EE6604
Personal & Mobile Communications

Lecture 27

OFDM Decision-Aided Reconstruction

and Residual ISI Cancellation

Clipped Signal Prior to Transmission

- **The N-point IFFT output sequence is**

$$X_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n \exp\left\{ j \frac{2\pi n k}{N} \right\}, \quad 0 \leq k \leq N-1$$

- **Suppose we clip the IFFT output sequence as**

$$Y_k = \begin{cases} X_k & , \quad |X_k| \leq A \\ A \exp\left\{\arg(X_k)\right\} & , \quad |X_k| > A \end{cases} \qquad 0 \leq k \leq N-1$$

  – **Note that the amplitude is clipped but the phase is maintained.**

- **The clipping ratio (CR) is defined as**

$$\mathrm{CR} = 20 \log \frac{\mathrm{A}}{\sigma} \ \ \mathrm{dB} \qquad \sigma = \sqrt{\mathrm{E}\left[|\mathrm{X_n}|^2\right]}$$

where σ is the rms signal power of x_k.

- **A guard interval is added as**

$$Y_k^g = Y_{(k+N-G)_N}, \quad 0 \leq k \leq N+G-1$$

Demodulated Signal with Clipping

- **The received samples after removal of the guard interval are**

$$R_k = \sum_{m=0}^{M} h_m Y_{(k-m)_N} + n_k, \quad 0 \le k \le N-1$$

where h_m is the channel impulse response at lag m.

- **Performing an FFT on $\{R_k\}$ gives**

$$\begin{aligned} Z_n &= \eta_n y_n + w_n \\ &= \alpha_n \eta_n x_n + q_n, \qquad 0 \le k \le N-1 \end{aligned}$$

where the complex channel gain is

$$\eta_n = \sum_{m=0}^{M} h_m \exp\left\{-j\frac{2\pi nm}{N}\right\}$$

Decision-Aided Reconstruction

1. **From the demodulated samples $\{Z_n\}_{n=0}^{N-1}$, an estimate of the clipped signal, $\{\hat{Y}_k\}$, is obtained and stored in memory by performing IFFT on $\{\hat{y}_n\}$, where**

$$\hat{y}_n = \frac{Z_n}{\eta_n}, \qquad 0 \le n \le N-1$$

2. **Decisions on the transmitted symbols are made in the frequency domain as**

$$\hat{x}_n^{(I)} = \min_{\{x\}} \left| Z_n^{(I)} - \eta_n x \right|, \qquad 0 \le n \le N-1$$

where I represents an iteration number with an initial value of $I = 0$, and $Z_n^{(0)} = Z_n$.

3. **The decisions in Step 2 are converted back to the time domain using an IFFT, yielding $\{\hat{X}_k^{(I)}\}$.**

4. **The clipped samples are detected and new sequence, $\{\hat{Y}_k^{(I)}\}$, is generated as**

$$\hat{Y}_k^{(I)} = \begin{cases} \hat{Y}_k & , \quad \left|\hat{X}_k^{(I)}\right| \le A \\ \hat{X}_k^{(I)} & , \quad \left|\hat{X}_k^{(I)}\right| > A \end{cases}, \qquad 0 \le k \le N-1$$

where the sequence $\{\hat{Y}_k\}$ is *always* retrieved from the memory in Step 1.

Decision-Aided Reconstruction (cont'd)

5 The sequence $\{\hat{Y}_k^{(I)}\}$ is converted to the frequency domain yielding $\{\hat{y}_n^{(I)}\}$.

6 Increment the index number $I \leftarrow I+1$ and determine $\{Z_n^{(I)}\}$ by

$$Z_n^{(I)} = \eta_n \hat{y}_n^{(I-1)}, \qquad 0 \leq n \leq N-1 \quad .$$

7 Go back to Step 2. Decisions are made yielding $\{\hat{x}_n^{(I)}\}$. This completes the Ith iteration of the DAR algorithm.

