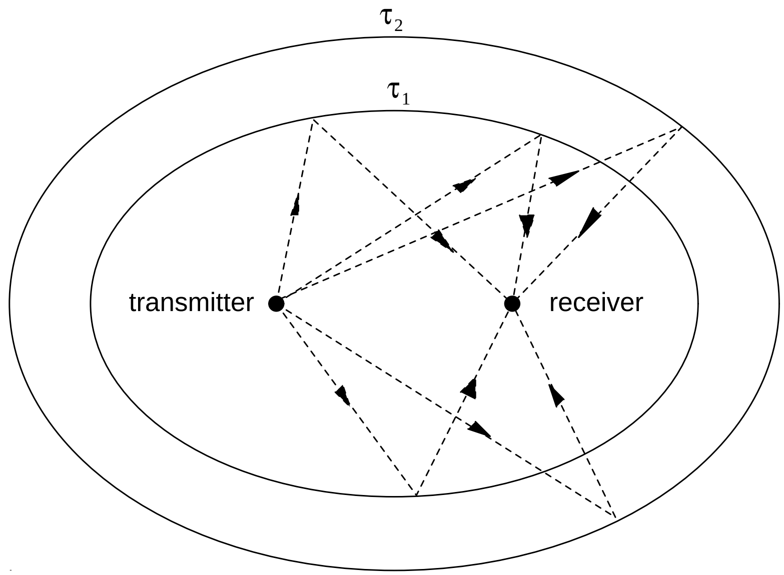
EE6604

Personal & Mobile Communications

Lecture 9

Statistical Channel Modeling, COST207 Models

Scattering Mechanism for Wideband Channels



Concentric ellipses model for frequency-selective fading channels.

- **Frequency-selective (wide-band) channels have strong scatterers that are located on several ellipses such that the corresponding differential path delays $\tau_i - \tau_j$ for some i, j, are significant compared to the modulated symbol period T.**

Transmission Functions

- **Multipath fading channels are time-variant linear filters, whose inputs and outputs can be described in the time and frequency domains.**
- **There are four possible transmission functions**
  - **Time-variant channel impulse response** $g(t, \tau)$
  - **Output Doppler spread function** $H(f, \nu)$
  - **Time-variant transfer function** $T(f, t)$
  - **Doppler-spread function** $S(\tau, \nu)$

Time-variant channel impulse response, $g(t, \tau)$

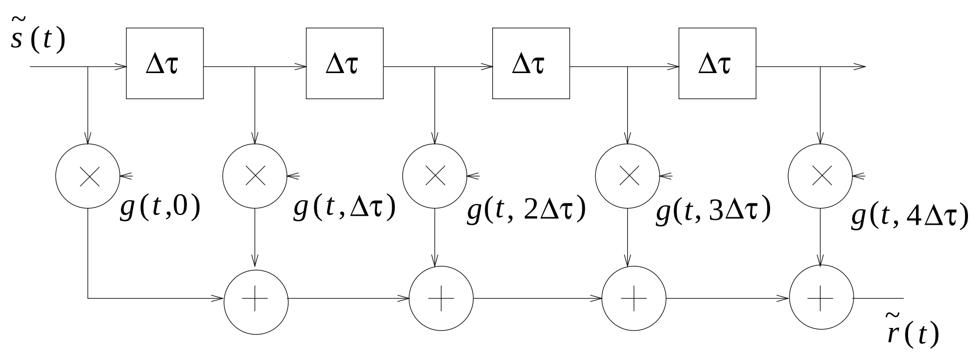
- **Also known as the input delay spread function.**
- **The time varying complex channel impulse response relates the input and output time domain waveforms**

$$\tilde{r}(t) = \int_{-\infty}^{\infty} \tilde{s}(t-\tau) g(t,\tau) d\tau$$

  - **In physical terms, $g(t, \tau)$ can be interpreted as the channel response at time t due to an impulse applied at time $t - \tau$. Since a physical channel is causal, $g(t, \tau) = 0$ for $\tau < 0$ and, therefore, the lower limit of integration in the convolution integral is zero.**

- **The convolution integral can be written in the discrete form**

$$\tilde{r}(t) = \sum_{m=0}^{n} \tilde{s}(t - m\Delta\tau) g(t, m\Delta\tau) \Delta\tau$$



Discrete-time tapped delay line model for a multipath-fading channel.

