

**ECE6604**  
**PERSONAL & MOBILE COMMUNICATIONS**

**Lecture 4**

**Capacity, Flat Fading**

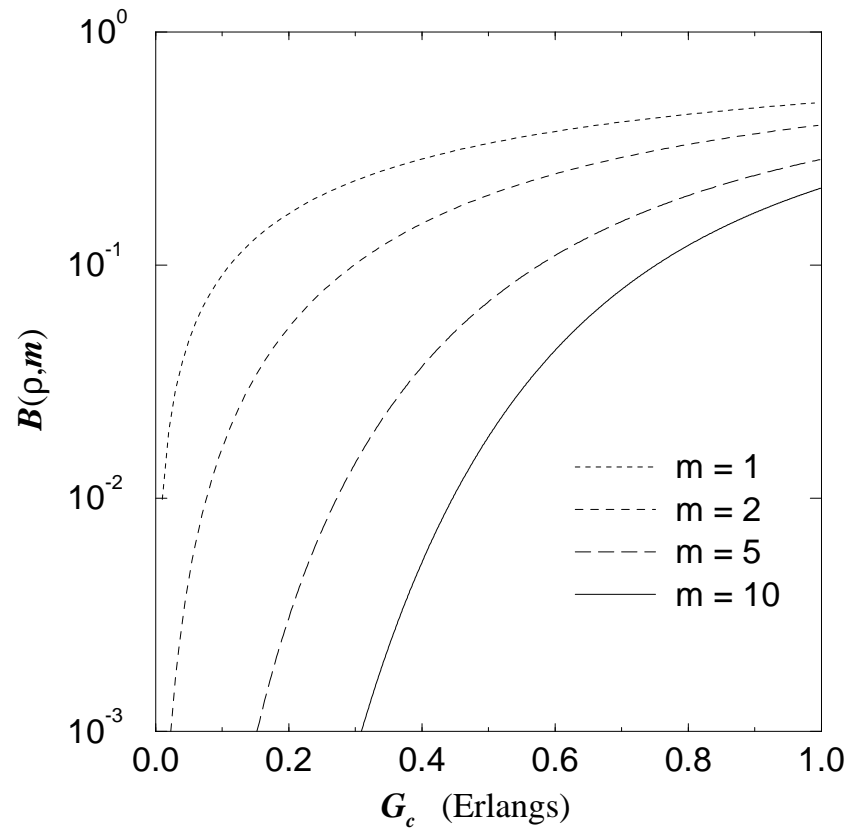
# Capacity

- The capacity of a cellular system is often measured in terms of two quantities
  1. the cell capacity or sector capacity equal to the number of available voice channels per cell or cell sector.
  2. the cell Erlang capacity equal to the traffic carrying capacity of a cell (in Erlangs) for a specified call blocking probability. One Erlang is the traffic intensity in a channel that is continuously occupied, so that a channel occupied for  $x\%$  of the time carries  $x/100$  Erlangs. The Erlang capacity can be calculated using the famous Erlang-B formula

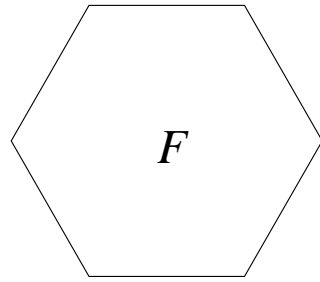
$$B(\rho, m) = \frac{\rho^m}{m! \sum_{k=0}^m \frac{\rho^k}{k!}}$$

where  $B(\rho, m)$  is the call blocking probability,  $m$  is the total number of channels in the trunk and  $\rho = \lambda\mu$  is the total offered traffic in Erlangs ( $\lambda$  is the call arrival rate and  $\mu$  is the mean call duration).

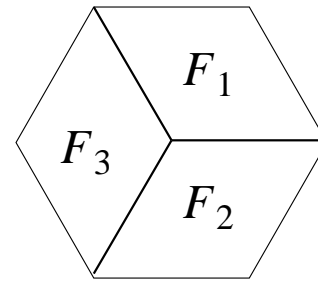
- The Erlang capacity accounts for the trunking efficiency, a phenomenon where larger groups of channels are able to carry more traffic per channel for a given blocking probability than smaller groups of channels.



