EE6604

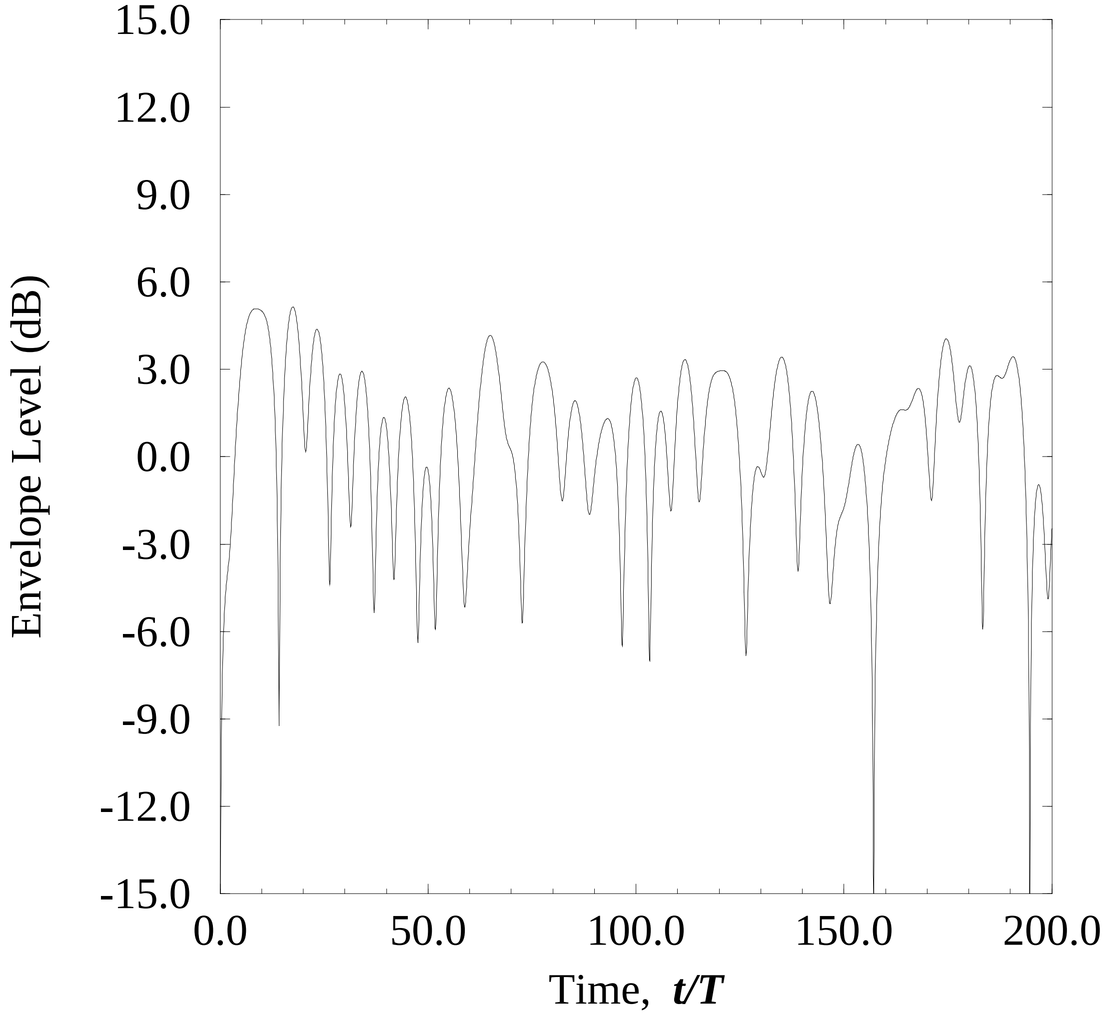
Personal & Mobile Communications

Lecture 8

level Crossing Rate, Average Fade Duration, Zero Crossing Rate

Level Crossing Rate and Average Fade Duration

- The level crossing rate (LCR) is the rate at which the received envelope crosses a specified level in the positive (or negative) going direction.
  - The LCR can be used to estimate velocity.
- The average fade duration (AFD) is the average length of time that the envelope remains below a specified level.
  - The AFD is impacts the outage probability and quality of service.
- Both the LCR and AFD are second-order statistics that depend on the mobile station velocity, as well as the scattering environment.
- The LCR and AFD have been derived by Rice in the context of a sinusoid in narrowband Gaussian noise.



