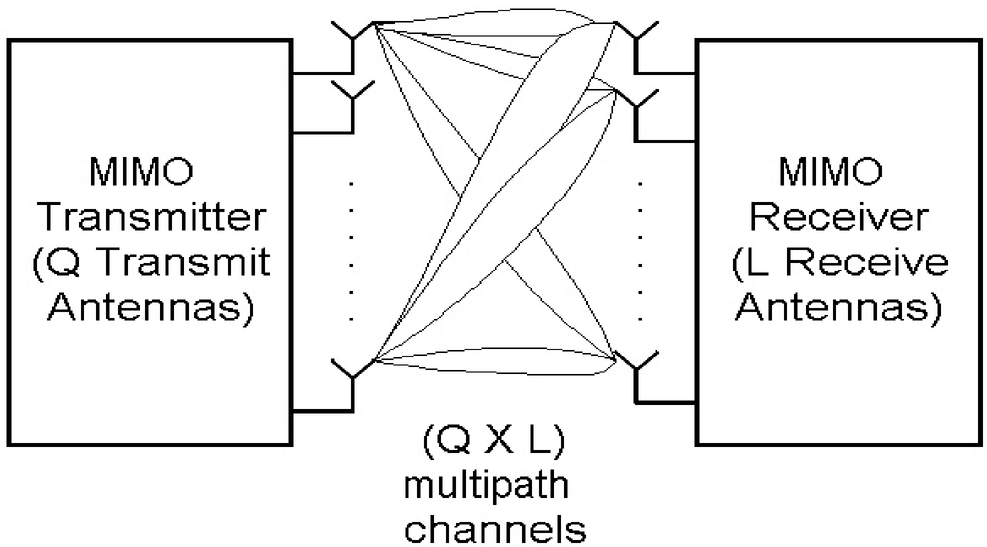
ECE 6604

Personal & Mobile Communications

Lecture 28

MIMO OFDM

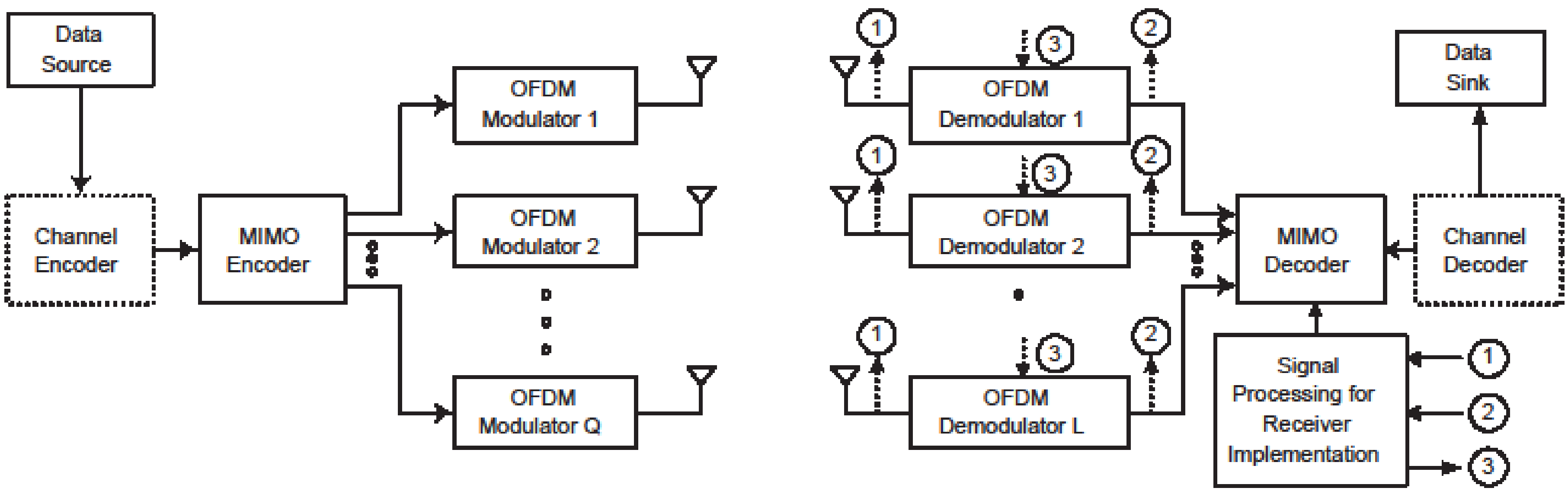
A MULTI INPUT MULTI OUTPUT (MIMO) OFDM SYSTEM