Transfer Function, $T(f,t)$

- **The transfer function relates the input and output frequencies:**

$$\tilde{R}(f,t) = \tilde{S}(f)T(f,t)$$

- **By using an inverse Fourier transform, we can also write**

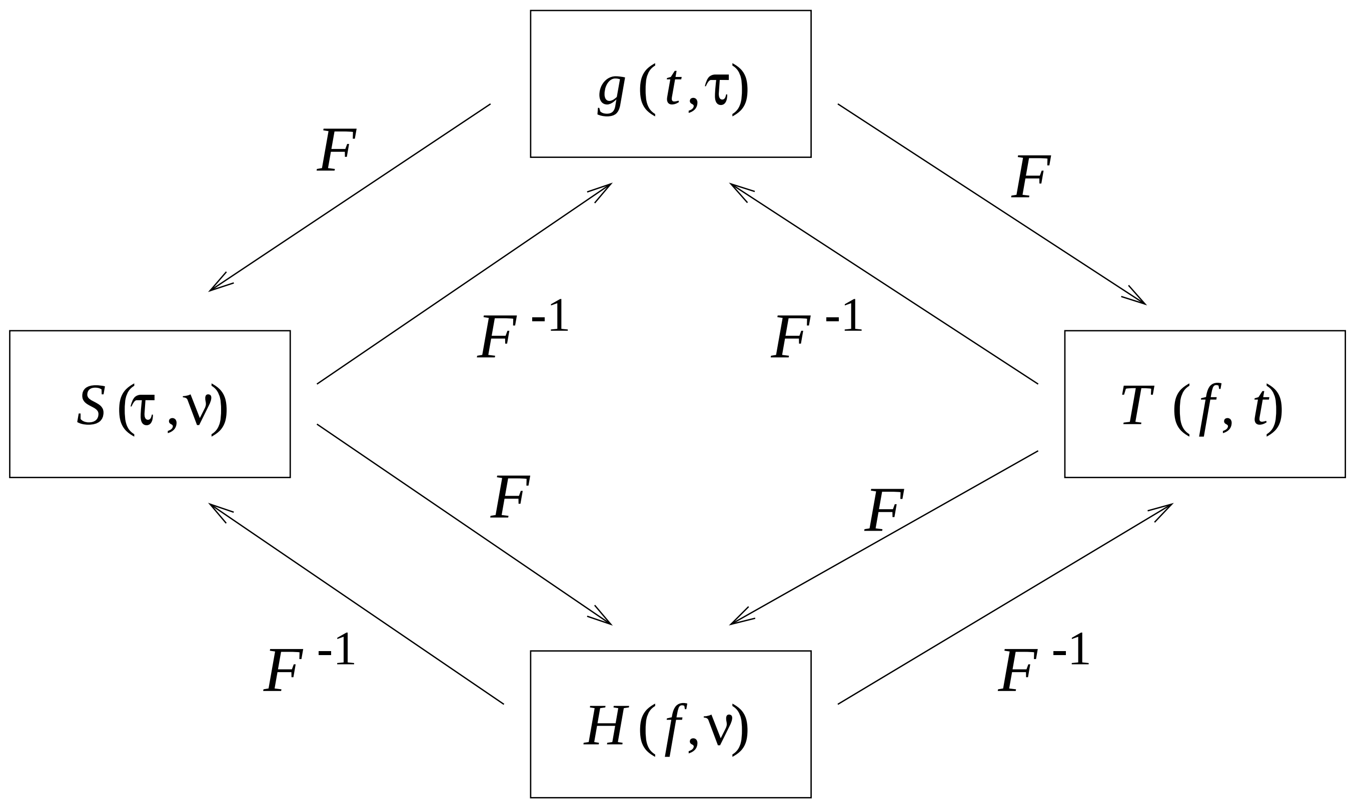
$$\tilde{r}(t) = \int_{-\infty}^{\infty} \tilde{S}(f)T(f,t)e^{j2\pi ft}df$$

- **The time-varying channel impulse response and time-varying channel transfer function are related through the Fourier transform:**

$$g(t,\tau) \Longleftrightarrow T(f,t)$$

  – **Note: the Fourier transform pair is with respect to the time-delay variable τ. The Fourier transform of $g(t,\tau)$ with respect to the time variable t gives the Doppler spread function.**

Fourier Transforms



Fourier transform relations between the system functions.

Statistical Correlation Functions

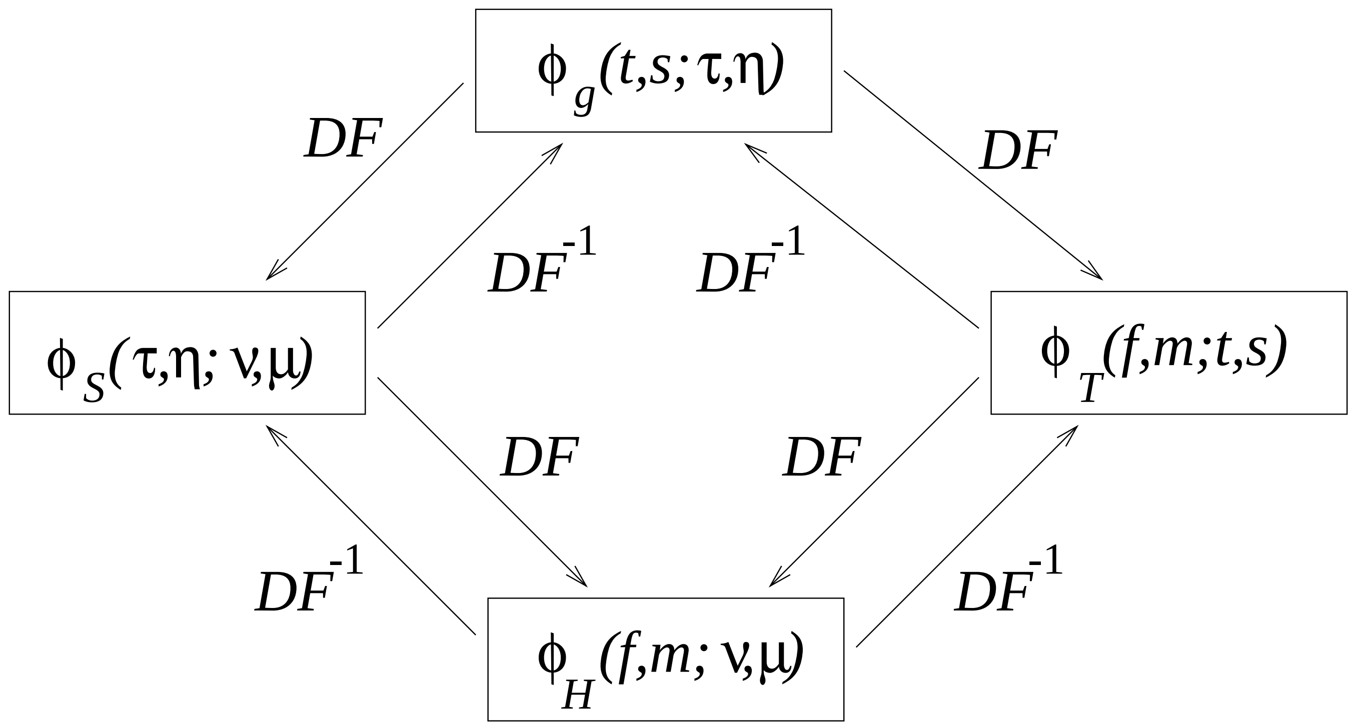
- **Similar to flat fading channels, the channel impulse response $g(t,\tau) = g_I(t,\tau) + jg_Q(t,\tau)$ of frequency-selective fading channels can be modelled as a complex Gaussian random process, where the quadrature components $g_I(t,\tau)$ and $g_Q(t,\tau)$ are Gaussian random processes.**
- **The transmission functions are all random processes. Since the underlying process is Gaussian, a complete statistical description of these transmission functions is provided by their means and autocorrelation functions.**
- **Four correlation functions can be defined**