Rayleigh faded envelope with 2-D isotropic scattering.

Level Crossing Rate

- **Obtaining the level crossing rate requires the joint pdf, $p(\alpha, \dot{\alpha})$, of the envelope level $\alpha = |g|$ and the envelope slope $\dot{\alpha}$.**
- **In terms of $p(\alpha, \dot{\alpha})$, the expected amount of time spent in the interval $(R, R+ d\alpha)$ for a given envelope slope $\dot{\alpha}$ and time duration dt is**

$$p(R, \dot{\alpha}) d\alpha d\dot{\alpha} dt$$

- **The time required to cross the level α once for a given envelope slope $\dot{\alpha}$, in the interval $(R, R + d\alpha)$ is**

$$d\alpha / \dot{\alpha}$$

- **The ratio of the above two quantities is the expected number of crossings of the envelope α within the interval $(R, R + d\alpha)$ for a given envelope slope $\dot{\alpha}$ and time duration dt, i.e.,**

$$\dot{\alpha} p(R, \dot{\alpha}) d\dot{\alpha} dt$$

- **The expected number of crossings of the envelope level R for a given envelope slope $\dot{\alpha}$ in a time interval of duration T is**

$$\int_0^T \dot{\alpha} p(R, \dot{\alpha}) d\dot{\alpha} dt = \dot{\alpha} p(R, \dot{\alpha}) d\dot{\alpha} T$$

- **The expected number of crossings of the envelope level R with a positive slope is**

$$N_R = T \int_0^\infty \dot{\alpha} p(R, \dot{\alpha}) d\dot{\alpha} \quad .$$

- **Finally, the expected number of crossings of the envelope level R per second, or the level crossing rate, is**

$$L_R = \int_0^\infty \dot{\alpha} p(R, \dot{\alpha}) d\dot{\alpha}$$

- **This is a general result that applies to any random process.**

- **Rice has derived the joint pdf $p(\alpha, \dot{\alpha})$ for a sine wave plus Gaussian noise. A Rician fading channel consists of LoS or specular (sine wave) component plus a scatter (Gaussian noise) component. For the case of a Rician fading channel,**

$$\begin{aligned} p(\alpha, \dot{\alpha}) \;&=\; \frac{\alpha (2\pi)^{-3/2}}{\sqrt{B b_0}} \int_{-\pi}^{\pi} d\theta \\ &\quad \times \exp\left\{ -\frac{1}{2 B b_0} \left[B\left(\alpha^2 - 2\alpha s \cos\theta + s^2 \right) + \left(b_0 \dot{\alpha} + b_1 s \sin\theta \right)^2 \right] \right\} \end{aligned}$$

  where s is the non-centrality parameter in the Rice distribution, and $B = b_0 b_2 - b_1^2$, where b_0, b_1, and b_2 are constants that depend on the scattering environment.

- **Suppose that the specular or LoS component of the complex envelope $g(t)$ has a Doppler frequency equal $f_q = f_m \cos\theta_0$, where $0 \le |f_q| \le f_m$. Then**

$$\begin{aligned} b_n &= (2\pi)^n \int_{-f_m}^{f_m} S_{gg}^c(f)(f - f_q)^n df \\ &= (2\pi)^n b_0 \int_0^{2\pi} \hat{p}(\theta)\left(f_m \cos\theta - f_q\right)^n d\theta \end{aligned}$$

where $\hat{p}(\theta)$ is the azimuth distribution (pdf) of the *scatter* **component and $S_{gg}^c(f)$ is the corresponding continuous portion of the Doppler power spectrum.**