*Blocking probability  $B(\rho, m)$  against offered traffic per channel  $G_c = \rho/m$ .*



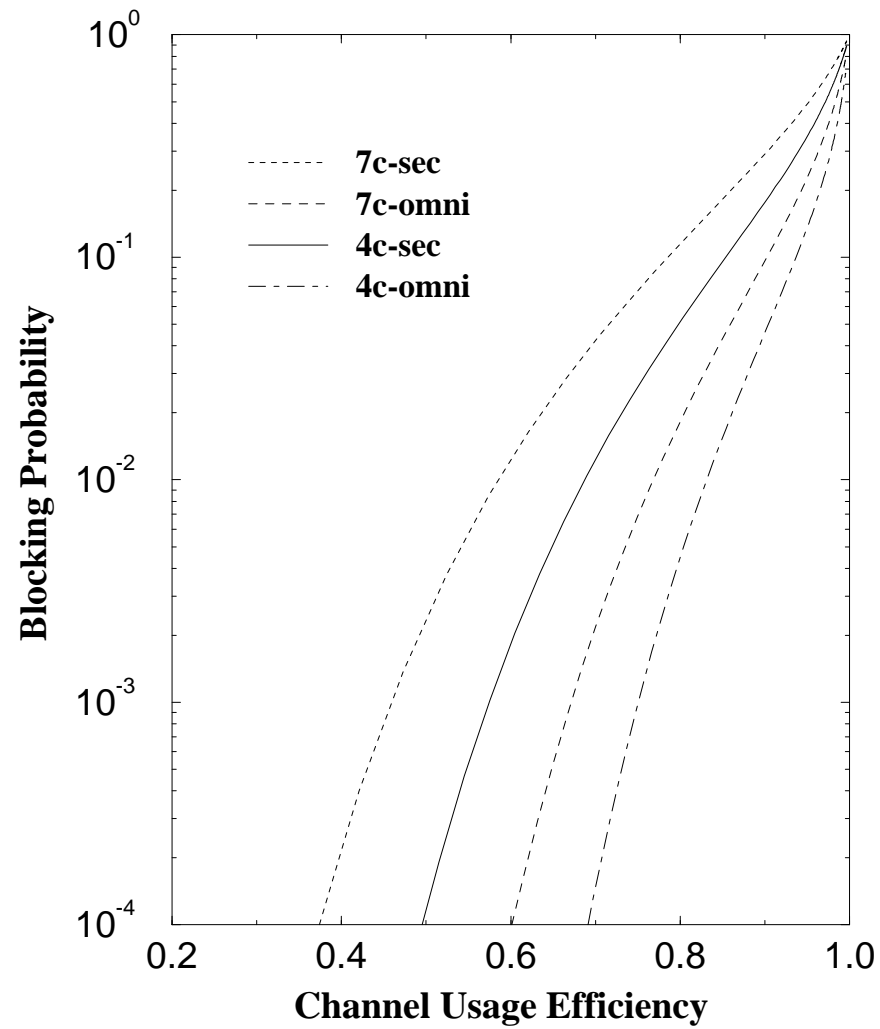
omni



3-sector

$$F = F_1 + F_2 + F_3$$

*Trunkpool schemes.*

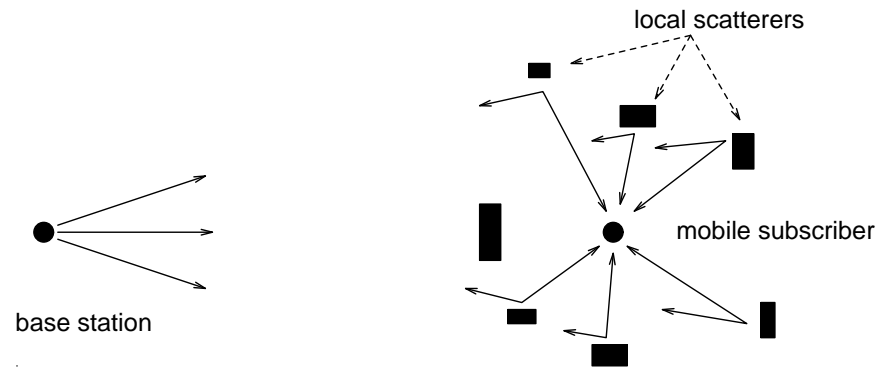


*Channel usage efficiency  $\eta_C = \rho(1 - B(\rho, m))/m$  for different trunkpool schemes;  
416 channels.*