8 Continue iterations by repeating Steps 3–7.

- **When the clipping noise is large compared to the AWGN, the performance is limited by an error floor due to the clipping noise.**
- **The clipping noise is mitigated significantly when the decisions are made in the frequency domain; when the symbol decisions are converted back to the time domain, the OFDM signal samples that had a large amplitude and were clipped in the transmitter will actually re-grow. Although these samples will still be somewhat distorted by clipping, they are now much less affected by the clipping noise.**

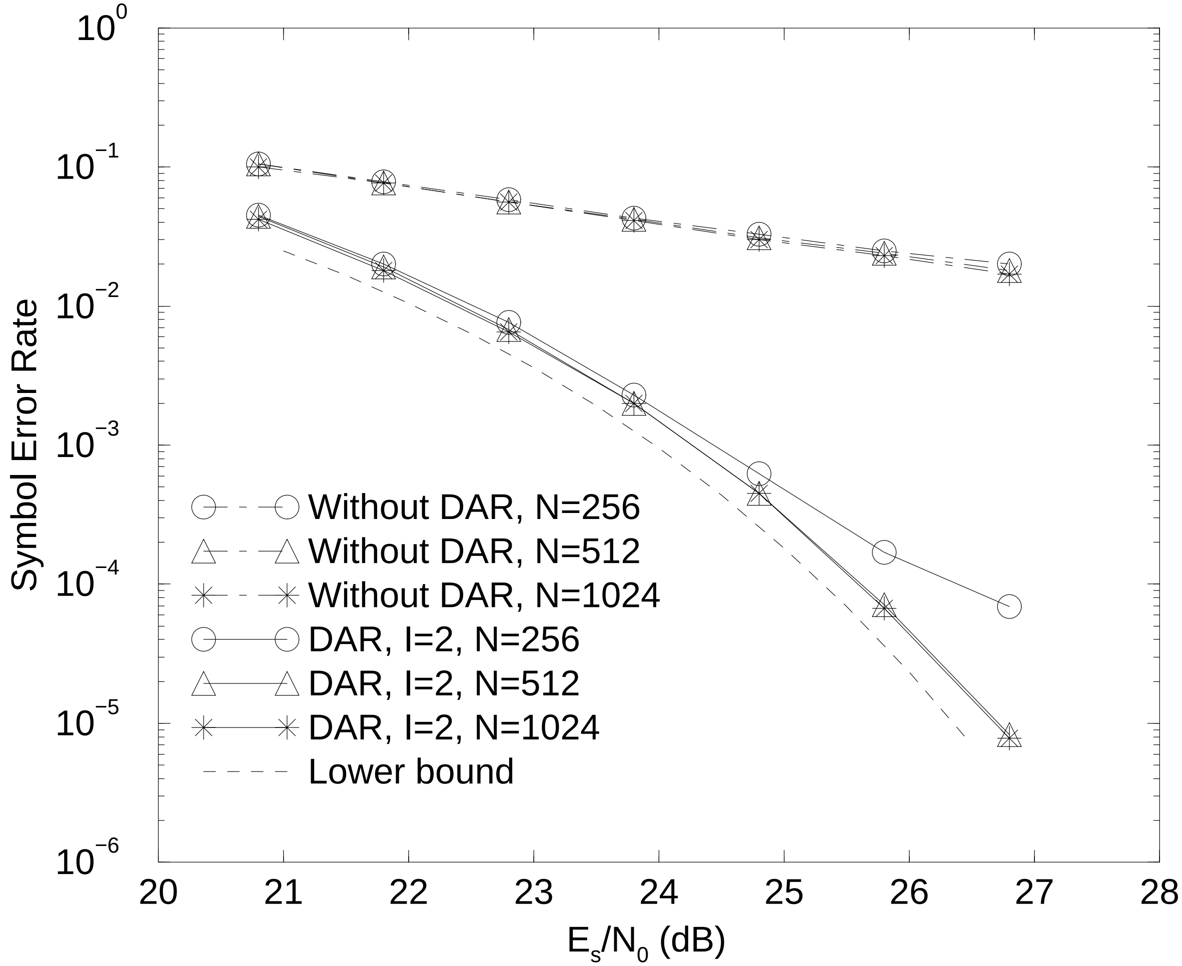
Simulations

- **Consider using M-QAM with sample spacing $T_s^g = 0.2\ \mu$s.**
- **The tap coefficients for the static ISI channel are listed below. This channel has several deep subchannel nulls, and the smallest subchannel power is -23.3 dB below the average subchannel power.**
- **A guard interval of length $10\ \mu$s is used, which is longer than the maximum excess delay of the channel.**

