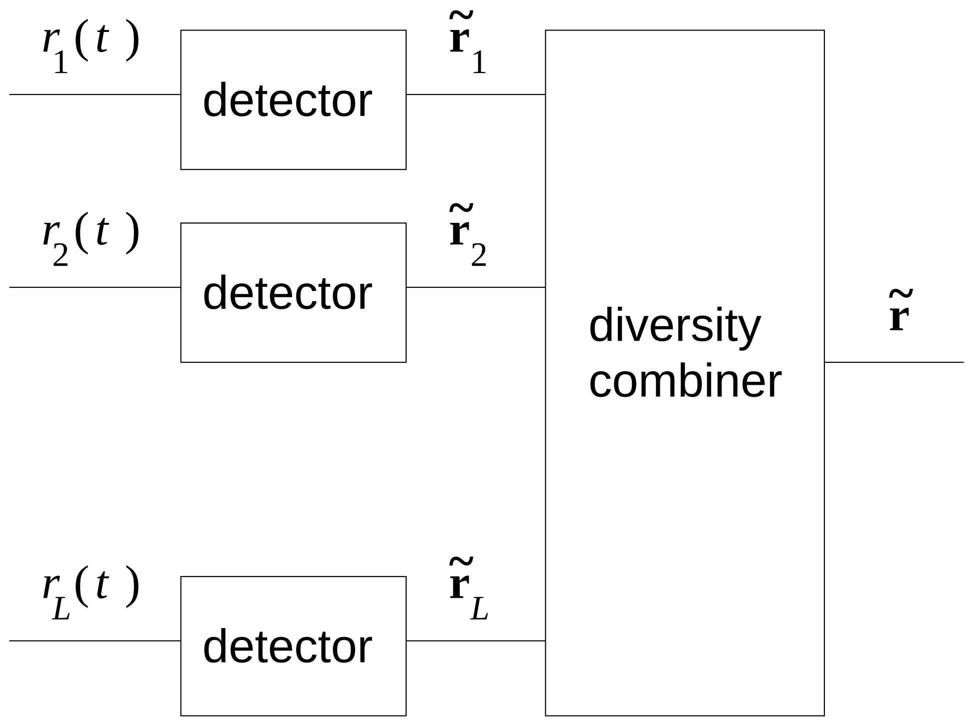
Diversity Techniques

Diversity Methods

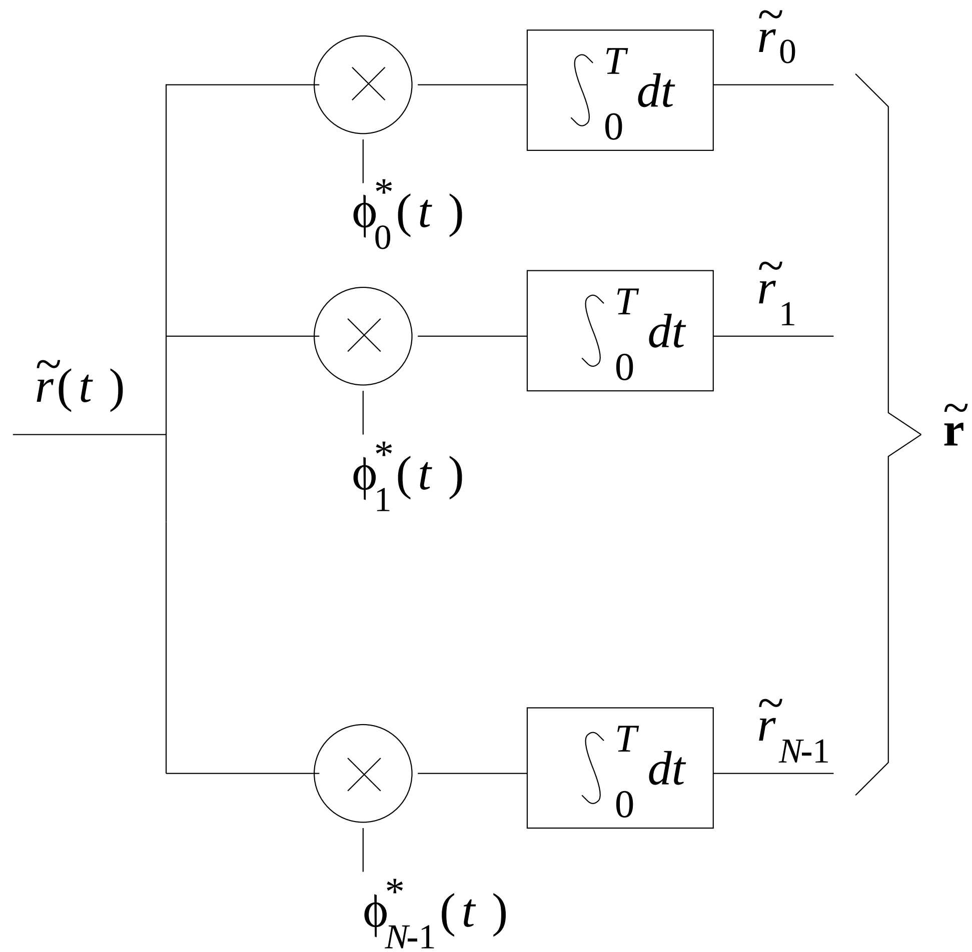
- **Diversity combats fading by providing the receiver with multiple uncorrelated replicas of the same information bearing signal.**
- **There are several types of receiver diversity methods:**
  - **spatial, angle, polarization, frequency, time, and multipath diversity**
- **There are different methods to combine the diversity branches.**
  - **maximal ratio, equal gain, switched, and selective combining.**
- **If the signal $\tilde{s}_m(t)$ is transmitted, the received complex envelopes on the different diversity branches are**