$$\begin{aligned}
\phi_g(t,s;\tau,\eta) &= \frac{1}{2}\mathrm{E}[g(t,\tau)g^*(s,\eta)] \\
\phi_T(f,m;t,s) &= \frac{1}{2}\mathrm{E}[T(f,t)T^*(m,s)] \\
\phi_H(f,m;\nu,\mu) &= \frac{1}{2}\mathrm{E}[H(f,\nu)H^*(m,\mu)] \\
\phi_S(\tau,\eta;\nu,\mu) &= \frac{1}{2}\mathrm{E}[S(\tau,\nu)S^*(\eta,\mu)] \quad .
\end{aligned}$$

- **Double Fourier transforms**

$$\begin{aligned}
\phi_S(\tau,\eta;\nu,\mu) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi_g(t,s;\tau,\eta)e^{-j2\pi(\nu t-\mu s)}dtds \qquad \text{Correction!} \\
\phi_g(t,s;\tau,\eta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi_S(\tau,\eta;\nu,\mu)e^{j2\pi(\nu t-\mu s)}d\nu d\mu \qquad \text{Correction!}
\end{aligned}$$

Fourier Transforms and Correlation Functions



Double Fourier transform relations between the channel correlation functions.

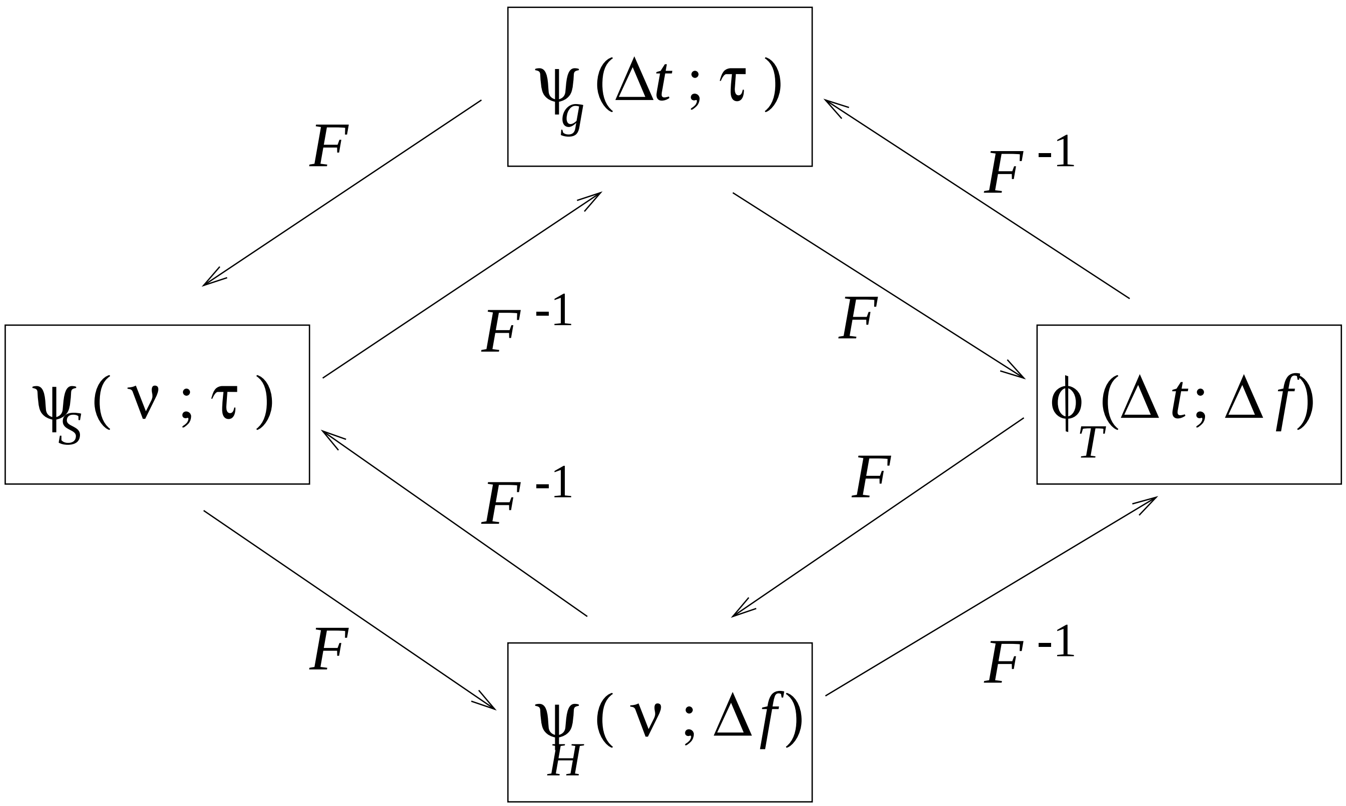
WSSUS Channels

- **Uncorrelated scattering in both the time-delay and Doppler shift domains.**
- **Practical radio channels are characterized by this behavior.**
- **Due to uncorrelated scattering in time-delay and Doppler shift, the channel correlation functions become:**

$$\begin{aligned}
\phi_g(t, t+\Delta t; \tau, \eta) &= \psi_g(\Delta t; \tau)\delta(\eta - \tau) \\
\phi_T(f, f+\Delta f; t, t+\Delta t) &= \phi_T(\Delta f; \Delta t) \\
\phi_H(f, f+\Delta f; \nu, \mu) &= \psi_H(\Delta f; \nu)\delta(\nu - \mu) \\
\phi_S(\tau, \eta; \nu, \mu) &= \psi_S(\tau, \nu)\delta(\eta - \tau)\delta(\nu - \mu) \quad .
\end{aligned}$$

- **Note the singularities $\delta(\eta - \tau)$ and $\delta(\nu - \mu)$ with respect to the time-delay and Doppler shift variables, respectively.**
- **Some correlation functions are more useful than others. The most useful functions:**
  - $\psi_g(\Delta t; \tau)$**: channel correlation function**
  - $\phi_T(\Delta f; \Delta t)$**: spaced-time spaced-frequency correlation function**
  - $\psi_S(\tau, \nu)$**: scattering function**

Fourier Transforms for WSSUS Channels



Power Delay Profile

- **The autocorrelation function of the time varying impulse response is**