Table 1: Tap Coefficients for the ISI Channel

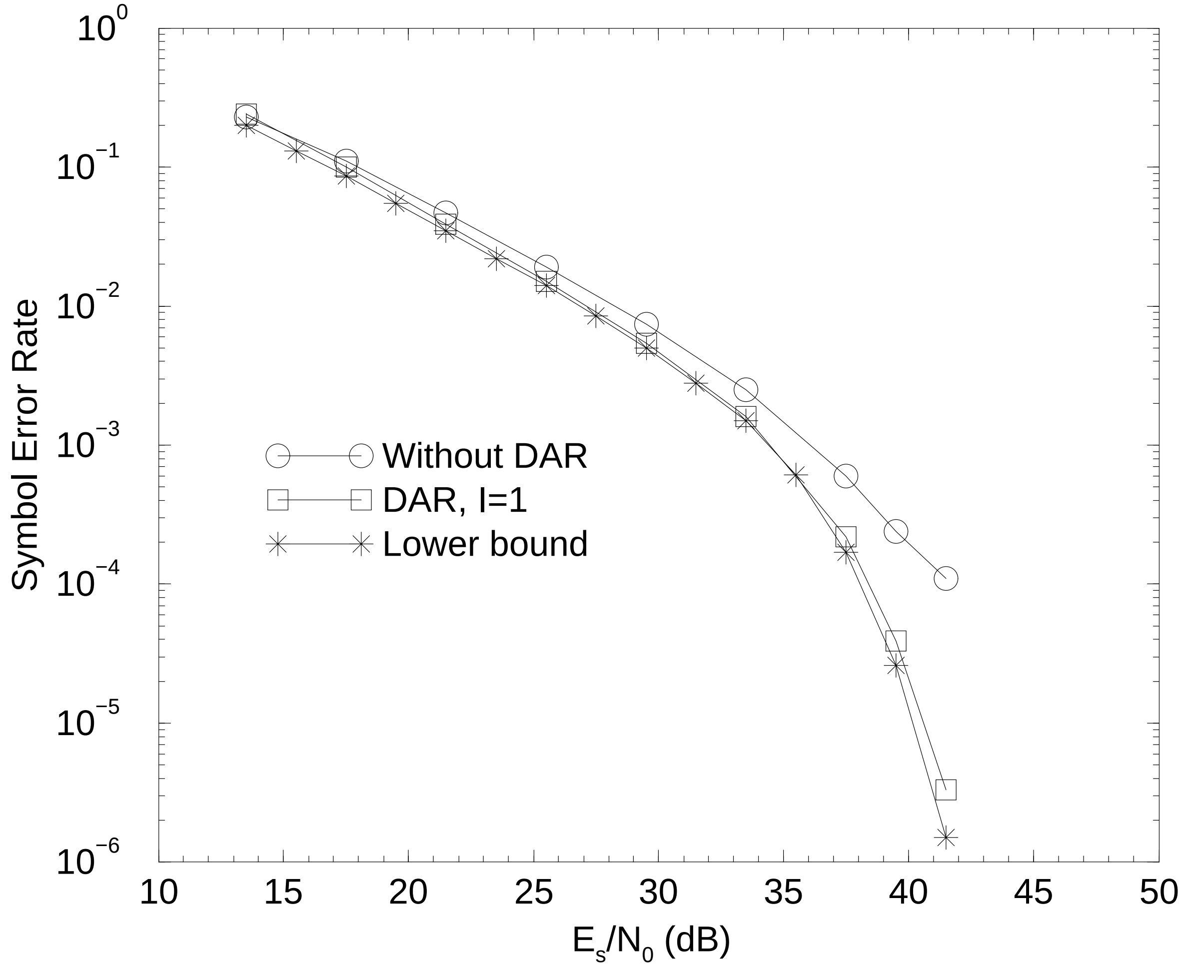
tap	delay (μs)	tap coeff.
h_0	0.0	0.405
h_1	0.2	0.541
h_2	1.0	0.383
h_3	1.6	0.307
h_4	5.0	0.430
h_5	6.6	0.342