$$\tilde{r}_k(t) = g_k \tilde{s}_m(t) + \tilde{n}_k(t), \quad k = 1, \ldots, L$$

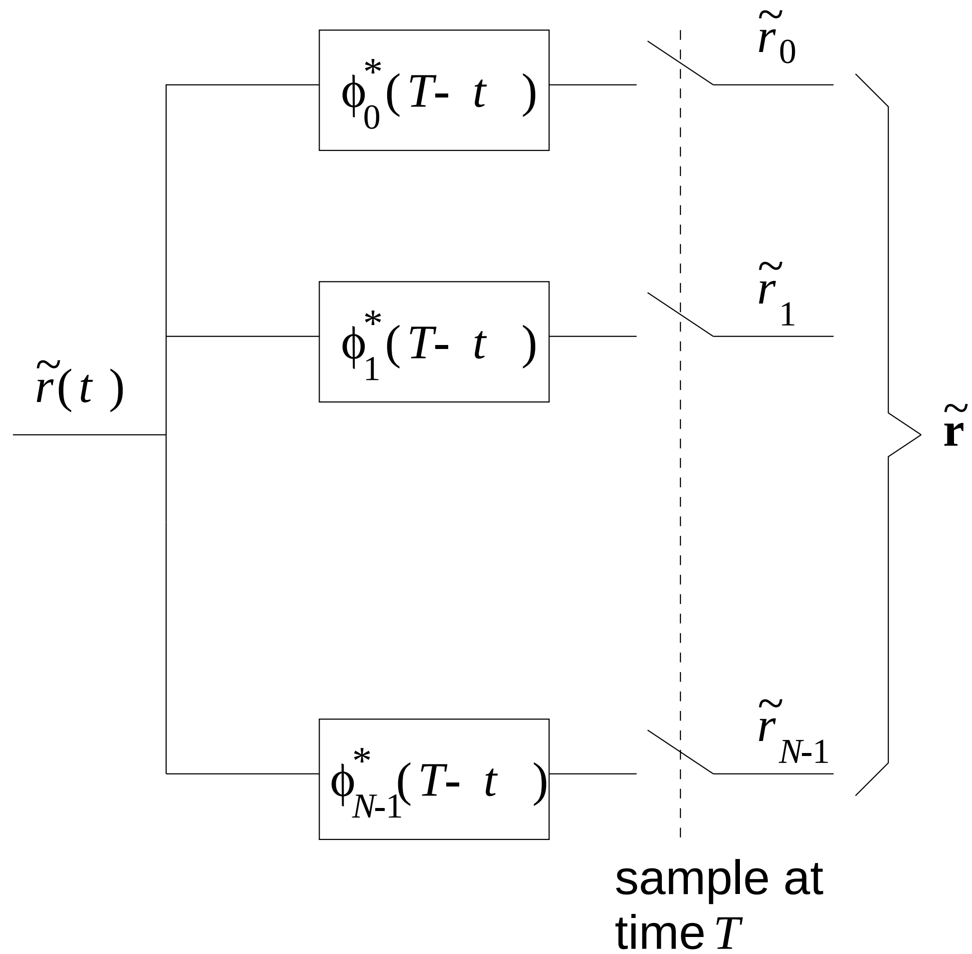
  - **L is the number of diversity branches.**
  - **$g_k = \alpha_k e^{-j\phi_k}$ is the fading gain associated with the k^{th} branch.**
  - **The AWGN processes $\tilde{n}_k(t)$ are independent from branch to branch.**



Postdetection diversity receiver.



Correlator detector



Matched filter detector

Maximal Ratio Combining (MRC)

- **The vector**

$$\tilde{\mathbf{r}} = \mathrm{vec}\,(\tilde{\mathbf{r}}_1,\ \tilde{\mathbf{r}}_2,\ \cdots,\ \tilde{\mathbf{r}}_L)$$

has the multivariate complex Gaussian distribution

$$p(\tilde{\mathbf{r}}|\mathbf{g}, \tilde{\mathbf{s}}_m) == \frac{1}{(2\pi N_o)^{LN}} \exp\left\{-\frac{1}{2N_o}\sum_{k=1}^{L} \|\tilde{\mathbf{r}}_k - g_k\tilde{\mathbf{s}}_m\|^2\right\}$$