$$\begin{aligned}\phi_g(t, t+\Delta t, \tau, \eta) &= \frac{1}{2}\mathrm{E}\left[g(t,\tau)g^*(t+\Delta t, \eta)\right] \\ &= \psi_g(\Delta t; \tau)\delta(\eta - \tau)\end{aligned}$$

- **The function $\psi_g(0;\tau) \equiv \psi_g(\tau)$ is called the multipath intensity profile or power delay profile.** Small Correction!

- **The average delay μ_τ is the mean value of $\psi_g(\tau)$, i.e.,**

$$\mu_\tau = \frac{\int_0^\infty \tau \psi_g(\tau)\mathrm{d}\tau}{\int_0^\infty \psi_g(\tau)\mathrm{d}\tau} \qquad \text{SmallCorrection!}$$

- **The delay spread σ_τ is defined as the variance of $\psi_g(\tau)$, i.e.,**

$$\sigma_\tau = \sqrt{\frac{\int_0^\infty (\tau - \mu_\tau)^2 \psi_g(\tau)\mathrm{d}\tau}{\int_0^\infty \psi_g(\tau)\mathrm{d}\tau}} \qquad \text{SmallCorrection!}$$

Wideband Channel Models

- **Wide-band channel can be modeled by a tapped delay line with irregularly spaced tap delays. Each channel tap is the superposition of a large number of scattered plane waves that arrive with approximately the same delay. and, therefore, the channel taps will undergo fading.**

- **The wide-band channel has the time-variant impulse response**

$$g(t,\tau) = \sum_{i=1}^{n} g_i(t)\delta(\tau - \tau_i) \quad ,$$

where n is the number of channel taps, and the $\{g_i(t)\}$ and $\{\tau_i\}$ are the complex gains and path delays associated with the channel taps.

- **The channel can be described by the tap gain vector**

$$\mathbf{g}(t) = (g_1(t),\ g_2(t),\ \ldots,\ g_n(t))^{\mathrm{T}}$$

and the tap delay vector

$$\tau = (\tau_1, \tau_2,\ \ldots, \tau_n) \quad .$$

COST207 Models

- **The COST207 models were developed for the GSM system. COST207 specifies four different Doppler spectra, $S_{gg}(f)$. Define**

$$G(A, f_1, f_2) = A \exp\left\{ -\frac{(f - f_1)^2}{2f_2^2} \right\}$$

The following types are defined;

a) **CLASS is used for path delays less than 500 ns ($\tau_i \leq 500$ ns);**

$$\text{(CLASS)} \quad S_{gg}(f) = \frac{A}{\sqrt{1 - (f/f_m)^2}} \quad |f| \leq f_m$$

b) **GAUS1 is used for path delays from 500 ns to $2\ \mu$s; ($500 \text{ ns} \leq \tau_i \leq 2\mu\text{s}$)**

$$\text{(GAUS1)} \quad S_{gg}(f) = G(A, -0.8f_m, 0.05f_m) + G(A_1, 0.4f_m, 0.1f_m)$$

where A_1 is 10 dB below A.