# Some Elements for High Capacity

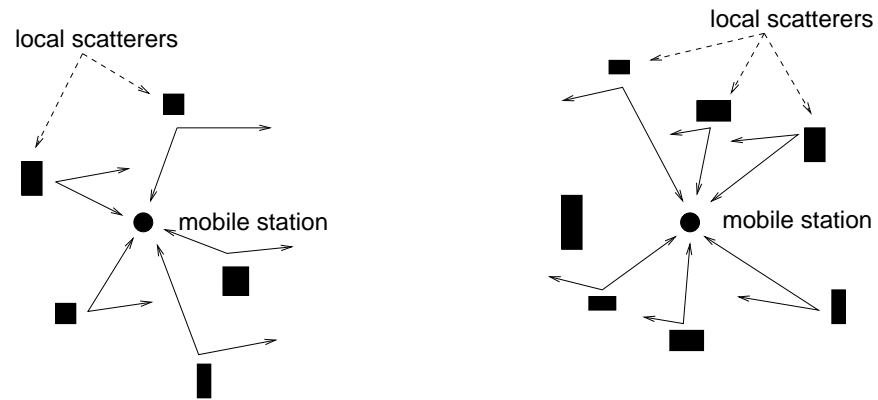
- Our emphasis is on physical wireless communications
- At the physical layer, some of the key elements to high capacity frequency reuse systems are
  - adaptive power and bandwidth efficient modulation
  - multipath-fading mitigation (transmit and receiver diversity, error control coding)
  - techniques to mitigation time delay spread (OFDM, equalizers, RAKE receivers)
  - co-channel interference cancellation (single and multi-antenna interference cancellation)
  - coding modulation (Turbo trellis coding, bit interleaved coded modulation)
  - co-channel interference control (handoffs, power control, space-division multiple access)