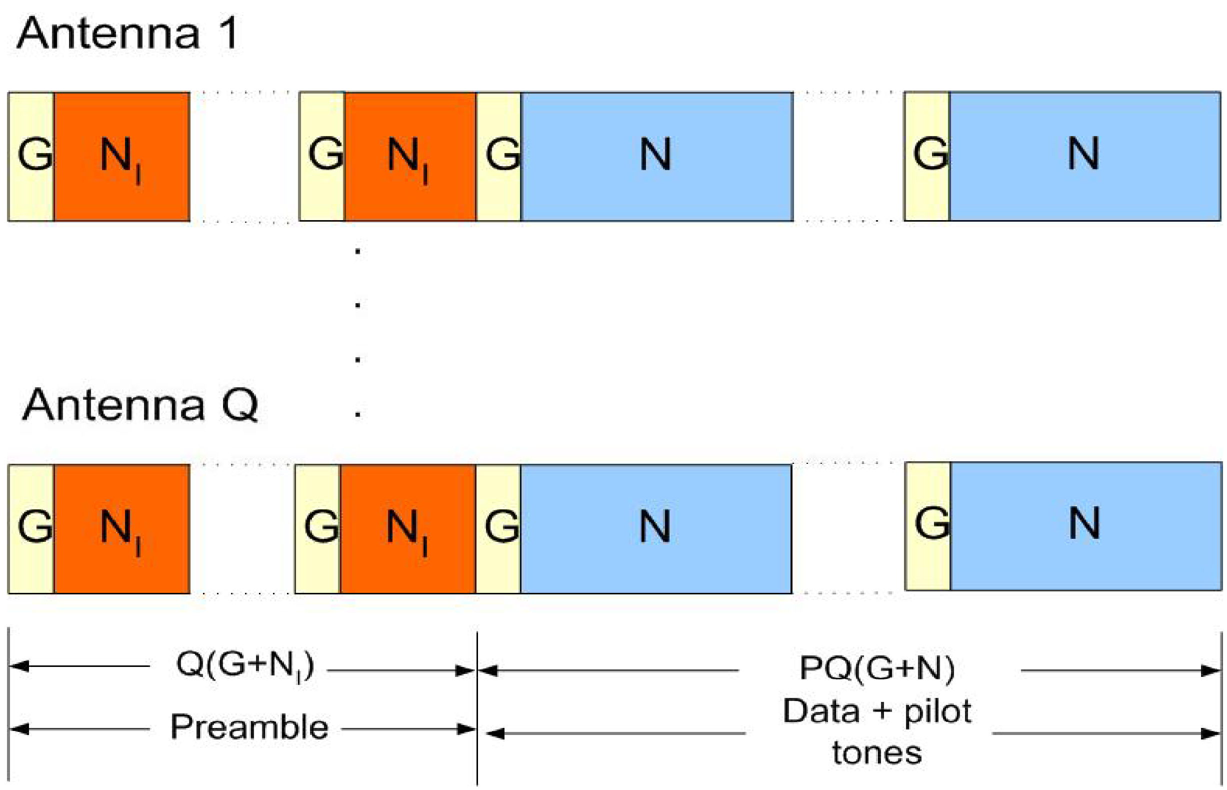
- **A MIMO system uses *Q* Transmit antennas and *L* Receive Antennas.**

- **A *Q* x *L* MIMO arrangement can provide a diversity gain of order (*Q* X *L*) using space-time coding techniques, or the data rate can be increased by a factor of min {*Q*, *L*} using spatial multiplexing techniques.**

Q-TRANSMIT L-RECEIVE MIMO OFDM SYSTEM



FRAME STRUCTURE



- **Preamble consists of *Q* OFDM symbols, and each OFDM symbol has length N_l, where $N_l = N/l$, $l = 1,2,4..$**

- **Data symbols consist of *P* blocks of Q OFDM symbols, where each OFDM symbol has length *N*.**

- **Each symbol is preceded by a length-*G* cyclic prefix.**

MIMO OFDM PREAMBLE CONSTRUCTION

- The preamble sequences of length N_I can be constructed by
  - exciting every I th sub-channel of an N point sequence in the frequency domain using some known alphabet.
  - Taking an N-point IFFT of the sequence.
  - Keep the first N_I samples and discarding the rest.
- The time domain training sequence (minus the guard interval) that is transmitted during the d^{th} training symbol period from the q^{th} transmit antenna is given by

$$s_n^{(d,q)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k^{(d,q)} \exp\left\{ j \frac{2\pi n k}{N} \right\}, \; 0 \le n \le N_I - 1 \; .$$

- Add a length-G cyclic prefix to the sequence $\underline{s}^{(d,q)}$ and transmit the resulting sequence $\underline{t}^{(d,q)}$ from the q^{th} transmit antenna.

RECEIVED SIGNALS

- **At the epoch *n*, the sample stream at the ℓ^{th} receive antenna is**

$$r_n^{(d,\ell)} = \sum_{q=1}^{Q} t_n^{(d,q)} * h_n^{(q,\ell)} + w_n^{\ell}$$

- **The received samples $\{r_n^{(d,\ell)}\}_{n=0}^{N_I-1}$ after removal of the guard interval are repeated *I* times and demodulated using an *N*-point FFT.**

$$\begin{aligned} R_k^{d,\ell} = & \sum_{q=1}^{Q} S_k^{(d,q)} \eta_k^{q,\ell} \exp\left\{\frac{j2\pi}{N}[d(N+G)(k\beta+\gamma+\gamma\beta)]\right\} \\ & \frac{1}{N}\frac{\sin[\pi(k\beta+\gamma+\gamma\beta)]}{\sin[\frac{\pi}{N}(k\beta+\gamma+\gamma\beta)]}\exp\left\{j\pi(k\beta+\gamma+\gamma\beta)(1-\frac{1}{N})\right\} \\ & + W_k^{d,\ell} \end{aligned}$$

$$\begin{aligned} k &= 0,\ldots,N-1 \\ d &= \text{running index of the OFDM symbol,} \\ \ell &= \text{receiver antenna index,} \\ \beta &= \text{normalized sampling frequency offset } (T'-T)/T, \\ \gamma &= \text{fractional RF oscillator frequency offset.} \end{aligned}$$