Simulation Results



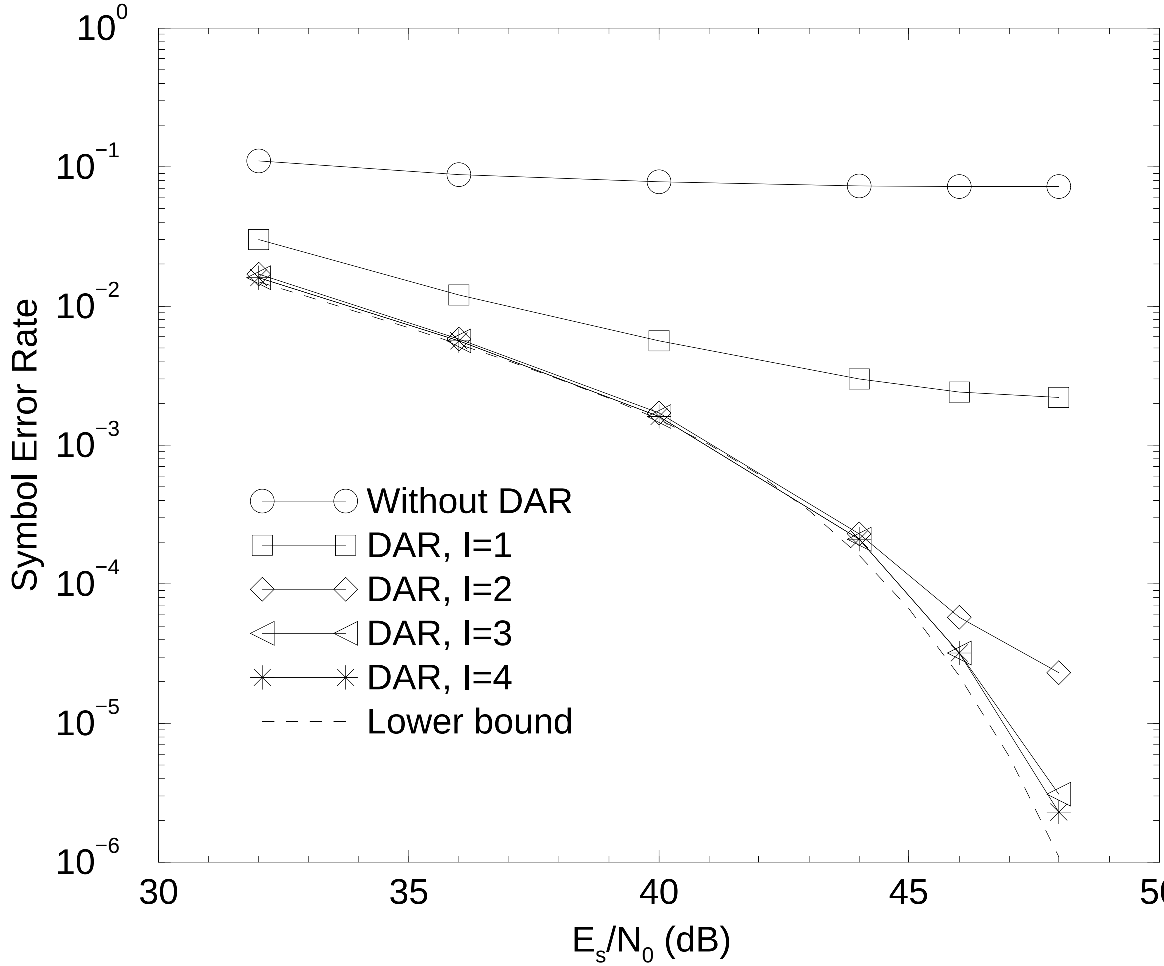
On AWGN channel with 64-QAM; CR=5 dB.
The lower bound is achieved when clipping is not used.

Simulation Results



On static ISI channel with 16-QAM; CR=4 dB, $N = 1024$.