- **Note that $S_{gg}^c(f)$ is given by the Fourier transform of $\phi_{gg}^c(\tau) = \phi_{g_I g_I}^c(\tau) + j\phi_{g_I g_Q}^c(\tau)$ where**

$$\begin{aligned} \phi_{g_I g_I}^c(\tau) &= \frac{\Omega_p}{2} \int_0^{2\pi} \cos(2\pi f_m \tau \cos\theta)\hat{p}(\theta)d\theta \\ \phi_{g_I g_Q}^c(\tau) &= \frac{\Omega_p}{2} \int_0^{2\pi} \sin(2\pi f_m \tau \cos\theta)\hat{p}(\theta)d\theta \end{aligned}$$

- **In some special cases, the psd $S_{gg}^c(f)$ is symmetrical about the frequency $f_q = f_m \cos\theta_0$. This condition occurs, for example, when $f_q = 0$ ($\theta_0 = 90^{\mathrm{o}}$) and $\hat{p}(\theta) = 1/(2\pi), -\pi \leq \theta \leq \pi$.**

- **In this case, $b_n = 0$ for all odd values of n (and in particular $b_1 = 0$) so that the joint pdf $p(\alpha, \dot{\alpha})$ reduces to the convenient product form**

$$
\begin{aligned}
p(\alpha, \dot{\alpha}) &= \sqrt{\frac{1}{2\pi b_2}} \exp\left\{-\frac{\dot{\alpha}^2}{2b_2}\right\} \cdot \frac{\alpha}{b_0} \exp\left\{-\frac{(\alpha^2 + s^2)}{2b_0}\right\} I_0\left(\frac{\alpha s}{b_0}\right) \\
&= p(\dot{\alpha}) \cdot p(\alpha) \quad .
\end{aligned}
$$

- **Since $p(\alpha, \dot{\alpha}) = p(\dot{\alpha}) \cdot p(\alpha)$, it follows that α and $\dot{\alpha}$ are independent.**

- **When $f_q = 0$ and $\hat{p}(\theta) = 1/(2\pi)$, a closed form expression can be obtained for the envelope level crossing rate.**
- **We have that**

$$b_n = \begin{cases} b_0(2\pi f_m)^n \frac{1\cdot 3\cdot 5 \ \cdots \ (n-1)}{2\cdot 4\cdot 6 \ \cdots \ n} & n \textbf{ even} \\ 0 & n \textbf{ odd} \end{cases} .$$

- **Therefore, $b_1 = 0$ and $b_2 = b_0(2\pi f_m)^2/2$, and**

$$L(R) = \sqrt{2\pi(K+1)} f_m \rho e^{-K-(K+1)\rho^2} I_0\left(2\rho\sqrt{K(K+1)}\right)$$