c) **GAUS2 is used for path delays exceeding $2\ \mu$s; ($\tau_i > 2\ \mu$s)**

$$\text{(GAUS2)} \quad S_{gg}(f) = G(B, 0.7f_m, 0.1f_m) + G(B_1, -0.4f_m, 0.15f_m)$$

where B_1 is 15 dB below B.

d) **RICE is a sometimes used for the direct ray;**

$$\text{(RICE)} \quad S_{gg}(f) = \frac{0.41}{2\pi f_m \sqrt{1 - (f/f_m)^2}} \ + \ 0.91\delta(f - 0.7f_m) \quad |f| \leq f_m$$

COST207 Models

- **The COST 207 specifies a number of continuous power delay profiles, $\psi_g(\tau)$. Here, they are normalized so that $\int_0^\infty \psi_g(\tau)d\tau = 1$.**
- **For rural (non-hilly) areas (RA) the power delay profile is:**

$$\psi_g(\tau) = \begin{cases} \frac{9.2}{1-e^{-6.44}} e^{-9.2\tau} \;, & 0 \le \tau \le 0.7 \\ 0 & \text{elsewhere} \end{cases}$$

- **For typical urban (TU) (non-hilly) areas the power delay profile is:**

$$\psi_g(\tau) = \begin{cases} \frac{1}{1-e{-7}} e^{-\tau} \;, & 0 \le \tau \le 7 \\ 0 & \text{elsewhere} \end{cases}$$

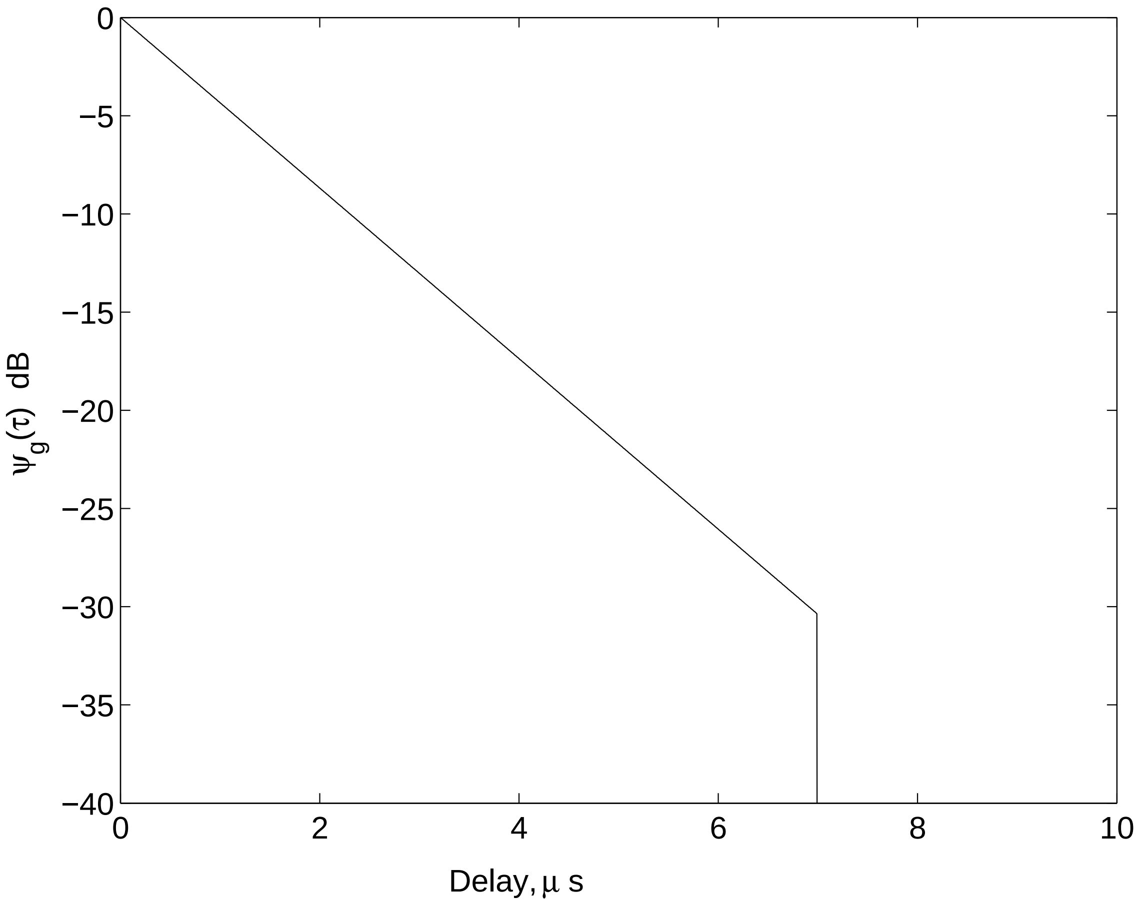
- **For bad urban (BU) (non-hilly) areas the power delay profile is:**

$$\psi_g(\tau) = \begin{cases} \frac{2}{3(1-e^{-5})} e^{-\tau} \;, & 0 \le \tau \le 5 \\ \frac{2}{3(1-e^{-5})} * 0.5 * e^{5-\tau} \;, & 5 \le \tau \le 10 \\ 0 & \text{elsewhere} \end{cases}$$