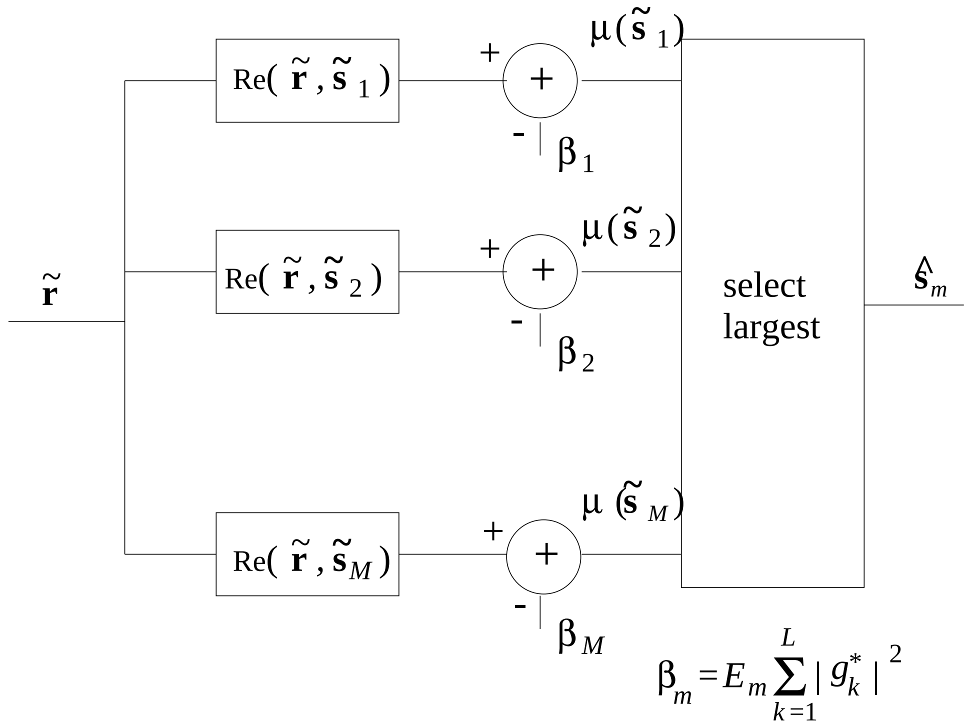
where $\mathbf{g} = (g_1, g_2, \ldots, g_L)$ **is the channel vector.**

- **The ML receiver chooses the message vector** $\tilde{\mathrm{s}}_m$ **that minimizes the metric**

$$\begin{aligned}\mu(\tilde{\mathbf{s}}_m) &= \sum_{k=1}^{L} \|\tilde{\mathbf{r}}_k - g_k\tilde{\mathbf{s}}_m\|^2 \\ &= \sum_{k=1}^{L} \left\{\|\tilde{\mathbf{r}}_k\|^2 - 2\mathrm{Re}\,(g_k^*\tilde{\mathbf{r}}_k, \tilde{\mathbf{s}}_m) + |g_k|^2\|\tilde{\mathbf{s}}_m\|^2\right\} \quad .\end{aligned}$$

- **This is equivalent to maximizing the metric**

$$\begin{aligned}\mu_2(\tilde{\mathbf{s}}_m) &= \sum_{k=1}^{L} \mathrm{Re}\,(g_k^*\tilde{\mathbf{r}}_k, \tilde{\mathbf{s}}_m) - E_m\sum_{k=1}^{L}|g_k|^2 \\ &= \mathrm{Re}\left(\sum_{k=1}^{L} g_k^*\tilde{\mathbf{r}}_k, \tilde{\mathbf{s}}_m\right) - E_m\sum_{k=1}^{L}|g_k|^2\end{aligned}$$



Metric computer for maximal ratio combining.

MRC Performance

- **From (1), the MRC receiver generates the sum**

$$\tilde{\mathbf{r}} = \sum_{k=1}^{L} g_k^* \tilde{\mathbf{r}}_k$$

and chooses m to maximize

$$\mu(\tilde{\mathbf{s}}_m) = \mathrm{Re}\,(\tilde{\mathbf{r}}, \tilde{\mathbf{s}}_m) - E_m \sum_{k=1}^{L} |g_k|^2$$

- **To evaluate the performance gain with MRC, we note that**

$$\begin{aligned} \tilde{\mathbf{r}} &= \sum_{k=1}^{L} g_k^* \left(g_k \tilde{\mathbf{s}}_m + \tilde{\mathbf{n}}_k \right) \\ &= \left(\sum_{k=1}^{L} \alpha_k^2 \right) \tilde{\mathbf{s}}_m + \sum_{k=1}^{L} g_k^* \tilde{\mathbf{n}}_k \\ &\equiv \alpha_{\mathrm{M}}^2 \tilde{\mathbf{s}}_m + \tilde{\mathbf{n}}_{\mathrm{M}} \quad , \end{aligned} \tag{1}$$

- **The first term in (1) is the signal component with average energy $\frac{1}{2}\mathrm{E}[\alpha_{\mathrm{M}}^4 \|\tilde{\mathbf{s}}_m\|^2] = \alpha_{\mathrm{M}}^4 E_{\mathrm{av}}$, where E_{av} is the average symbol energy in the signal constellation.**

