## Mathcad Tutorial

Example 1. Inverting Systems of Equations. Consider the following problem arising from the analysis of a circuit with two mesh currents.

$$
\left(\begin{array}{cc}
\mathrm{R}_{1} & -\mathrm{R}_{2} \\
-\mathrm{R}_{2} & \mathrm{R}_{3}
\end{array}\right) \cdot\binom{\mathrm{I}_{1}}{\mathrm{I}_{2}}=\binom{\mathrm{E}_{1}}{\mathrm{E}_{2}}
$$

Given the Rs and Es determine the Is. Solution: First specify the numerical values of
the Rs and Es using the colon on the keyboard, viz. R period 1 colon 1000

$$
\mathrm{R}_{1}:=1000 \quad \mathrm{R}_{2}:=2000 \quad \mathrm{R}_{3}:=800 \quad \mathrm{E}_{1}:=10 \quad \mathrm{E}_{2}:=20
$$

Mathcad has the ability to add units of Ohms, volts, and amps but for beginners the numbers are enough.
The numbers below and to the right of the Rs and Es are subscripts. To get these type a period on the keyboard after the R or E .

Create the matrices with the Insert Matrix Command, the inverse with the carrot on the keyboard and a-1 as the exponent which yields

$$
\binom{\mathrm{I}_{1}}{\mathrm{I}_{2}}:=\left(\begin{array}{cc}
\mathrm{R}_{1} & -\mathrm{R}_{2} \\
-\mathrm{R}_{2} & \mathrm{R}_{3}
\end{array}\right)^{-1} \cdot\binom{\mathrm{E}_{1}}{\mathrm{E}_{2}}
$$

where the operator after the I vector is the colon on the keyboard or the second entry on the Evaluation Toolbar

To see the result simply type the I vector and the equal on the keyboard (no colon) which is the first entry on the Evaluation Toolbar

$$
\binom{\mathrm{I}_{1}}{\mathrm{I}_{2}}=\binom{-0.015}{-0.013}
$$

Remember that Mathcad works from top to bottom. A variable must be defined before it is used. Otherwise it will turn red. One of the neat things about Mathcad is that if the user changes the numerical value of a variable the result will be automatically recomputed.

To insert units simply multiply the variable (asterisk on keyboard) and then click on the icon for the measuring cup on the top of the command bar. Select the appropriate unit and click on it. For instance
$\mathrm{R}_{1}:=1 \cdot \mathrm{k} \Omega \quad$ introduces the unit of k Ohms
If the correct units are used for the Rs and Es the result for the Is will be expressed as amps.

## Example 2. Two Dimensional Plots

A topic of paramount importance to all ECE 3041 students is the step function response of the first order RC circuit which is given by

$$
\mathrm{v}_{\mathrm{C}}(\mathrm{t})=\mathrm{E}\left(1-\exp \left(\frac{-\mathrm{t}}{\tau}\right)\right) \mathrm{u}(\mathrm{t})
$$

where $E$ is the amplitude of the step function of voltage, $u(t)$ is the unit step function,
$\tau=\mathrm{RC} \quad$ is the time constant, R is the resistance, C the capacitance, and $\quad \mathrm{V}_{\mathrm{C}}(\mathrm{t}) \quad$ is the capacitor voltage. To plot this begin by defining the values of the parameters with the colon definition on the keyboard.
$\mathrm{E}:=10 \quad \mathrm{R}:=10^{4}$
$C:=0.1 \cdot 10^{-6}$
$\mathrm{u}(\mathrm{t}):=\Phi(\mathrm{t}) \quad \tau:=\mathrm{R} \cdot \mathrm{C} \quad$ where $\Phi(\mathrm{t})$
is the unit
step function Now define the function with the keyboard colon

$$
\mathrm{v}_{\mathrm{C}}(\mathrm{t}):=\mathrm{E} \cdot\left(1-\exp \left(\frac{-\mathrm{t}}{\tau}\right)\right) \cdot \mathrm{u}(\mathrm{t})
$$

Start the 2D graph by clicking on the icon for the 2D graph on the graph toolbar or use the keyboard shortcut shift @


Fill in the place tools (which are red) with the desired numerical values. The dependent variable goes on the $y$ axis and the independent variable on the $x$ axis in the middle. The start and stop values are on the extreme positions on the axes. Click outside of the plot and the plot will be made. Use the positioning tools to make the plot the desired size.

To make multiple plots the capacitor voltage will now be plotted for several different values of $R$

Define the capacitor voltage to be a function of both time and the resistance.

$$
\mathrm{v}_{\mathrm{C}}(\mathrm{t}, \mathrm{R}):=\mathrm{E} \cdot\left(1-\exp \left(\frac{-\mathrm{t}}{\mathrm{R} \cdot \mathrm{C}}\right)\right) \cdot \mathrm{u}(\mathrm{t})
$$

## Start the 2D graph again



Time

Type the expression $\quad v_{C}(t, R) \quad$ with the desired value of $R$ followed by a comma. The
Place marker will drop down for another function. The legend for the plot is shown below each and is also color coded.

To add labels to the plot put the pointer in the plot and double click and select label.