MATRIX REPRESENTATION

- **The received samples have the matrix representation**

$$\mathbf{R}_k^d = A_k^d C_k \mathbf{\Lambda}_k \mathbf{S}_k^d \cdot \mathbf{H}_k^d + \mathbf{W}_k$$

$$\begin{aligned}
A_k^d &= \exp\{j\,[2\pi(k\beta+\gamma)d(1+G/N)]\} \\
C_k &= \frac{1}{N}\frac{\sin[\pi(k\beta+\gamma+\gamma\beta)]}{\sin[\frac{\pi}{N}(k\beta+\gamma+\gamma\beta)]}\exp\{j\pi(k\beta+\gamma+\gamma\beta)(1-1/N)\}
\end{aligned}$$

$$\begin{aligned}
k &= \text{subcarrier index} \\
\mathbf{R}_k &= \text{demodulated OFDM sample matrix of dimension } (Q\times L) \\
\mathbf{S}_k &= \text{transmitted sample matrix of dimension } (Q\times Q) \\
\boldsymbol{\eta}_k &= \text{channel coefficient matrix of dimension } (Q\times L) \\
\mathbf{W}_k &= \text{additive white Gaussian noise matrix of dimension } (Q\times L)
\end{aligned}$$

$$\mathbf{\Lambda}_k = \begin{bmatrix} \exp\left\{\frac{j2\pi(k\beta+\gamma)0(N+G)}{N}\right\} & 0 & \cdots \\ 0 & \ddots & 0 \\ 0 & \cdots & \exp\left\{\frac{j2\pi(k\beta+\gamma)(Q-1)(N+G)}{N}\right\} \end{bmatrix}.$$

GENERALIZED PREAMBLE DESIGN

- Desired characteristics of sequences constituting the preamble:
  - Low peak to average power ratio (PAPR) for undistorted signal reception,
  - Good time correlation properties for time synchronization,
  - Suitable for RF oscillator frequency offset estimation over a wide range,
  - Suitable for channel estimation.
    - For optimum channel estimation, the training sequences must have a flat spectrum, where the spectral flatness is measured in terms of spectral max-to-min ratio (SMMR) given by

$$\xi(\underline{S}^{d,q}) = \frac{\left\{ \max|S_k^{d,q}| : 0 \leq k \leq N \right\}}{\left\{ \min|S_k^{d,q}| : 0 \leq k \leq N \right\}}$$

- Low receiver computational complexity and low overhead, but high accuracy in estimating parameters.

GENERATION OF LENGTH 256 SEQUENCE $N=256$, $I=1$

Example: $N_I=256$

S = [0 1 -1 1 -1 -1 1 -1 1 -1 1 1 1 -1 1 1 -1 -1 1 -1 1 -1
1 -1 1 1 1 -1 -1 -1 1 -1 1 -1 1 1 -1 1 -1 -1 1 -1 -1 -1 -1
1 -1 1 1 1 1 1 -1 -1 -1 1 -1 1 -1 -1 -1 -1 1 -1 -1 -1 1 -1
-1 1 -1 1 1 1 -1 1 -1 1 -1 -1 1 1 -1 -1 1 -1 -1 -1 -1 1
1 1 1 1 -1 -1 -1 -1 1 1 -1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 -1 1
-1 -1 -1 -1 1 1 -1 1 -1 -1 1 -1 1 1 -1 1 -1 -1 1 1 1 1 -
1 1 -1 1 1 -1 1 -1 -1 -1 -1 -1 -1 1 -1 1 1 1 1 1 1 -1 1
1 -1 -1 1 1 1 -1 -1 1 -1 1 1 -1 -1 1 -1 1 1 1 -1 -1 1 1
1 1 1 -1 1 -1 1 -1 1 1 1 1 -1 -1 -1 1 -1 -1 1 1 1 1 1
-1 -1 -1 1 1 -1]