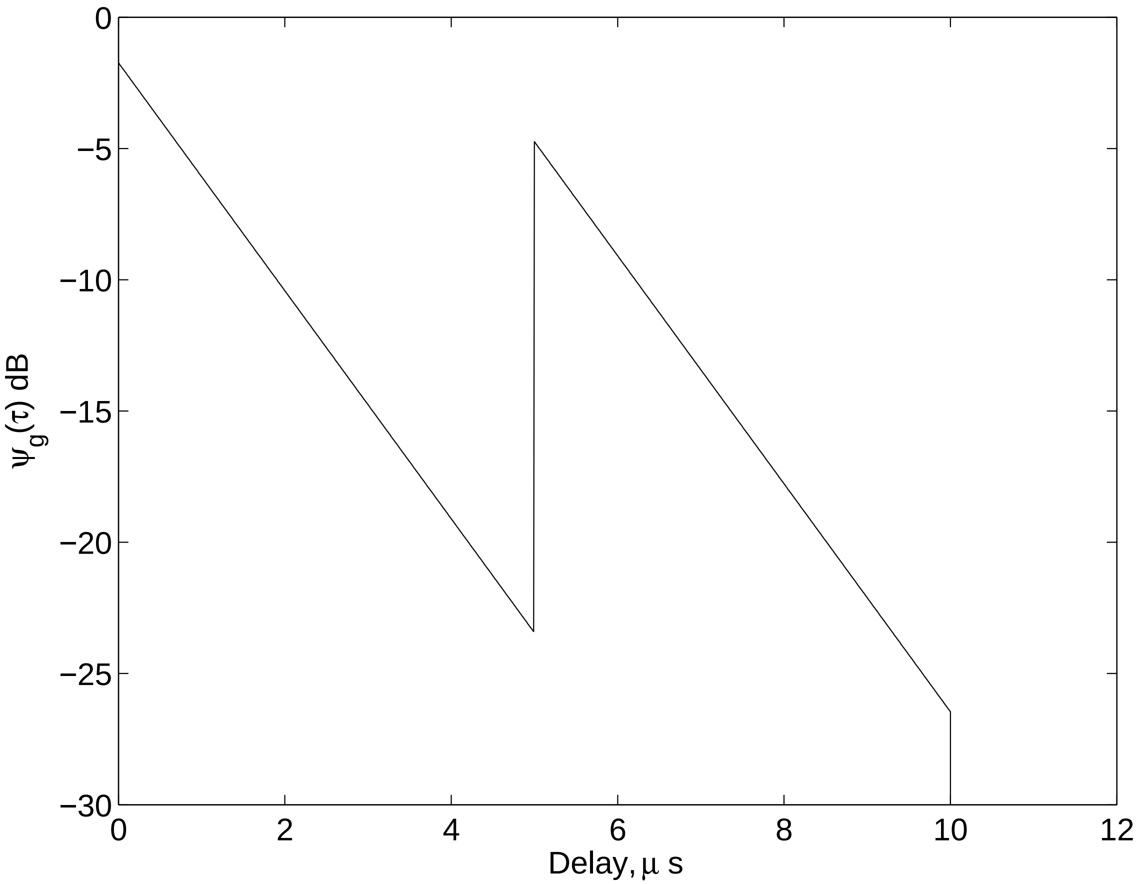
- **For hilly terrain (HT) areas the power delay profile is:**

$$\psi_g(\tau) = \begin{cases} \frac{1}{(1-e^{-7})/3.5+0.1(1-e^{-5})} e^{-3.5\tau} \;, & 0 \le \tau \le 2 \\ \frac{1}{(1-e^{-7})/3.5+0.1(1-e^{-5})} * 0.1 * e^{15-\tau} \;, & 15 \le \tau \le 20 \\ 0 & \text{elsewhere} \end{cases}$$

COST 207 typical case for urban (non-hilly) area (TU)



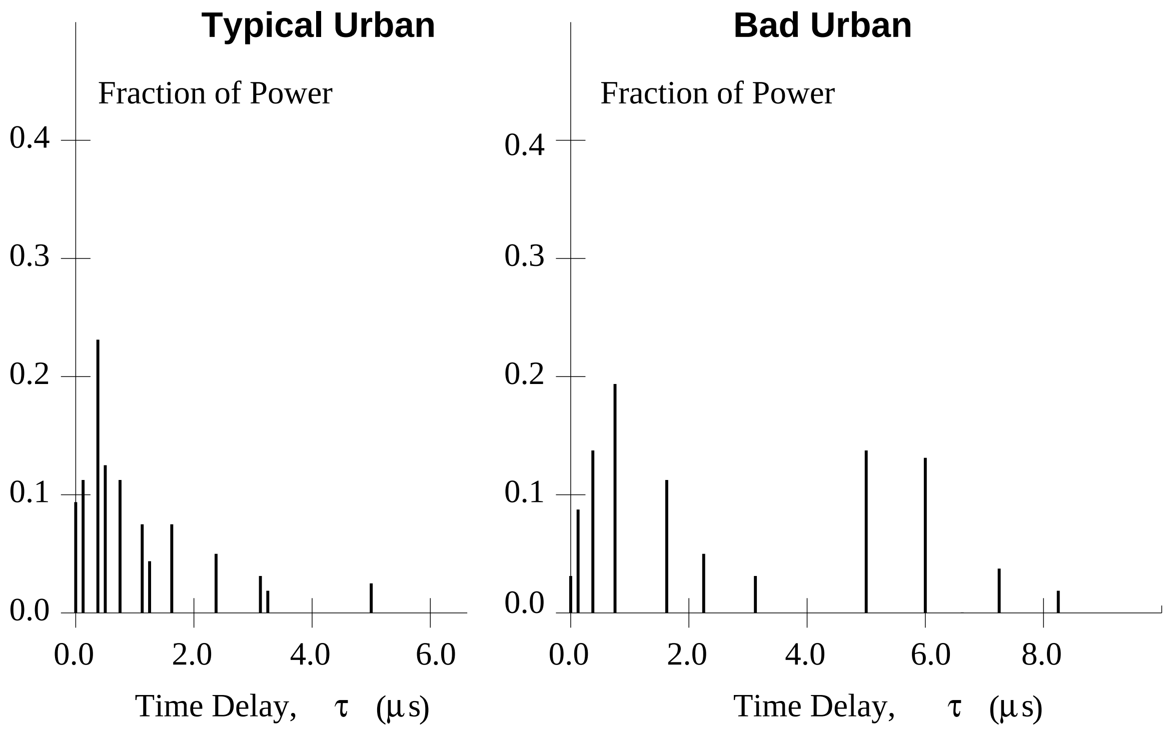
COST 207 typical case for bad urban (non-hilly) area (BU)