where

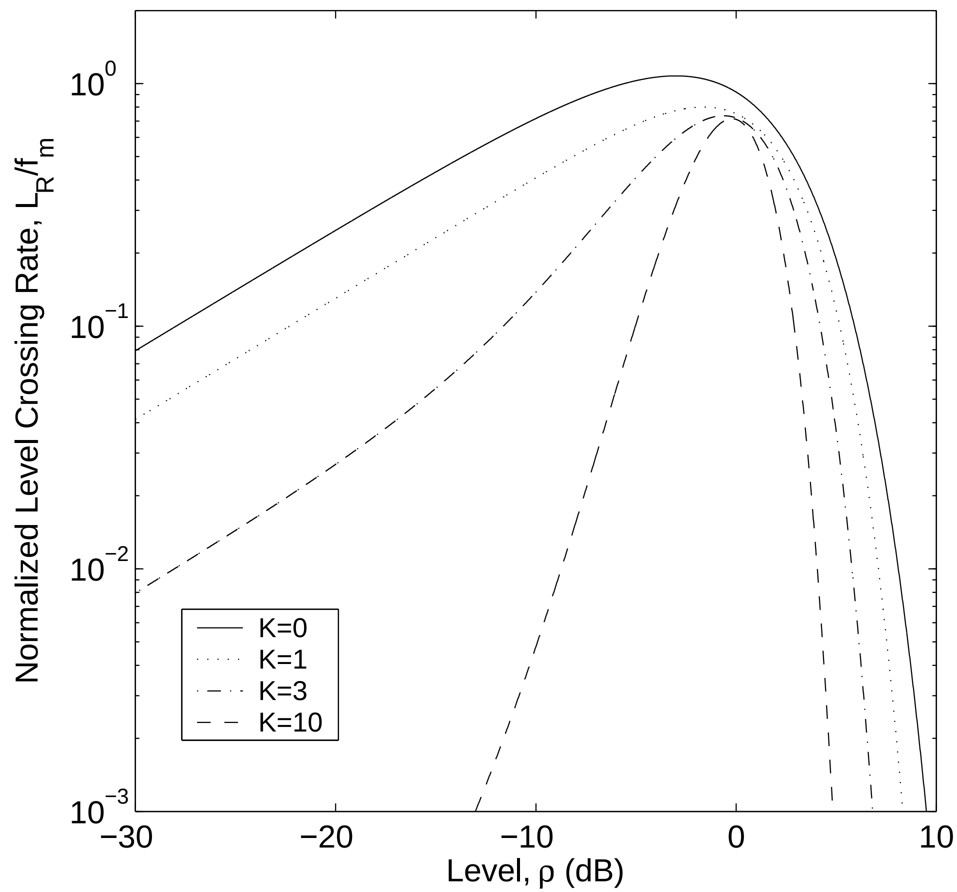
$$\rho = \frac{R}{\sqrt{\Omega_p}} = \frac{R}{R_{\rm rms}}$$

and $R_{\rm rms} \triangleq \sqrt{{\rm E}[\alpha^2]}$ is the *rms* envelope level.

- **Under the further condition that $K = 0$ (Rayleigh fading)**

$$L(R) = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

- **Notice that the level crossing rate is directly proportional to the maximum Doppler frequency f_m and, hence, the MS speed $v = f_m/\lambda_c$.**



Normalized level crossing rate for Rician fading. A specular component arrives with angle $\theta_0 = 90^{\circ}$ and there is 2-D isotropic scattering of the scatter component.

Average Fade Duration

- No known expression exists for the duration of fades; an open problem! Therefore, we consider the "**average fade duration**".
- Consider a very long time interval of length T, and let t_i be the duration of the ith fade below the level R.
- The probability that the received envelope is less than R is

$$\Pr[r \leq R] = \frac{1}{T} \sum_i t_i$$

- The average fade duration is equal to

$$\bar{t} = \frac{1}{TL(R)} \sum_i t_i = \frac{\Pr[r \leq R]}{L(R)}$$

- **If the envelope is Rician distributed, then**

$$\Pr[r \leq R] = \int_0^R p(r)dr = 1 - Q\left(\sqrt{2K}, \sqrt{2(K+1)\rho^2}\right)$$

where $Q(a,b)$ is the Marcum Q function.

- **if we again assume that $f_q = 0$ and $\hat{p}(\theta) = 1/(2\pi)$, we have**