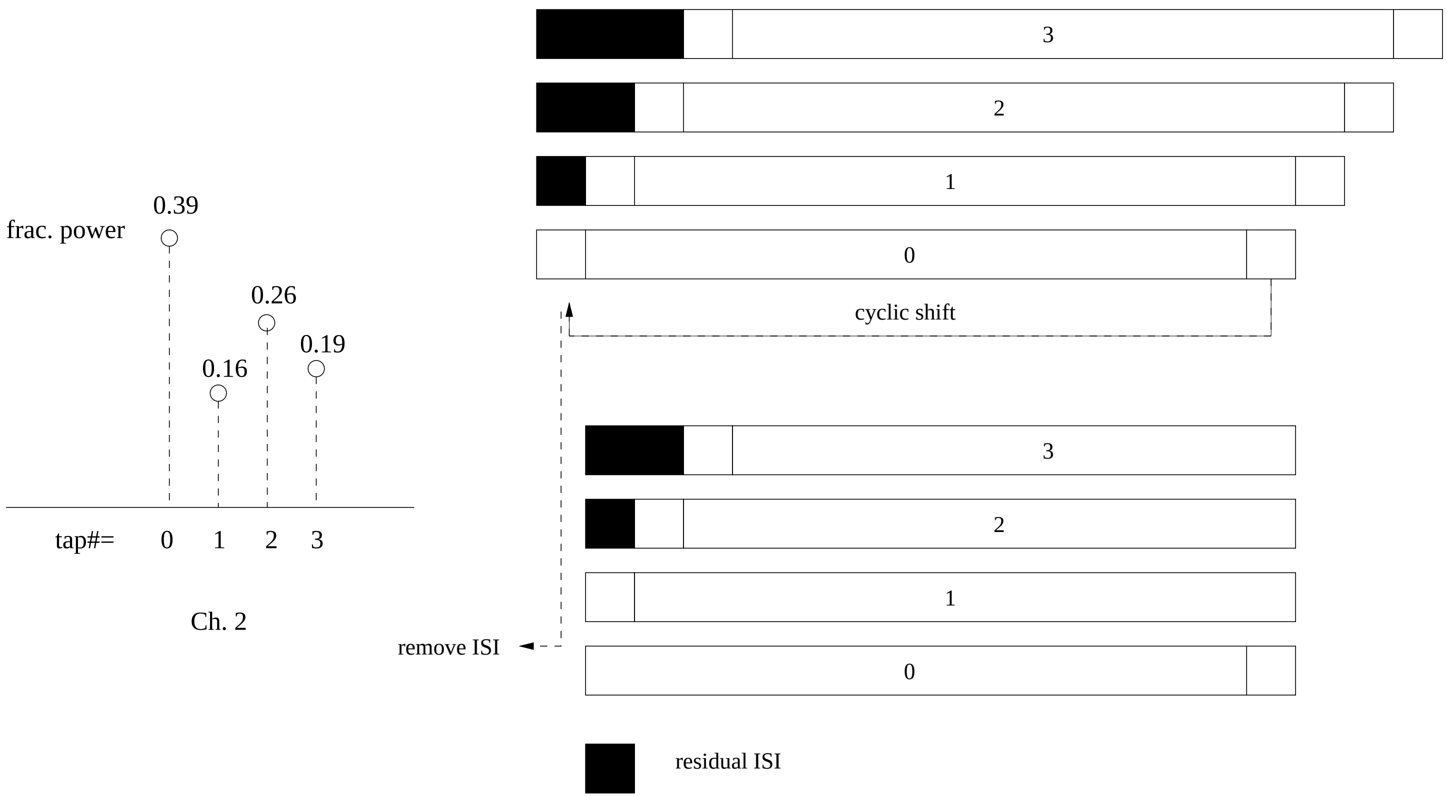
Simulation Results



On static ISI channel with 64-QAM; CR=4 dB, $N = 1024$.

Mitigation of ISI Using Guard Interval

- **Assume a guard interval of length $G = 1$ and a channel of length $L = 3$. In this case, there will be residual ISI.**



Residual ISI Cancellation (RISIC)

- **RISIC algorithm is carried out by executing two steps,** *tail cancellation* **and** *cyclic reconstruction***.**

  1. **Decisions on transmitted symbols** $\{\hat{x}_{i-1,n}\}_{n=0}^{N-1}$ **from block** $i-1$ **are obtained for use in tail cancellation. These symbols are converted back to the time domain using an IFFT giving** $\{\hat{X}_{i-1,k}\}_{k=0}^{N-1}$**.**
  2. **For block** i**, we perform tail cancellation by calculating residual ISI and subtract it from** $R_{i,k}$ **as**

  $$\tilde{R}_{i,k}^{(0)} = R_{i,k} - \sum_{m=G+1}^{L} h_m \hat{X}_{i-1,(k-m+G)_N}(1 - u(k-m+G)) \qquad 0 \le k \le N-1$$

  3. $\{\tilde{R}_{i,k}^{(0)}\}_{k=0}^{N-1}$ **obtained in Step 2 are converted to frequency domain using FFT and decisions are made. Afterwards, decisions are converted back to time domain to give** $\{\hat{X}_{i,k}^{(0)}\}_{k=0}^{N-1}$**.**