PAPR = 5.34 dB

55 0's come from IEEE802.16a spectral requirements

GENERATION OF LENGTH 128 SEQUENCE N=256, I=2

Example: N_I=128

**S = [0 0 -1 0 -1 0 1 0 -1 0 -1 0 1 0 1 0 1 0 -1 0
1 0 1 0 1 0 -1 0 -1 0 -1 0 -1 0 -1 0 -1 0 1 0
-1 0 -1 0 -1 0 -1 0 -1 0 -1 0 1 0 1 0 1 0 -1 0
1 0 -1 0 1 0 1 0 -1 0 1 0 1 0 1 0 -1 0 -1 0 -1
0 -1 0 -1 0 1 0 -1 0 -1 0 1 0 -1 0 -1 0 1 0 -1
{55 0's} -1 0 1 0 1 0 1 0 1 0 -1 0 -1 0 1 0 -1 0
1 0 -1 0 -1 0 1 0 1 0 -1 0 1 0 -1 0 1 0 -1 0 1
0 -1 0 1 0 1 0 -1 0 1 0 -1 0 -1 0 1 0 -1 0 -1
0 -1 0 1 0 1 0 -1 0 1 0 1 0 1 0 -1 0 1 0 1 0
-1 0 -1 0 -1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0]**

PAPR = 4.31 dB

GENERATION OF LENGTH 64 SEQUENCE N=256, I=4

Example: For N_I=64

S = [0 0 0 0 +1+j 0 0 0 -1-j 0 0 0 +1+j 0 0 0 +1-j 0 0 0 +1+j 0 0 0
+1+j 0 0 0 -1+j 0 0 0 +1+j 0 0 0 -1-j 0 0 0 +1+j 0 0 0 +1+j 0 0 0
+1-j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 +1-j 0 0 0 -1-j 0 0 0 +1+j 0 0 0 -1-j
0 0 0 -1-j 0 0 0 -1+j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 +1-j 0 0 0 -1-j 0 0 0
-1+j 0 0 0

-1+j {55 0's} +1+j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 -1+j 0 0 0 +1+j 0 0 0
-1-j 0 0 0 -1+j 0 0 0 -1-j 0 0 0 -1-j 0 0 0 -1-j 0 0 0 +1+j 0 0 0 -1+j 0
0 0 +1+j 0 0 0 -1-j 0 0 0 +1-j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 +1+j 0 0 0
-1-j 0 0 0 +1-j 0 0 0 +1+j 0 0 0 -1-j 0 0 0 +1-j 0 0 0 +1+j 0 0 0 -1+j
0 0 0 +1-j 0 0 0]

PAPR = 3.00 dB

SIGNAL TRANSMISSION MATRIX DESIGN

- The signal transmission matrix S_k must be i) unitary and ii) have at least rank Q for least squares (LS) channel estimation to be possible.

- The simplest unitary structure is obtained when the signal transmission matrix is diagonal
  - Direct extension of SISO
  - The transmitted power needs to be increased by a factor of Q in the training phase. Hence, it requires power amplifiers with an increased dynamic range.

Time

$$S_D = \begin{bmatrix} \underline{s}_1 & 0 & 0 & 0 \\ 0 & \underline{s}_2 & 0 & 0 \\ 0 & 0 & \underline{s}_3 & 0 \\ 0 & 0 & 0 & \underline{s}_4 \end{bmatrix}$$

Antenna

SIGNAL TRANSMISSION MATRIX DESIGN

- **If the channel is sufficiently static, then the MIMO channel can be estimated over a number of OFDM symbols, e.g. IEEE 802.11a, IEEE 802.16 (2004) fixed wireless access.**

- **Transmission of signal from all the antennas improves the channel estimation performance and, hence, the data transmission accuracy.**