MRC Performance

- **The second term is the noise component with variance**

$$\sigma^2_{\tilde{n}_{\rm M}} = \frac{1}{2}{\rm E}[\|\tilde{\mathbf{n}}_{\rm M}\|^2] = N_o \sum_{k=1}^{L} \alpha_k^2 = N_o \alpha_{\rm M}^2 \quad .$$

- **The ratio of the two gives the symbol energy-to-noise ratio**

$$\gamma_s^{\rm mr} = \frac{\frac{1}{2}{\rm E}[\alpha_{\rm M}^4 \|\tilde{\mathbf{s}}_m\|^2]}{\sigma^2_{\tilde{n}_{\rm M}}} = \frac{\alpha_{\rm M}^2 E_{\rm av}}{N_o} = \sum_{k=1}^{L} \frac{\alpha_k^2 E_{\rm av}}{N_o} = \sum_{k=1}^{L} \gamma_k$$

where $\gamma_k = \alpha_k^2 E_{\rm av}/N_o$**.**

- **If the branches are balanced (which is a reasonable assumption with antenna diversity) and uncorrelated, then** $\gamma_b^{\rm mr}$ **has a chi-square distribution with** $2L$ **degrees of freedom**

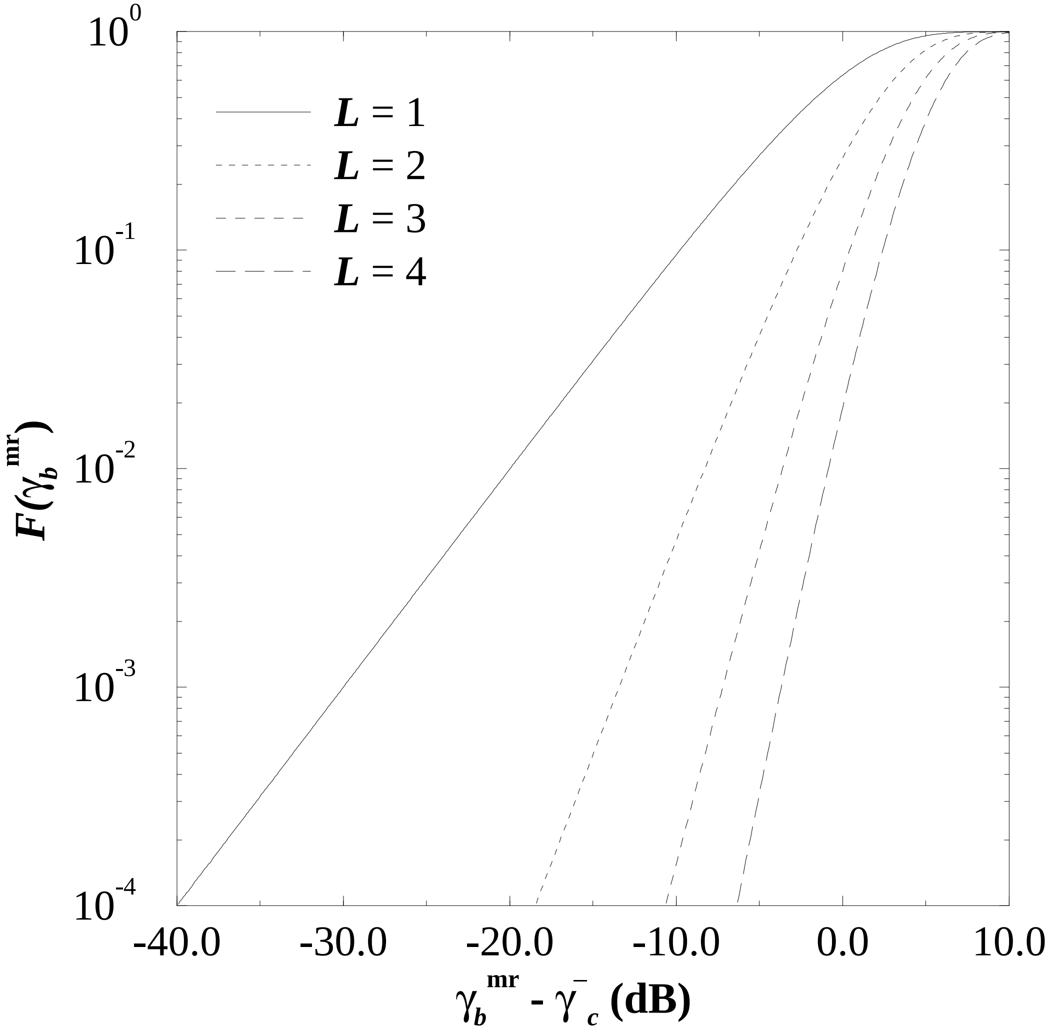
$$p_{\gamma_b^{\rm mr}}(x) = \frac{1}{(L-1)!(\bar{\gamma}_c)^L} x^{L-1} e^{-x/\bar{\gamma}_c}$$

where

$$\bar{\gamma}_c = {\rm E}[\gamma_k] \qquad k = 1, \ \ldots, \ L$$

- **The cdf of** $\gamma_s^{\rm mr}$ **is**

$$F_{\gamma_s^{\rm mr}}(x) = 1 - e^{-x/\bar{\gamma}_c} \sum_{k=0}^{L-1} \frac{1}{k!} \left(\frac{x}{\bar{\gamma}_c}\right)^k \quad .$$