Typical Urban (TU) and Bad Urban (BU) 12-ray models

Typical Urban (TU)			Bad Urban (BU)		
delay μs	Fractional Power	Doppler Category	delay μs	Fractional Power	Doppler Category
0.0	**0.092**	CLASS	**0.0**	**0.033**	CLASS
0.1	**0.115**	CLASS	**0.1**	**0.089**	CLASS
0.3	**0.231**	CLASS	**0.3**	**0.141**	CLASS
0.5	**0.127**	CLASS	**0.7**	**0.194**	GAUS1
0.8	**0.115**	GAUS1	**1.6**	**0.114**	GAUS1
1.1	**0.074**	GAUS1	**2.2**	**0.052**	GAUS2
1.3	**0.046**	GAUS1	**3.1**	**0.035**	GAUS2
1.7	**0.074**	GAUS1	**5.0**	**0.140**	GAUS2
2.3	**0.051**	GAUS2	**6.0**	**0.136**	GAUS2
3.1	**0.032**	GAUS2	**7.2**	**0.041**	GAUS2
3.2	**0.018**	GAUS2	**8.1**	**0.019**	GAUS2
5.0	**0.025**	GAUS2	**10.0**	**0.006**	GAUS2

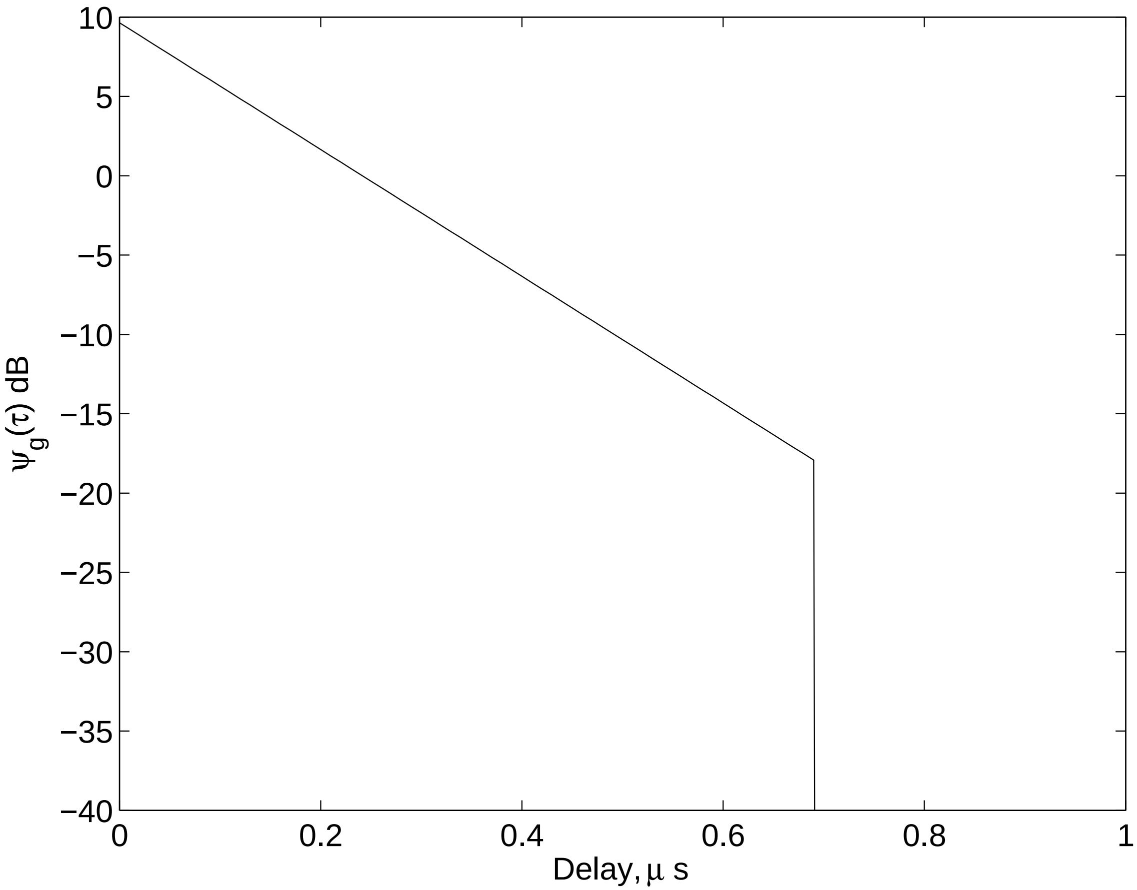
Typical Urban (TU) and Bad Urban (BU) 12-ray models