SIGNAL TRANSMISSION MATRIX DESIGN

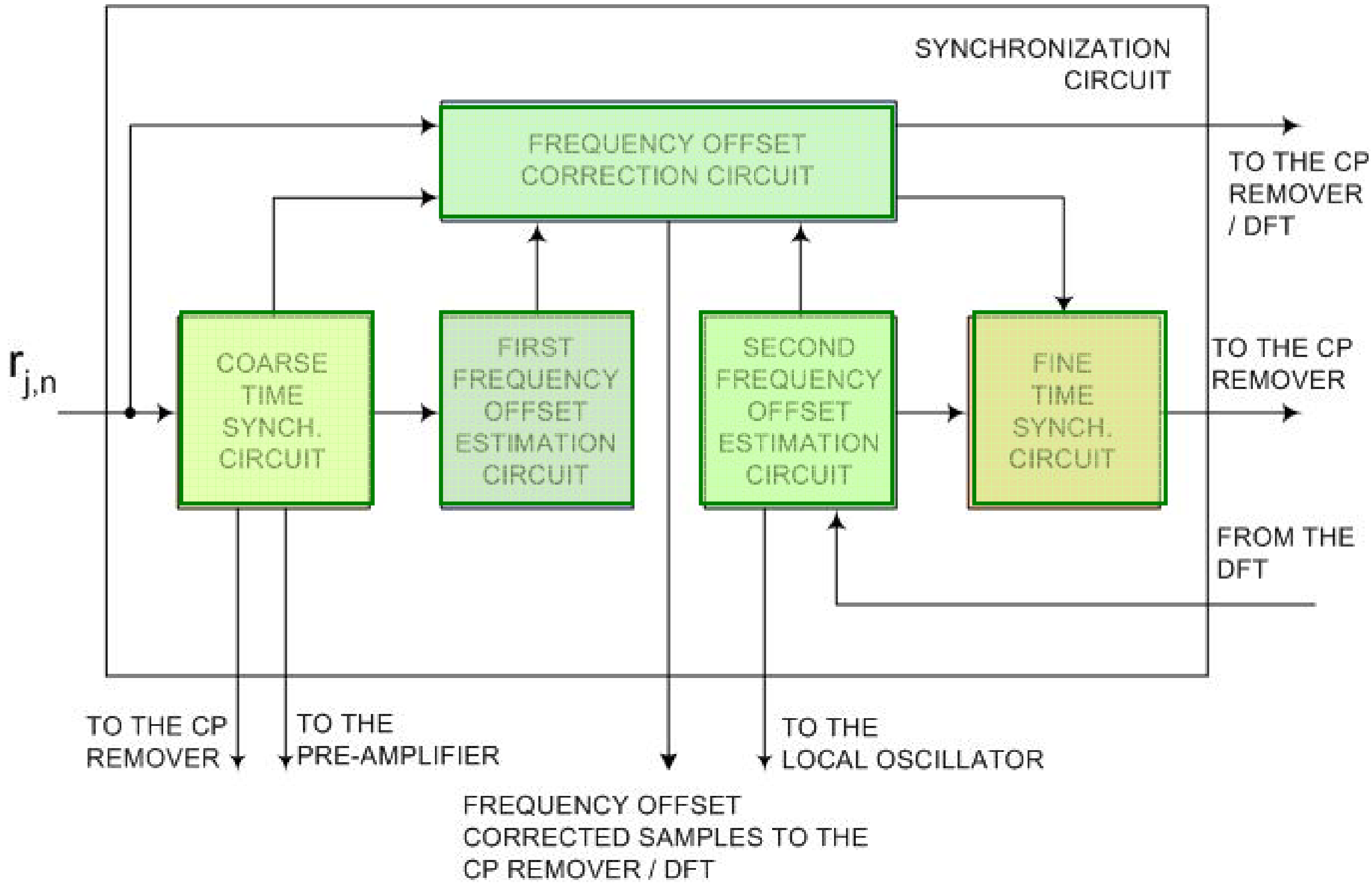
- For Q=2, Alamouti' s structure is optimal

$$\mathrm{S}_A = \begin{bmatrix} \underline{s}_1 & \underline{s}_2 \\ \underline{-s}_2^* & \underline{s}_1^* \end{bmatrix} \qquad \mathrm{S}_{AS} = \begin{bmatrix} \underline{s}_1 & \underline{s}_1 \\ \underline{-s}_1^* & \underline{s}_1^* \end{bmatrix}$$

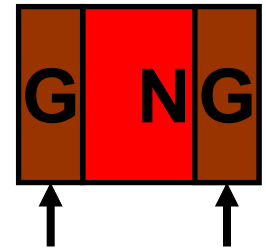
- When the number of transmit antennas Q is 2, 4, 8, etc., then orthogonal signal structures may be used to form the S_k, e.g. for Q = 4,

$$\mathrm{S}_{TS} = \begin{bmatrix} \underline{s}_1 & \underline{s}_1 & \underline{s}_1 & \underline{s}_1 \\ \underline{-s}_1 & \underline{s}_1 & \underline{-s}_1 & \underline{s}_1 \\ \underline{-s}_1 & \underline{s}_1 & \underline{s}_1 & \underline{-s}_1 \\ \underline{-s}_1 & \underline{-s}_1 & \underline{s}_1 & \underline{s}_1 \end{bmatrix}$$

RECEIVER ARCHITECTURE – TIME AND FREQUENCY SYNCHRONIZATION



SIGNAL ACQUISITION PHASE



Step I – Coarse time synchronization

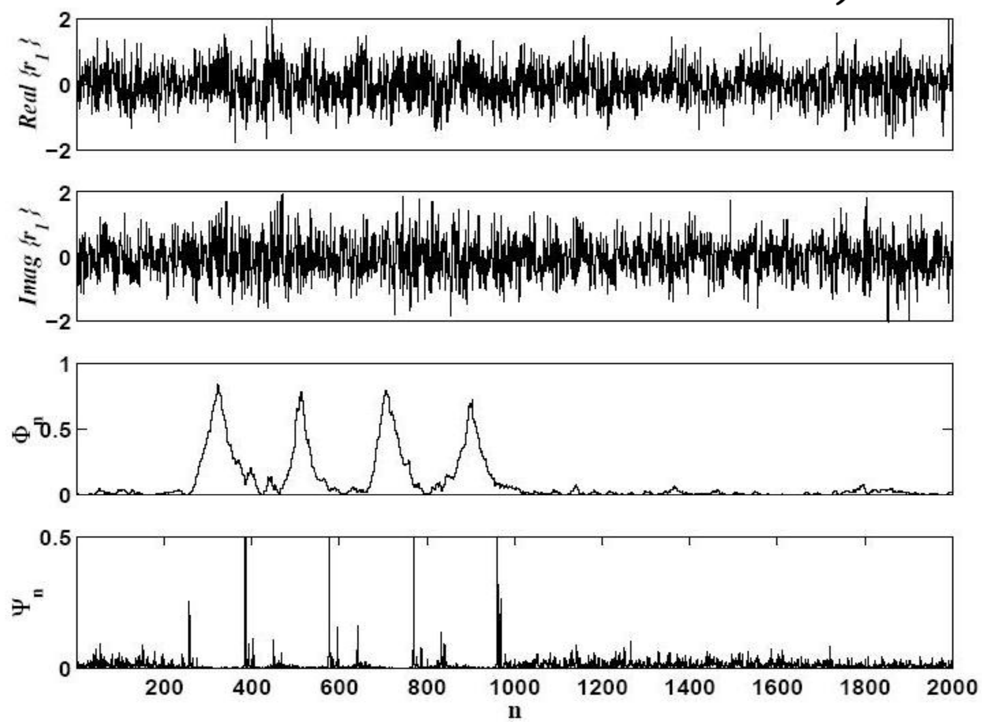
- **Exploiting repeated samples due to the cyclic prefix**