Example 3. Bode Plots. Bode plots are plots of the magnitude of a complex transfer function in decibels versus frequency and the phase in degrees as a function of frequency. The frequency is always plotted on a log scale and the gain or magnitude of the complex transfer function and the phase shift are plotted on a linear scale. Consider the complex transfer function for the first-order high-pass filter

$$
\mathrm{T}(\mathrm{~s})=\mathrm{K} \cdot \frac{\mathrm{~s} \tau}{1+\mathrm{s} \tau}
$$

where $K$ is the high frequency gain, $s$ the complex frequency variable, $T(s)$ is the ratio of the output complex phasor to the input complex phasor, and T is the time constant which is the product of $R$ times $C$. To make the Bode plot, define the parameters, and express $T(s)$ as a function of either $T(f)$ or $T(\omega)$. The -3 dB frequency for this filter will be picked to be 1 kHz which fixes $T$

$$
\begin{array}{cc}
\mathrm{f}_{3}:=1000 & \tau:=\frac{1}{2 \cdot \pi \cdot \mathrm{f}_{3}} \quad \mathrm{~K}:=1 \quad \mathrm{j}:=\sqrt{-1} \\
\mathrm{~T}(\mathrm{f}):=\mathrm{K} \cdot \frac{(\mathrm{j} \cdot 2 \cdot \pi \cdot \mathrm{f} \cdot \tau)}{1+(\mathrm{j} \cdot 2 \cdot \pi \cdot \mathrm{f} \cdot \tau)} & \mathrm{M}(\mathrm{f}):=20 \cdot \log (|\mathrm{~T}(\mathrm{f})|) \quad \phi(\mathrm{f}):=\frac{180}{\pi} \cdot \arg (\mathrm{~T}(\mathrm{f}))
\end{array}
$$

where $M(f)$ is the magnitude of $T(f)$ in decibels and $\quad \phi(f) \quad$ is the phase in degrees.
The absolute value operator is found on the calculator toolbar.
Both $M(f)$ and $\quad \phi(f) \quad$ can now be plotted as functions of $f$. However, instead of using
the default number of points and spacing, the number of points and logarithmic spacing will now be set by specifying the frequency as a vector which, of course, will make functions of it vectors.
$\mathrm{N}:=1000 \quad$ Define the number of points to be 1000
$\mathrm{i}:=0 . . \mathrm{N}-1 \quad$ Define i to be an index. It starts with $\mathrm{i}=0$ and goes to $\mathrm{i}=\mathrm{N}-1$ Mathcad starts with array index 0 . The keyboard sequence is i colon 0 semicolon $\mathrm{N}-1$

$$
\mathrm{f}_{\text {start }}:=100 \quad \mathrm{f}_{\text {stop }}:=10^{5}
$$

The start and stop are subscripts obtained with the period on the keyboard.
$\mathrm{f}_{\mathrm{i}}:=\mathrm{f}_{\text {start }} \cdot\left(\frac{\mathrm{f}_{\text {stop }}}{\mathrm{f}_{\text {start }}}\right)^{\frac{\mathrm{i}}{\mathrm{N}-1}}$
The i on the f is NOT a subscript. It is an array index. The keyboard shortcut is the left square bracket. On the Matrix toolbar is the the $x$ sub $n$ icon.

This defines N logarithmically spaced values of frequency.

Start the 2D graph with either the icon on the Graph toolbar or the @ on the keyboard. Plot $M(f)$ versus $f$ where the array index is entered instead of just $f$.


Note the plot is all scrunched up on the left. This is because the default setting for both axes is linear. Put the pointer inside the graph, double click and switch the $x$ axis to log


And viola!
Now a separate plot could be made for the phase. However, it's easier to double click on the graph and enable the secondary $y$ axis. Then enter the phase.


This is the Bode plot.

Suppose one wanted to take the results calculated by Mathcad and port them to another software package such as Excel. Simply type $\mathrm{f}=, \mathrm{M}(\mathrm{f})=$, etc.

$\mathrm{f}=$|  | 0 |
| :---: | ---: |
| 0 | 100 |
| 1 | 100.694 |
| 2 | 101.393 |
| 3 | 102.096 |
| 4 | 102.804 |
| 5 | 103.518 |
| 6 | 104.236 |
| 7 | 104.959 |
| 8 | 105.688 |
| 9 | 106.421 |
| 10 | 107.159 |
| 11 | 107.903 |
| 12 | 108.652 |
| 13 | 109.405 |
| 14 | 110.165 |
| 15 | $\ldots$ |

$\mathrm{M}\left(\mathrm{f}_{\mathrm{i}}\right)=$

| -20.043 |
| ---: | ---: |
| -19.984 |
| -19.924 |
| -19.865 |
| -19.805 |
| -19.746 |
| -19.687 |
| -19.627 |
| -19.568 |
| -19.508 |
| -19.449 |
| -19.39 |
| -19.33 |
| -19.271 |
| -19.212 |
| $\ldots$. |


| $\phi\left(\mathrm{f}_{\mathrm{i}}\right)=$ |
| :--- |
| 84.289 |
| 84.25 |
| 84.21 |
| 84.171 |
| 84.13 |
| 84.09 |
| 84.049 |
| 84.008 |
| 83.967 |
| 83.925 |
| 83.884 |
| 83.841 |
| 83.799 |
| 83.756 |
| 83.713 |
| $\ldots$. |

Although only 16 values are shown on the screen all 1000 are available. Put the pointer inside the array and right click. Click Select All. Put the pointer inside the highlighted array right click and select Copy Selection. Minimize Mathcad Open Excel or Word and paste the results in.