Reduce typical Urban (RTU) and Reduced Bad Urban (RBU) 6-ray models

Typical Urban (TU)			Bad Urban (BU)		
delay μs	Fractional Power	Doppler Category	delay μs	Fractional Power	Doppler Category
0.0	**0.189**	CLASS	**0.0**	**0.164**	CLASS
0.2	**0.379**	CLASS	**0.3**	**0.293**	CLASS
0.5	**0.239**	CLASS	**1.0**	**0.147**	GAUS1
1.6	**0.095**	GAUS1	**1.6**	**0.094**	GAUS1
2.3	**0.061**	GAUS2	**5.0**	**0.185**	GAUS2
5.0	**0.037**	GAUS2	**6.6**	**0.117**	GAUS2

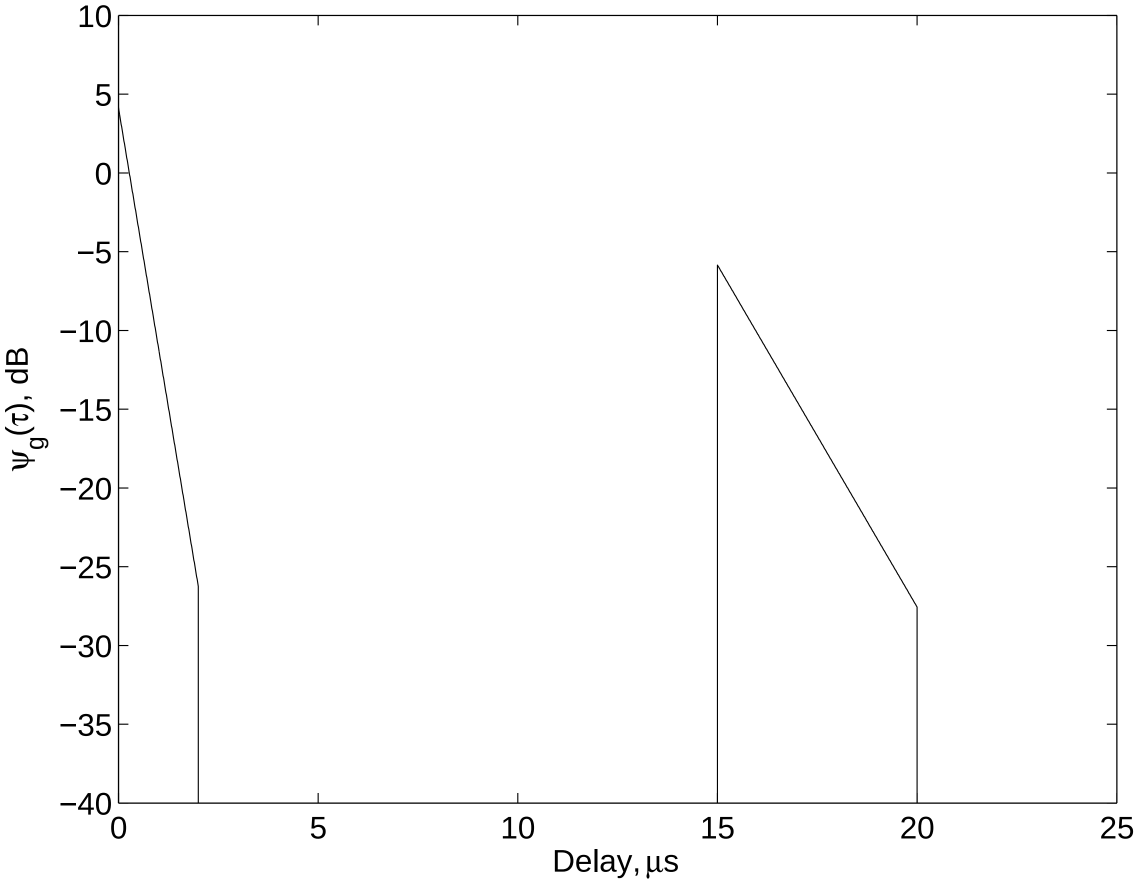
COST 207 typical case for rural (non-hilly) area (RA)



COST 207 typical case for rural (non-hilly) area (RA)

delay μ**s**	**Fractional Power**	**Doppler Category**
0.0	**0.602**	RICE
0.1	**0.241**	CLASS
0.2	**0.096**	CLASS
0.3	**0.036**	CLASS
0.4	**0.018**	CLASS
0.5	**0.006**	CLASS

COST 207 typical case for hilly terrain (HT)



COST 207 typical case for hilly terrain (HT)

delay μ**s**	**Fractional Power**	**Doppler Category**
0.0	**0.026**	CLASS
0.1	**0.042**	CLASS
0.3	**0.066**	CLASS
0.5	**0.105**	CLASS
0.7	**0.263**	GAUS1
1.0	**0.263**	GAUS1
1.3	**0.105**	GAUS1
15.0	**0.042**	GAUS2
15.2	**0.034**	GAUS2
15.7	**0.026**	GAUS2
17.2	**0.016**	GAUS2
20.0	**0.011**	GAUS2

Reduced Hilly Terrain (RHT)

delay μ**s**	**Fractional Power**	**Doppler Category**
0.0	**0.413**	CLASS
0.1	**0.293**	CLASS
0.3	**0.145**	CLASS
0.5	**0.074**	CLASS
15.0	**0.066**	GAUS2
17.2	**0.008**	GAUS2