Cdf of $\gamma_s^{\rm mr}$ *for maximal ratio combining;*
$\bar{\gamma}_c$ *is the average branch symbol energy-to-noise ratio.*

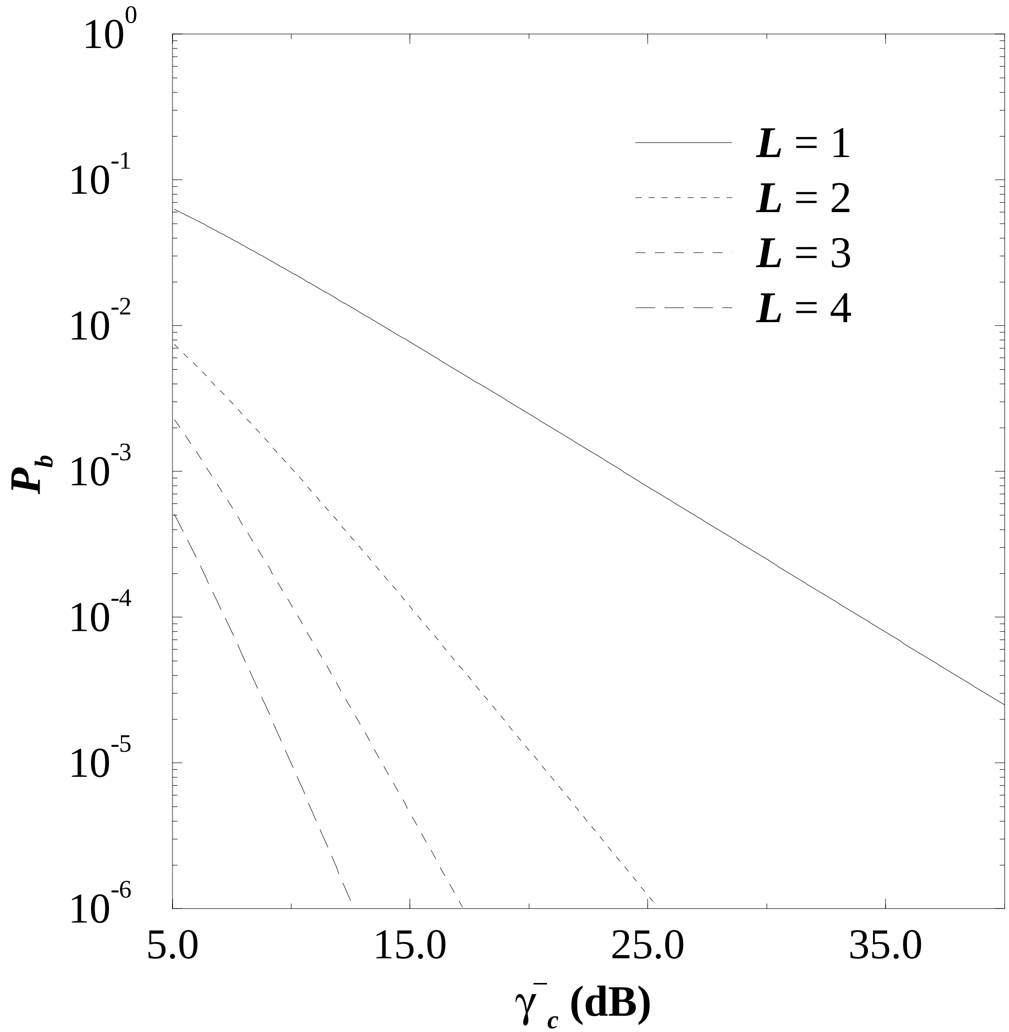
Performance of BPSK with MRC

- **The bit error probability with BPSK and MRC is**

$$
\begin{aligned}
P_b &= \int_0^\infty P_b(x) p_{\gamma_b^{\mathrm{mr}}}(x)dx \\
&= \int_0^\infty Q\left(\sqrt{2x}\right) \frac{1}{(L-1)!(\bar{\gamma}_c)^L} x^{L-1} e^{-x/\bar{\gamma}_c} \\
&= \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k
\end{aligned}
$$

where

$$
\mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}
$$



Bit error probability for BPSK with maximal ratio combining against the average branch bit energy-to-noise ratio.

Selective Combining (SC)

- **With ideal SC, the branch with the largest symbol energy-to-noise ratio is always selected so that the instantaneous symbol energy-to-noise ratio at the output of the selective combiner is**

$$\gamma_s^{\mathrm{s}} = \max\left\{\gamma_1,\ \gamma_2,\ \cdots,\ \gamma_L\right\} \quad ,$$

where L is the number of branches.