RISIC Algorithm

4. **Next we perform cyclic reconstruction by forming**

$$\tilde{R}_{i,k}^{(I)} = \tilde{R}_{i,k}^{(0)} + \sum_{m=G+1}^{L} h_m \hat{X}_{i,(k-m)_N}^{(I-1)} (1 - u(k - m + G)), \qquad 0 \le k \le N - 1$$

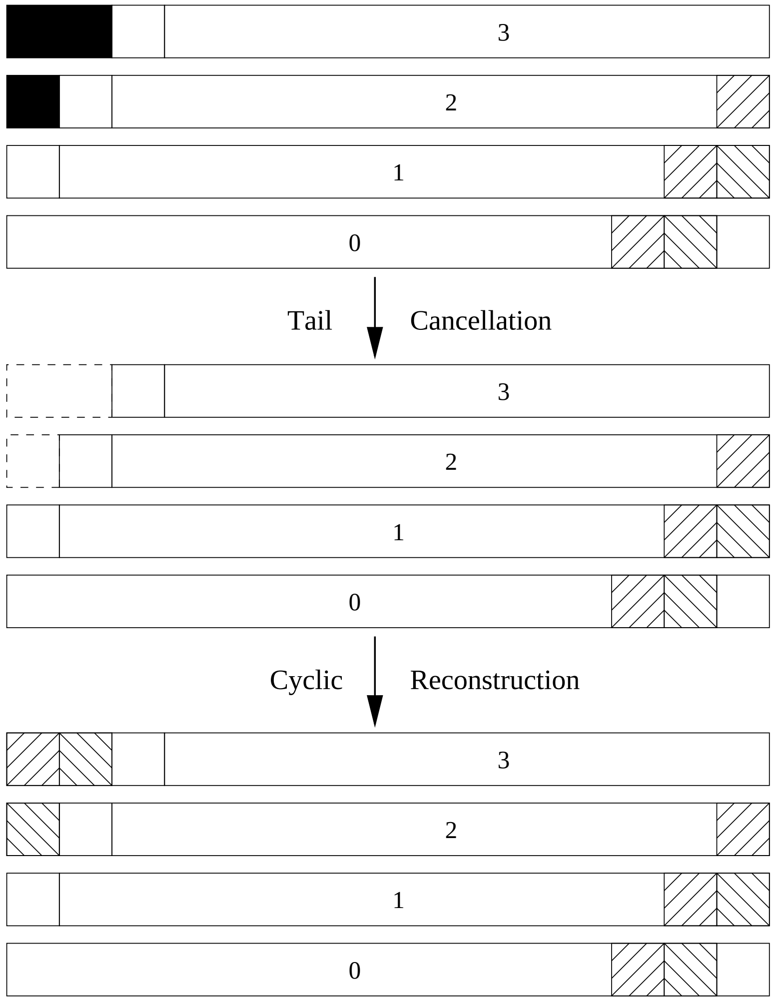
   where I represents an iteration number with an initial value of $I = 1$.

5. $\{\tilde{R}_{i,k}^{(I)}\}_{k=0}^{N-1}$ **are converted to frequency domain and decisions are made yielding** $\{\hat{x}_{i,n}^{(I)}\}_{n=0}^{N-1}$**. This completes the** I**th iteration in RISIC algorithm.**

6. **To continue iterations, convert** $\{\hat{x}_{i,n}^{(I)}\}_{n=0}^{N-1}$ **to** $\{\hat{X}_{i,k}^{(I)}\}_{k=0}^{N-1}$ **and repeat Steps 4 – 6 with** $I \leftarrow I + 1$ **.**