$$n^{\ell}_{\text{coarse}} = \left\{ \arg\max_{n} \left\{ \phi^{\ell}_{n} \right\} : |\phi^{\ell}_{n}| \geq \rho_{\text{coarse}} \cdot (P^{\ell}_{n} + P^{\ell}_{n+N_I}) \right\}.$$

where

$$\phi^{\ell}_{n} = \sum_{k=0}^{G-1} (r^{\ell\,*}_{n+k} \cdot r^{\ell}_{n+k+N_I}),$$

$$P^{\ell}_{n} = \sum_{k=0}^{G-1} |r^{\ell}_{n+k}|^2.$$



To minimize false alarms, we choose

$$\rho_{\text{coarse}} = 0.1$$

J. J. van de Beek et al. ML Estimation of Time and Frequency Offsets in OFDM – IEEE Trans. Signal Proc., July 1997

S. H. Müller-Weinfurtner – Optimality of metrics for coarse frame synchronization in OFDM, IEEE PIMRC ‘98.

SIGNAL ACQUISITION PHASE

Step II – Fractional frequency offset estimation in time domain

- Frequency offsets of up to ±*I*/2 sub-carrier spacings are reflected in the cyclic prefix and the posterior part of the OFDM symbol as a proportional phase shift.

- Estimate the frequency offset as $\hat{\gamma}^{\ell} = \frac{I}{2\pi} \angle \{ \phi^{\ell}_{n^{\ell}_{\text{coarse}}} \},$

- Remove the frequency offset from the sample stream on the ℓ^{th} receiver antenna according to

$$r_n^{1,\ell\,\text{c}} = r_n^{1,\ell} \exp\left\{ -j2\pi\hat{\gamma}^{\ell}[(d-1)(N+G)+n]/N \right\}$$

- Reducing the length of the preamble sub-sequences by a factor of *I* increases the frequency offset estimation range by a factor of *I*, but with a penalty in the MSE performance.

P. H. Moose, A Technique for OFDM Frequency Offset Correction, *IEEE Trans. on Comm*., Oct. 1994

SIGNAL ACQUISITION PHASE

Step III - Integer Frequency Offset Estimation

- The range of the maximum-likelihood frequency offset estimator is ± *I* / 2 sub-channel spacings.

- This frequency offset estimation/ correction range can be increased by using frequency domain processing.

- If the same sequence $s_n^{1,1}$, n=0,…,N_I-1 is transmitted from all the antennas during the first training sub-sequence (i.e., for d=1), a cyclic cross-correlation of the demodulated and fractional frequency offset corrected OFDM symbol with the original sequence can be used to estimate the integer frequency offset.

Integer Frequency Offset Estimation (cont'd)

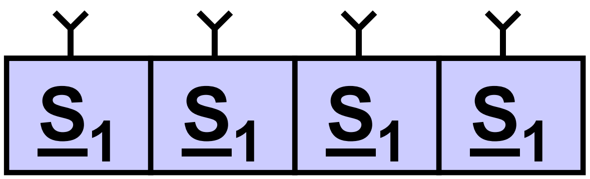
- **Sequence $s_n^{1,1}$ and the received frequency corrected samples**