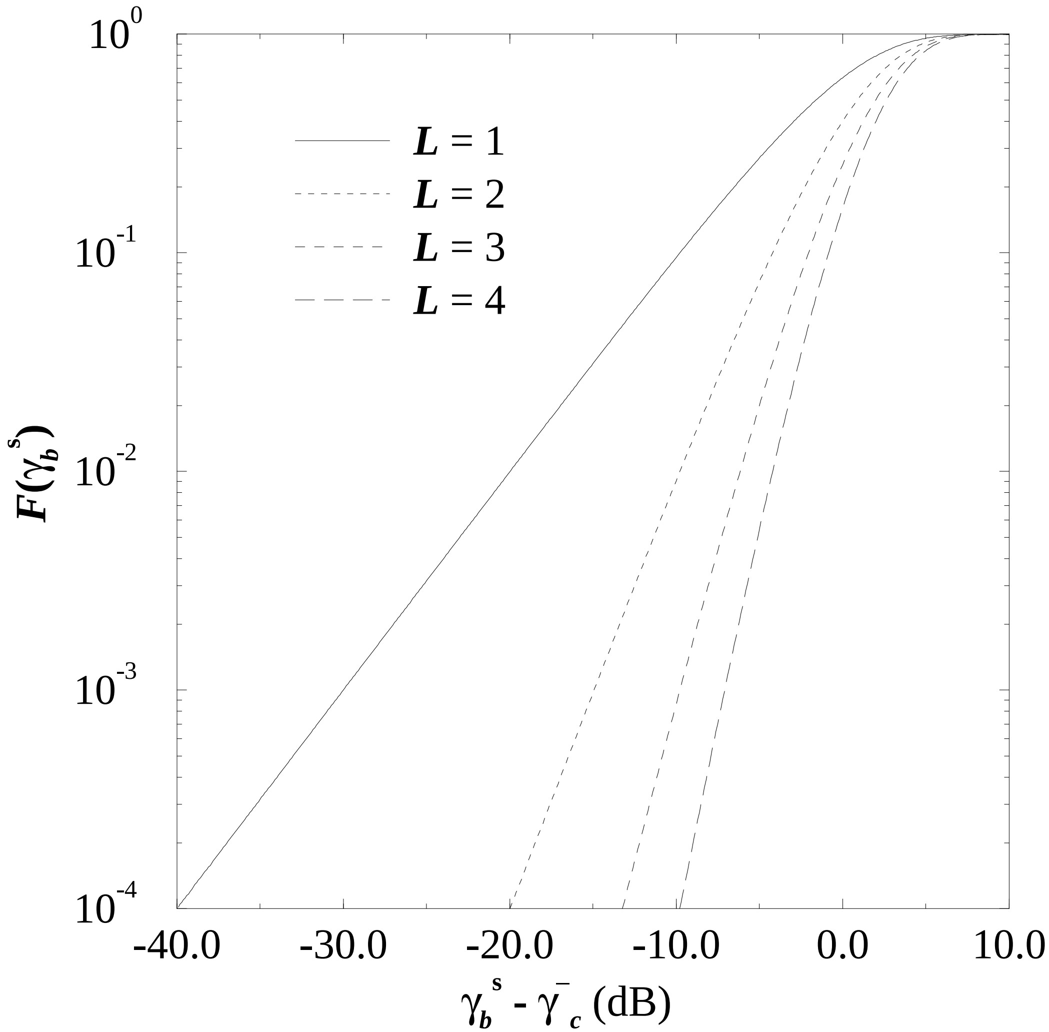
- **If the branches are independently faded, then order statistics gives the cumulative distribution function (cdf)**

$$F_{\gamma_s^{\mathrm{s}}}(x) = \Pr\left[\gamma_1 \le x, \gamma_2 \le x,\ \cdots, \gamma_L \le x\right] = \left[1 - e^{-x/\bar{\gamma}_c}\right]^L \quad .$$

- **Differentiating the above expression gives the pdf of the instantaneous output symbol energy-to-noise ratio as**

$$p_{\gamma_s^{\mathrm{s}}}(x) = \frac{L}{\bar{\gamma}_c}\left[1 - e^{-x/\bar{\gamma}_c}\right]^{L-1} e^{-x/\bar{\gamma}_c} \quad .$$