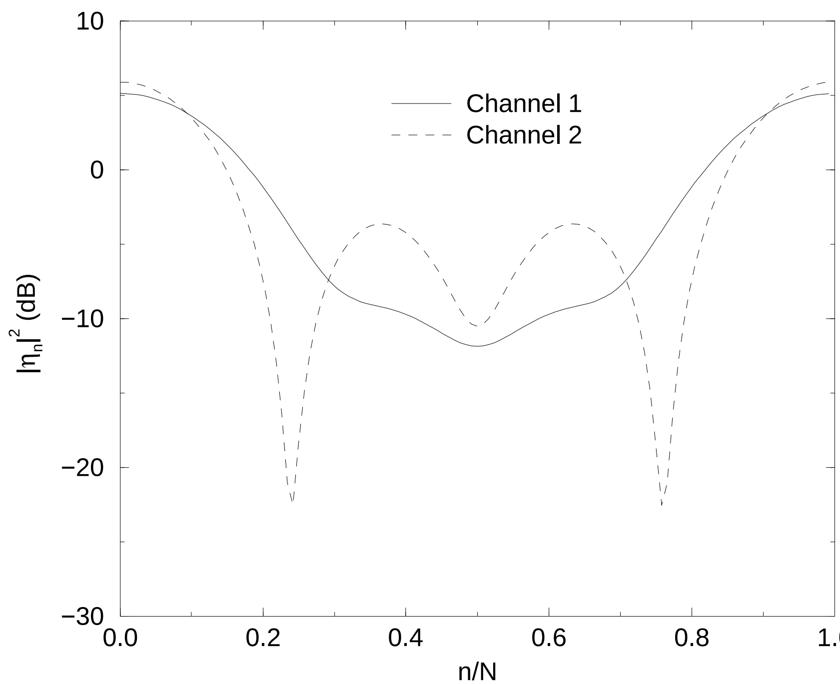
7. **End of RISIC algorithm for block** i**.**

RISIC Algorithm

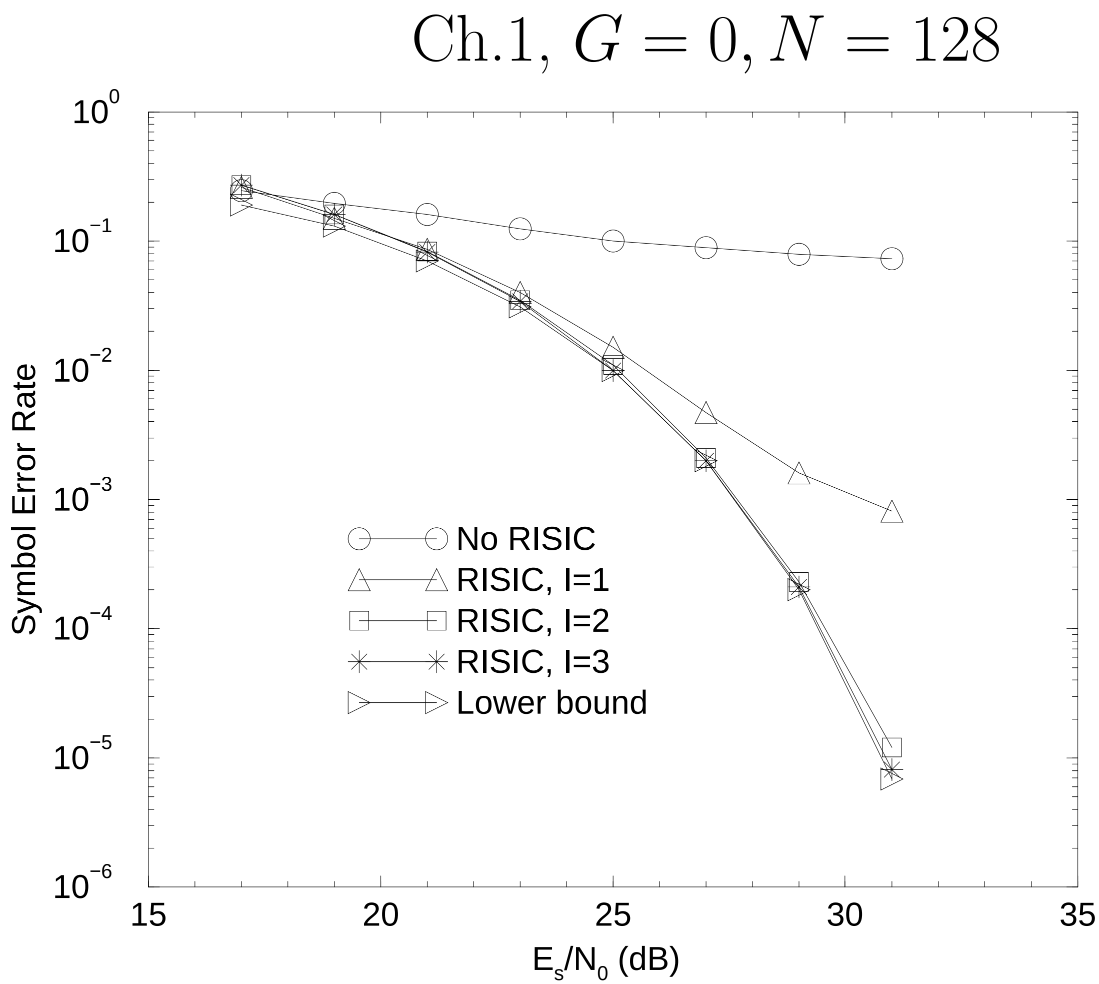


Static Channels ($N = 128$)

tap #	delay (μs)	Frac. Power (Ch. 1)	Frac. Power (Ch. 2)
0	0.0	0.15	0.39
1	0.2	0.65	0.16
2	0.4	0.15	0.26
3	0.6	0.05	0.19