# Multipath-Fading Mechanism

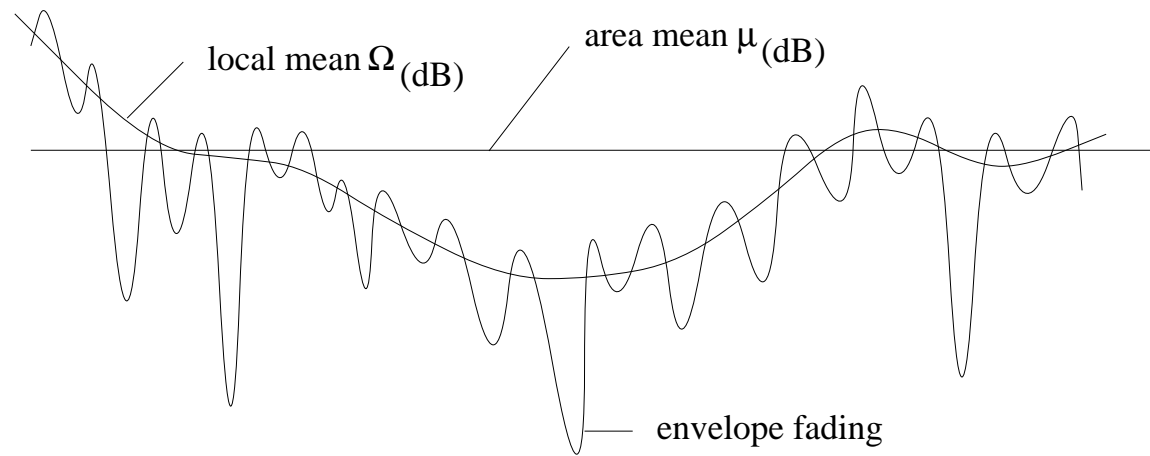


*A typical macrocellular mobile radio environment.*

# Multipath-Fading Mechanism

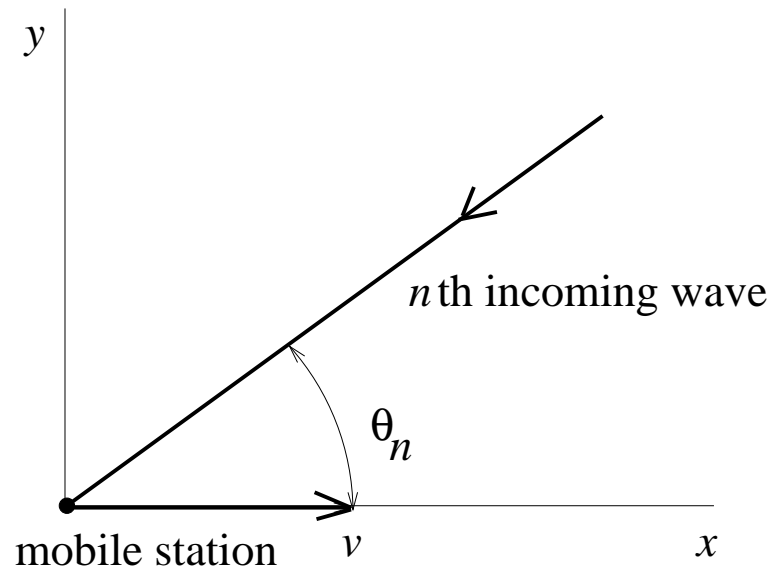


*Typical mobile-to-mobile radio propagation environment.*



*Path loss, shadowing, envelope fading.*

# Doppler Shift



*A typical wave component incident on a mobile station (MS).*

- The Doppler shift is  $f_{D,n} = f_m \cos \theta_n$ , where  $f_m = v/\lambda_c$  ( $\lambda_c$  is the carrier wavelength,  $v$  is the mobile station velocity).

# Multipath Propagation

- Consider the transmission of the band-pass signal

$$s(t) = \text{Re} \left\{ \tilde{s}(t) e^{j2\pi f_c t} \right\}$$

- At the receiver antenna, the  $n$ th plane wave arrives at angle  $\theta_n$  and experiences Doppler shift  $f_{D,n} = f_m \cos \theta_n$  and propagation delay  $\tau_n$ .
- If there are  $N$  propagation paths, the received bandpass signal is

$$r(t) = \text{Re} \left[ \sum_{n=1}^N C_n e^{j2\pi[(f_c + f_{D,n})(t - \tau_n)]} \tilde{s}(t - \tau_n) \right] \quad \tau_n = d_n/c$$

where  $d_n$  depends on the physical scattering geometry which we have yet to specify.

- The received bandpass signal  $r(t)$  has the form

$$r(t) = \text{Re} \left[ \tilde{r}(t) e^{j2\pi f_c t} \right]$$

where the received complex envelope is

$$\tilde{r}(t) = \sum_{n=1}^N C_n e^{j\phi_n(t)} \tilde{s}(t - \tau_n)$$

and

$$\phi_n(t) = 2\pi \left\{ f_{D,n} t - (f_c + f_{D,n}) \tau_n \right\}$$

## Flat Fading

- The channel can be modeled by a linear time-variant filter having the complex low-pass impulse response

$$g(t, \tau) = \sum_{n=1}^N C_n e^{j\phi_n(t)} \delta(\tau - \tau_n)$$

- If the differential path delays  $\tau_i - \tau_j$  are small compared to the duration of a modulated symbol,  $T$ , then the  $\tau_n$  are all approximately equal to their average value  $\hat{\tau}$ .
- The channel impulse response has the form

$$g(t, \tau) = g(t) \delta(\tau - \hat{\tau}) , \quad g(t) = \sum_{n=1}^N C_n e^{j\phi_n(t)} .$$

- The received complex envelope is

$$\tilde{r}(t) = g(t) \tilde{s}(t - \hat{\tau})$$

which experiences fading due to the time-varying complex channel gain  $g(t)$ .

- In the frequency domain, the received complex envelope is

$$\tilde{R}(f) = G(f) * \tilde{S}(f) e^{-j2\pi f \hat{\tau}}$$

Since the channel changes with time,  $G(f)$  has a finite non-zero width in the frequency domain. Due to the convolution operation, the output spectrum  $\tilde{R}(f)$  will be larger than the input spectrum  $\tilde{S}(f)$ . This broadening of the transmitted signal spectrum is caused by the channel time variations and is called “frequency spreading.”

# Channel Transfer Function - Flat Fading

- The corresponding time-variant channel transfer function is obtained by taking the Fourier transform of the time-variant channel impulse response  $g(t, \tau)$  with respect to the delay variable  $\tau$ , i.e.,

$$T(t, f) = g(t)e^{-j2\pi f\hat{\tau}} .$$

- Since the magnitude response is  $|T(t, f)| = |g(t)|$ , all frequency components in the received signal are subject to the same time-variant amplitude gain  $|g(t)|$ , while the phase response is a linear function of frequency with slope  $-2\pi\hat{\tau}$ .
- The received signal is said to exhibit “flat fading,” because the magnitude of the time-variant channel transfer function is constant (or flat) with respect to frequency variable  $f$ .