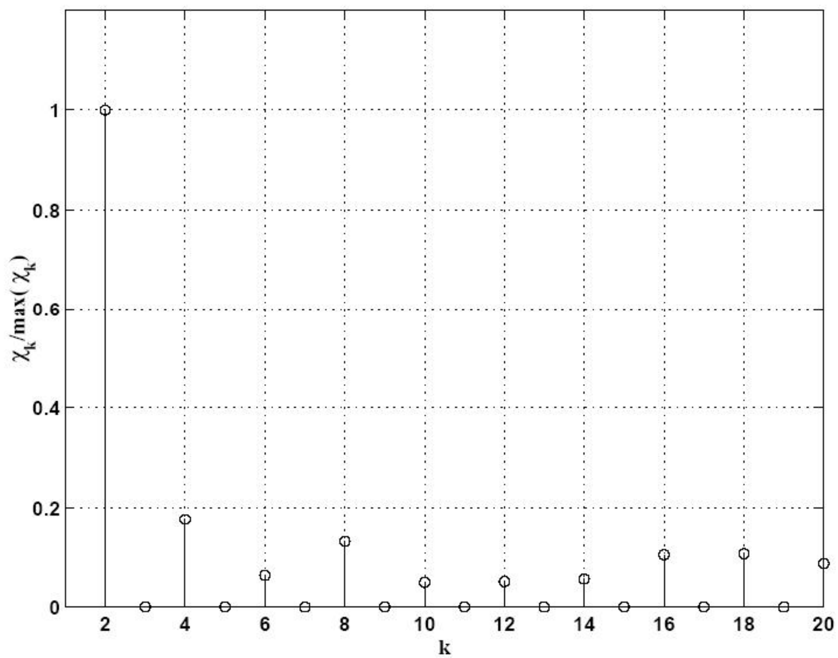
$$r_n^{1,\ell\ c} = r_n^{1,\ell} \exp\left\{-j2\pi\hat{\gamma}^{\ell}[(d-1)(N+G)+n]/N\right\}$$

corresponding to the preamble for *n*=0,1,…,*N*$_I$-1 are repeated *I* times and passed through an *N*-point FFT to obtain $S_n^{1,1}$ and $R_n^{1,lc}$

$$\chi_k = \sum_{\ell=1}^{L} \left| \sum_{n=0}^{uN-1} S_{(k+n)_{uN}}^{1,1\ *} R_n^{1,\ell\ c} \right|$$

$$\hat{\Gamma} = \text{argmax}\{\chi_k\}.$$





H. Zou *et al.*, Receiver implementation for high speed mobile OFDM system, *IEEE GLOBECOM 2001.*

A. N. Mody, G. L. Stüber, "Receiver Implementation for a MIMO OFDM System," *IEEE Global Communications Conference*, Taipei, Taiwan, Nov. 2002.

SIGNAL ACQUISITION PHASE

Step IV – Fine time synchronization

- **Fine time synchronization can be performed by cross-correlating the frequency compensated received samples of the complex envelope with the transmitted time domain preamble sequences.**

$$n^{\ell}_{\mathrm{fine}} = \left\{ \arg \underbrace{\max}_{n} \left\{ \psi^{\ell}_{n} \right\} : \psi^{\ell}_{n} \geq \rho_{\mathrm{fine}} \cdot P^{\ell}_{n} \right\},$$

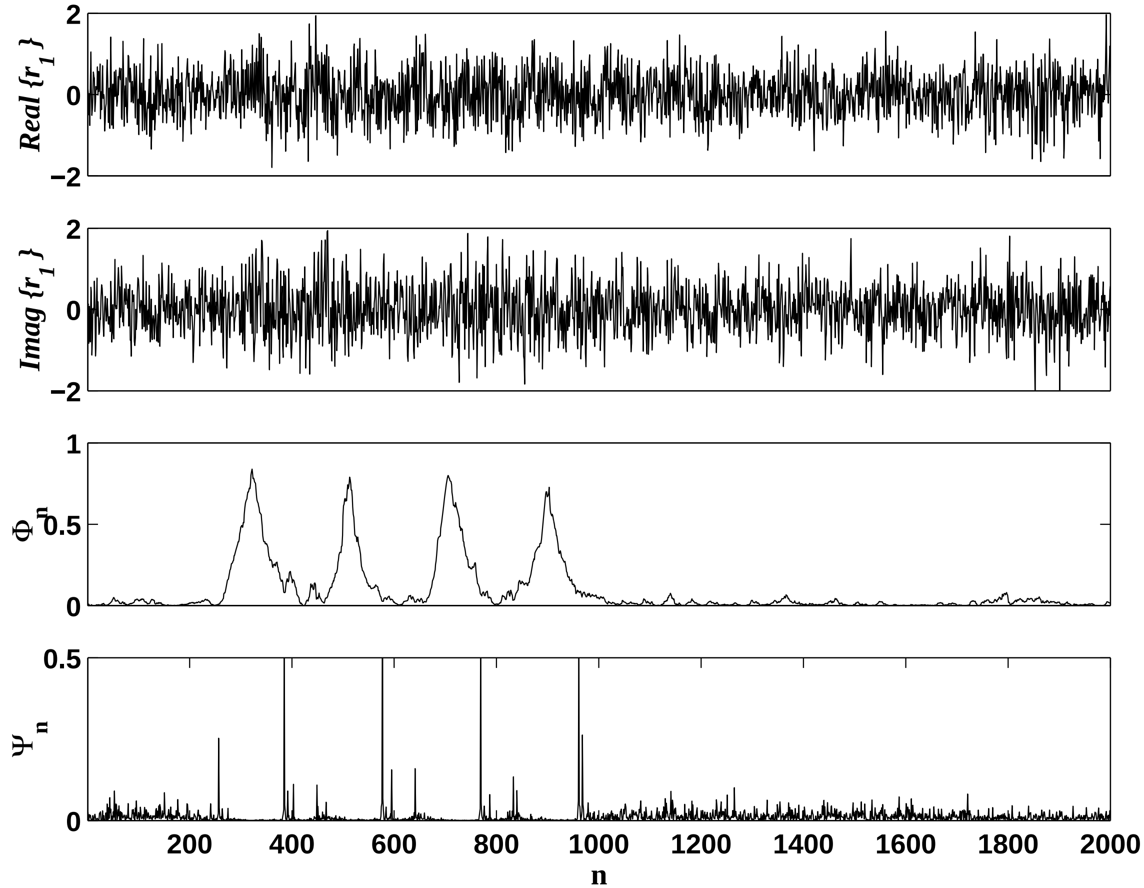
$$\psi^{\ell}_{n} = \sum_{q=1}^{Q} \left| \sum_{k=0}^{uN_I - 1} s_{k}^{1,q\,*} \cdot r^{\ell\ c}_{n+k} \right|$$