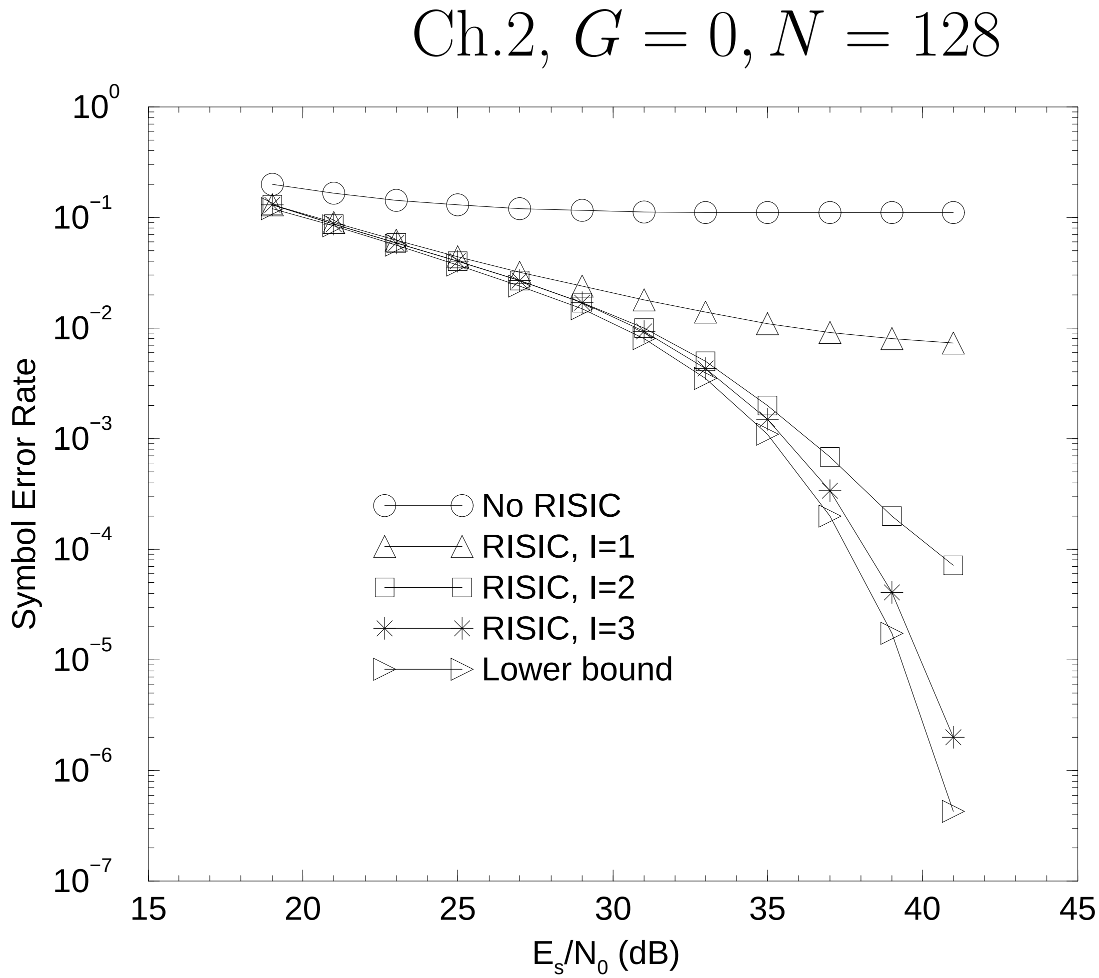


16-QAM on Static Channel; 20 Mbps



RISIC gives a severe error floor. RISIC recovers with $I = 2$ iterations.

16-QAM on Static Channel; 20 Mbps



RISIC gives a severe error floor. RISIC recovers with $I = 3$ iterations.