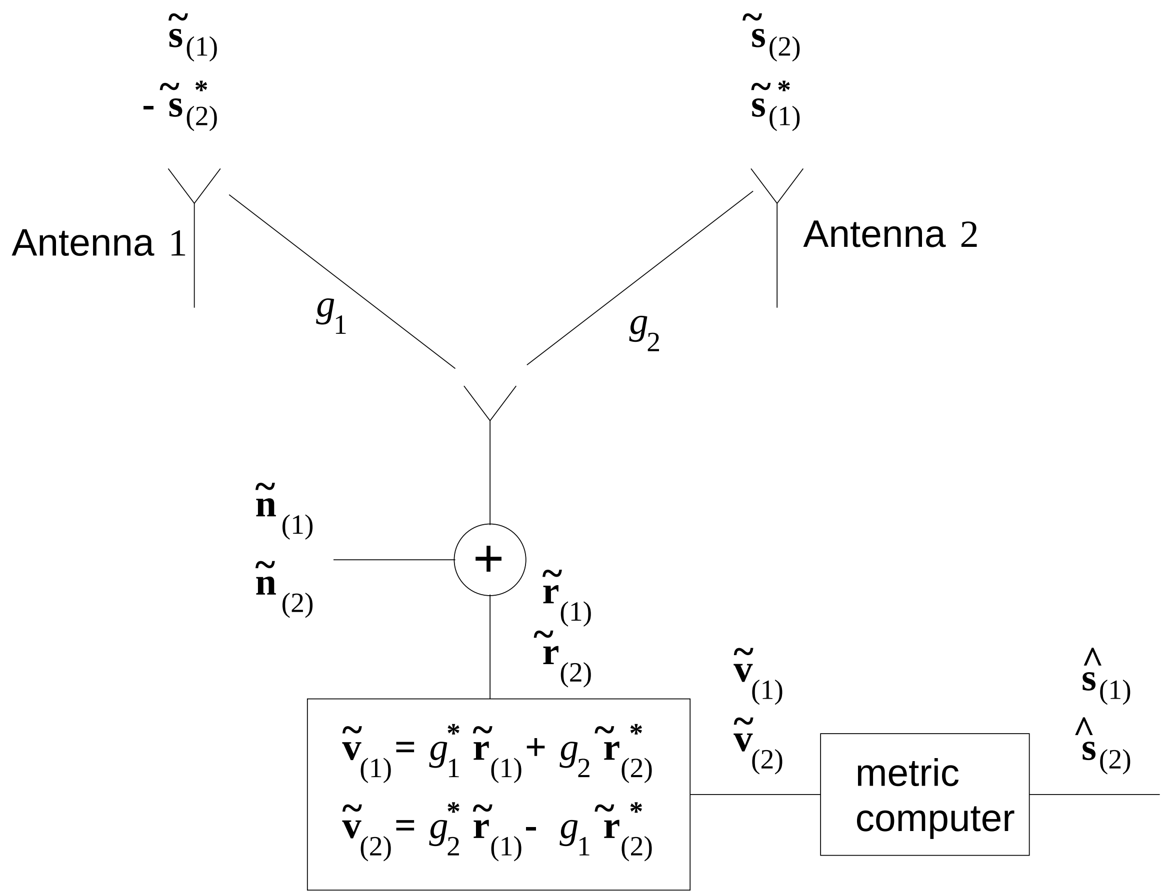
- **The above pdf can be used to evaluate the performance of various digital modulation schemes.**



Cdf of γ_b^{s} *for selective combining;*
$\bar{\gamma}_c$ *is the average branch symbol energy-to-noise ratio.*

Transmit Diversity

- **Transmitter diversity uses multiple transmit antennas to provide the receiver with multiple uncorrelated replicas of the same signal.**
- **The complexity of having multiple antenna is placed on the transmitter which may be shared among many receivers.**
- **Transmit diversity schemes require three functions:**
  - **the encoding and transmission of the information sequence at the transmitter**
  - **the combining scheme at the receiver**
  - **the decision rule for making decisions**
- **We consider a simple repetition transmit diversity scheme with maximum likelihood combining at the receiver.**



Space-time diversity receiver for 2×1 *diversity.*

- **The received complex vectors are**

$$\begin{aligned}\tilde{\mathbf{r}}_{(1)} &= g_1\tilde{\mathbf{s}}_{(1)} + g_2\tilde{\mathbf{s}}_{(2)} + \tilde{\mathbf{n}}_{(1)} \\ \tilde{\mathbf{r}}_{(2)} &= -g_1\tilde{\mathbf{s}}^*_{(2)} + g_2\tilde{\mathbf{s}}^*_{(1)} + \tilde{\mathbf{n}}_{(2)}\end{aligned}$$

$\tilde{\mathbf{r}}_{(1)}$ and $\tilde{\mathbf{r}}_{(2)}$ represent the received vectors at time t and $t+T$, respectively, and $\tilde{\mathbf{n}}_{(1)}$ and $\tilde{\mathbf{n}}_{(2)}$ are the corresponding noise vectors.

- **The combiner constructs the following two signal vectors**

$$\begin{aligned}\tilde{\mathbf{v}}_{(1)} &= g_1^*\tilde{\mathbf{r}}_{(1)} + g_2\tilde{\mathbf{r}}^*_{(2)} \\ \tilde{\mathbf{v}}_{(2)} &= g_2^*\tilde{\mathbf{r}}_{(1)} - g_1\tilde{\mathbf{r}}^*_{(2)}\end{aligned}$$

Afterwards, the receiver applies the vectors $\tilde{\mathbf{v}}_{(1)}$ and $\tilde{\mathbf{v}}_{(2)}$ in a sequential fashion to the metric computer, to make decisions by maximizing the metric

$$\begin{aligned}\mu(\tilde{\mathbf{s}}_{(1),m}) &= \mathrm{Re}\left(\tilde{\mathbf{v}}_{(1)}, \tilde{\mathbf{s}}_{(1),\mathrm{m}}\right) - E_m(|g_1|^2 + |g_2|^2) \\ \mu(\tilde{\mathbf{s}}_{(2),m}) &= \mathrm{Re}\left(\tilde{\mathbf{v}}_{(2)}, \tilde{\mathbf{s}}_{(2),\mathrm{m}}\right) - E_m(|g_1|^2 + |g_2|^2)\end{aligned}$$