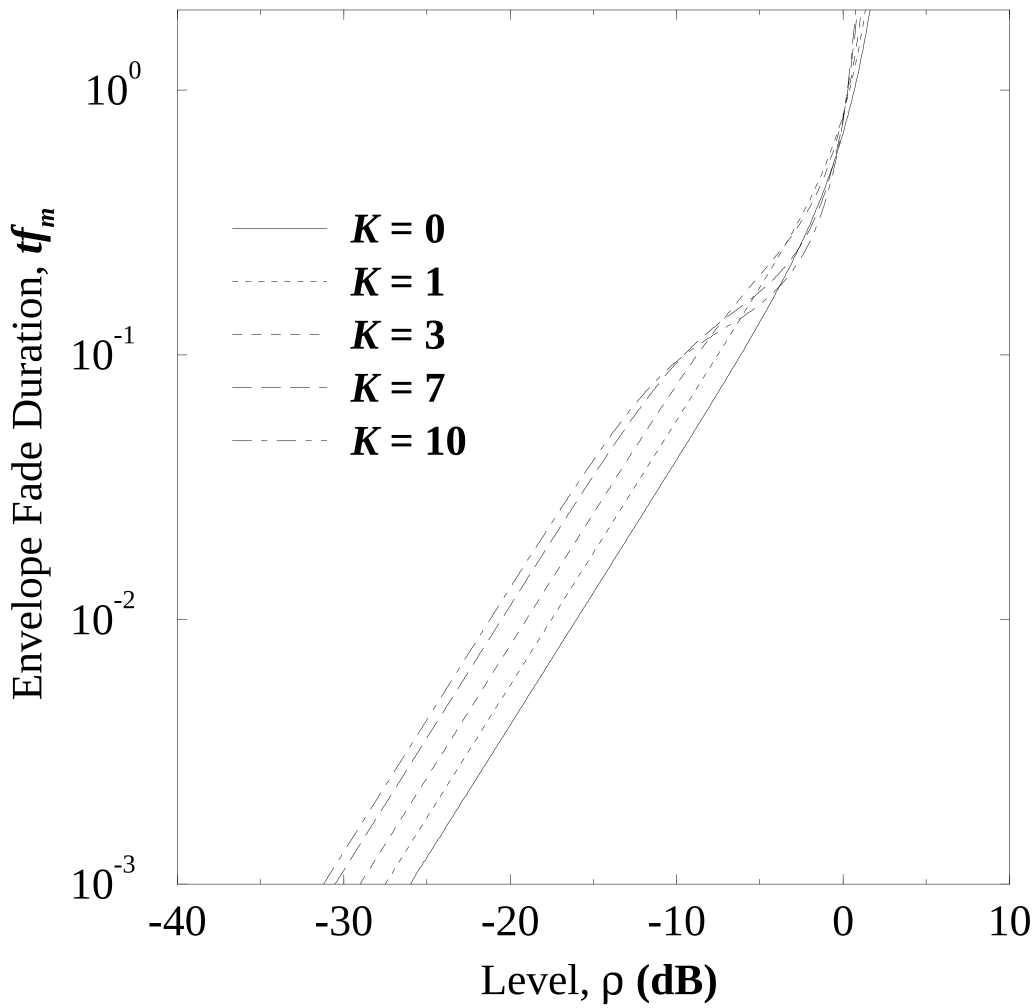
$$\bar{t} = \frac{1 - Q\left(\sqrt{2K}, \sqrt{2(K+1)\rho^2}\right)}{\sqrt{2\pi(K+1)}f_m \rho e^{-K-(K+1)\rho^2} I_0\left(2\rho\sqrt{K(K+1)}\right)}$$

- **If we further assume that $K = 0$ (Rayleigh fading), then**

$$\mathrm{P}(\alpha \leq R) = \int_0^R p(\alpha)d\alpha \ = \ 1 - e^{-\rho^2}$$

and

$$\bar{t} \ = \ \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} \ .$$



Normalized average fade duration with Ricean fading.

Zero Crossing Rate

- **Recall that received complex envelope $g(t) = g_I(t) + g_Q(t)$ is a complex Gaussian random process. If the channel is characterized by a specular or LoS component, then $g_I(t)$ and $g_Q(t)$ have mean values $m_I(t)$ and $m_Q(t)$, respectively. Here we are interested in the "zero crossing rate" of the zero-mean Gaussian random processes $\hat{g}_I(t) = g_I(t) - m_I(t)$ and $\hat{g}_Q(t) = g_Q(t) - m_Q(t)$.**

- **Rice has derived this zero crossing rate as**

$$L_Z = \frac{1}{\pi}\sqrt{\frac{b_2}{b_0}} \quad .$$

- **When the scatter component has the azimuth distribution $\hat{p}(\theta) = 1/(2\pi)$, $-\pi \leq \theta \leq \pi$, the zero crossing rate is**

$$L_Z = \sqrt{2} f_m \quad .$$

- **Similar to the level crossing rate, the zero crossing rate is directly proportional to the maximum Doppler frequency $f_m = v/\lambda_c$.**