$$P^{\ell}_{n} = \sum_{k=0}^{uN_I - 1} |r^{\ell\ c}_{n+k}|^2$$

where $r^{\ell\ c}_{n+k}$ are the received frequency offset corrected samples.

A. N. Mody and G. L. Stüber, Synchronization for MIMO OFDM Systems, *IEEE GLOBECOM 2001.*

TIME SYNCHRONIZATION



Coarse and fine time synchronization for a 4X4 system with N_I= 128, E_s/N_o = 10 dB and frequency offset 1.2 sub-channel spacings. Steps I and IV.

SIGNAL ACQUISITION PHASE

Step V – Channel estimation

In case the statistics of the channel and noise are not available then the least squares (LS) channel estimate at each subcarrier is given by

$$\hat{\mathrm{H}}_k = (\mathrm{B}_k^H \mathrm{B}_k)^{-1} \mathrm{B}_k^H \mathbf{R}_k \quad ,$$

$$\mathrm{B}_k = A_k^d C_k \mathbf{\Lambda}_p \mathbf{S}_k .$$

$$\mathsf{MSE}_{\mathbf{H}} = N_0$$

When *l*≠1, the initial channel coefficients obtained from the preamble must be interpolated/extrapolated to obtain the channel estimates for all the sub-carriers.

Ye Li et al. – Channel Estimation for OFDM with Transmit Receive Diversity – IEEE JSAC March 1999.

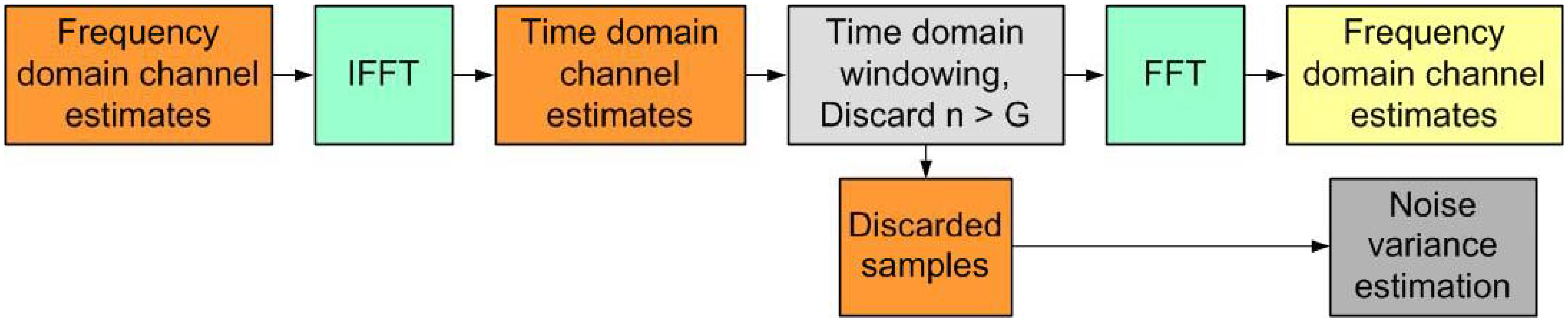
A. N. Mody, G. L. Stüber, "Parameter Estimation for MIMO OFDM Systems," *IEEE Vehicular Technology Conference*, Rhodes, Greece, May 2001.

Ye Li *et al.* – MIMO OFDM Wireless: Signal Det. with Enhanced Ch. Est. – IEEE Trans. On Comm. Sept. 2002

J. H. Kotecha, A. M. Sayeed, Optimal Signal Design for Estimation of Correlated MIMO Channels, IEEE ICC 2003.

Channel estimation (cont'd)

Reducing the MSE of the channel estimates:



$$\mathrm{MSE}_{\mathrm{LB}} = N_0 \frac{G}{N}$$

A. N. Mody, G. L. Stüber, "Parameter Estimation for MIMO OFDM Systems," *IEEE Vehicular Technology Conference*, Rhodes, Greece, May 2001.