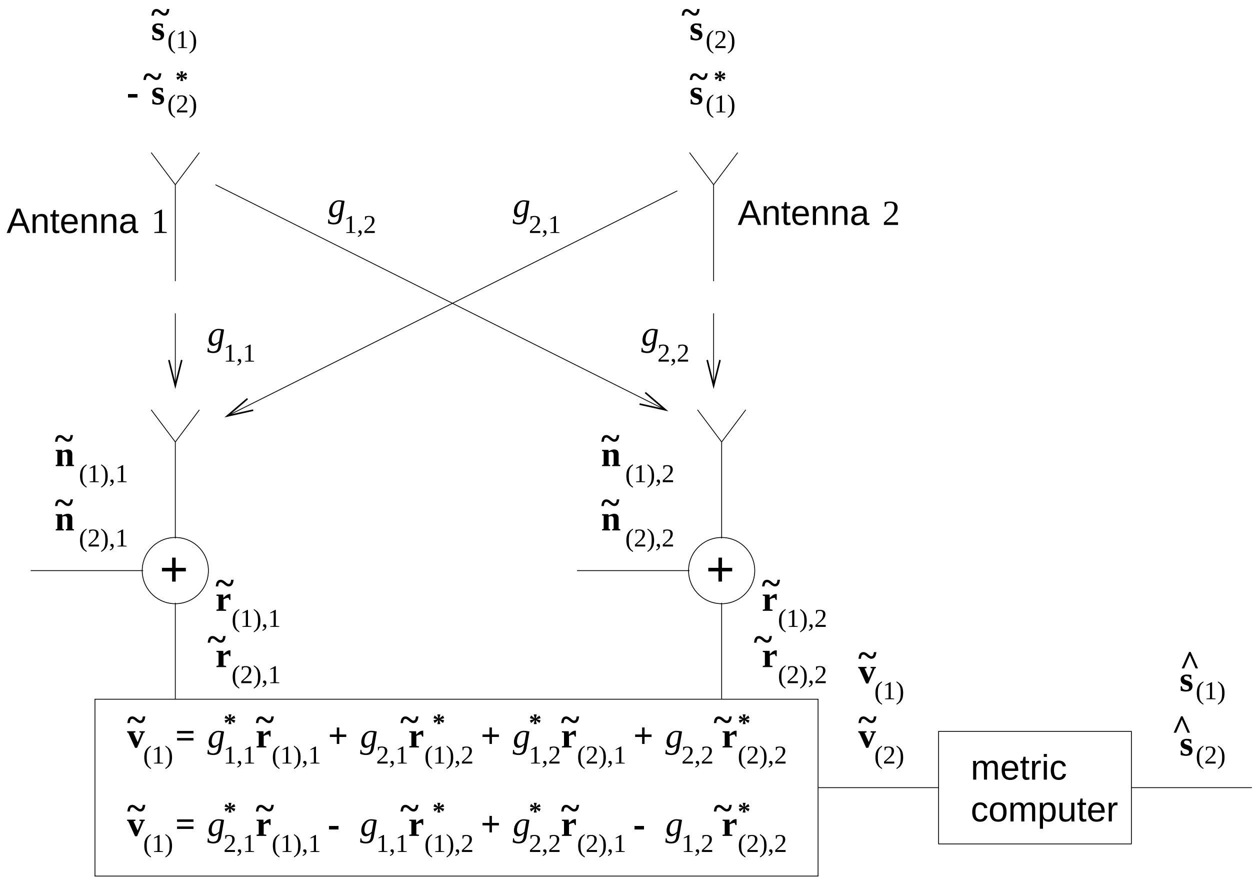
- **We have**

$$\begin{aligned}\tilde{\mathbf{v}}_{(1)} &= (\alpha_1^2 + \alpha_2^2)\tilde{\mathbf{s}}_{(1)} + g_1^*\tilde{\mathbf{n}}_{(1)} + g_2\tilde{\mathbf{n}}_{(2)}^* \\ \tilde{\mathbf{v}}_{(2)} &= (\alpha_1^2 + \alpha_2^2)\tilde{\mathbf{s}}_{(2)} - g_1\tilde{\mathbf{n}}_{(2)}^* + g_2^*\tilde{\mathbf{n}}_{(1)}\end{aligned}$$

- **Compare with the output of the MRC metric computer.**
- **With 1×2 diversity and MRC**

$$\begin{aligned}\tilde{\mathbf{r}} &= g_1^*\tilde{\mathbf{r}}_1 + g_2^*\tilde{\mathbf{r}}_2 \\ &= (\alpha_1^2 + \alpha_2^2)\tilde{\mathbf{s}}_m + g_1^*\tilde{\mathbf{n}}_1 + g_2^*\tilde{\mathbf{n}}_2\end{aligned}$$

The combined signals in each case are the same. The only difference is the phase rotations of the noise vectors which will not change the error probability.



Space-time diversity receiver for 2×2 diversity.