Measurement of the Electrical Constitutive Parameters of Materials Using Antennas, Part II

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Abstract—**The previously developed technique for measuring the electrical constitutive parameters of materials using a single monopole antenna is extended to include a monopole with closely spaced parasitic elements. The addition of the parasitic elements reduces the radiation from the antenna and, in turn, reduces the effect of reflections from material boundaries on the measurement. This allows smaller volumes of material and smaller image planes to be used when measuring materials with low dissipation. A simple measurement procedure is presented for a monopole with two symmetrically located parasitic elements. The procedure is based on the representation of the normalized impedance of the antenna by a rational function of order three. It is applicable for frequencies at which $|kh|$ (wavenumber length of antenna) in the medium is less than that for resonance in a lossless medium, $|kh| \lesssim \beta_r h$. The design of a practical probe for the *in situ* measurement of soils is described and its use is illustrated.**

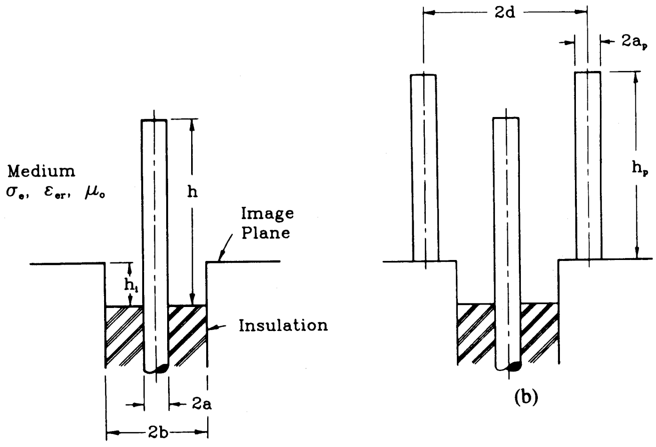

Fig. 1. (a) Cylindrical monopole antenna. (b) Cylindrical monopole antenna with two symmetrically located parasitic elements.

I. Introduction

In an earlier paper, a general procedure was presented for measuring the electrical constitutive parameters of materials using antennas, like the cylindrical monopole shown in Fig. 1(a) [1]. Briefly, this procedure consists of the following steps.

1) A *normalized impedance* is defined for the antenna:

$$Z_n(kh) \equiv \sqrt{\tilde{\epsilon}_r}\, Z(\omega, \tilde{\epsilon}_r) = \frac{kh}{k_0 h} Z(\omega, \tilde{\epsilon}_r). \tag{1}$$

Here, $Z(\omega, \tilde{\epsilon}_r)$ is the impedance, k_0 is the wavenumber in free space, $k = \beta - j\alpha$ is the complex wavenumber in a medium with effective conductivity σ_e, and effective relative permittivity ϵ_{er}, and

$$\tilde{\epsilon}_r \equiv \epsilon_{er} - j\sigma_e/\omega\epsilon_0. \tag{2}$$

The normalized impedance (1) is approximated by a rational function. For example, when a rational function of order three is used:

$$Z_n(kh) \approx jK \frac{1}{kh}\left[\frac{1 + jb_1(kh) + b_2(kh)^2}{1 + jb_1(kh) + a_2(kh)^2}\right]. \tag{3}$$

This particular rational function (3) was shown to be a very good representation for the normalized impedance provided $0 \leqslant |kh| \lesssim \beta_r h$, where β_r is wavenumber at which the antenna has its first resonance in a lossless medium. Note that b_1 appears in both the numerator and the denominator of (3). This is a consequence of requiring the radiation resistance of the antenna in free space to behave as $(k_0 h)^2$ in the limit $k_0 h \to 0$.

2) The real coefficients in the rational function (K, a_2, b_1, and b_2 in (3)) are determined from a measurement of the impedance $Z(\omega, \tilde{\epsilon}_{rs})$ over a range of frequencies in a *standard medium* with the known constitutive parameters σ_{es}, ϵ_{ers}. This step is performed once, and it can be viewed as a calibration of the antenna. A typical standard medium would be free space (air).

3) The unknown constitutive parameters σ_e, ϵ_{er} of a medium are determined by first measuring the impedance $Z(\omega, \tilde{\epsilon}_r)$ with the antenna immersed in the medium. At each frequency ω, the measured impedance and the coefficients of the rational function are used to form a polynomial in kh. For example, when the rational function is of order three, this polynomial will be of fourth degree:

$$(kh)^4 + j(b_1/a_2)(kh)^3 + [1/a_2 - jb_2 K k_0 h/a_2 Z](kh)^2 + (b_1 K k_0 h/a_2 Z)kh - jKk_0 h/a_2 Z = 0. \tag{4}$$

Now the constitutive parameters are determined from the appropriate root of this polynomial:

$$\tilde{\epsilon}_r(\omega) = (kh/k_0 h)^2. \tag{5}$$

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The value of kh must be checked at each frequency to determine if it is within the range for which the particular rational function applies. For the rational function of order three, $0 \leqslant |kh| \lesssim \beta_r h$.

This measurement technique is based on the normalized impedance (1) being a function of the electrical constitutive parameters $\tilde{\epsilon}_r$ of a *single* medium. Strictly speaking, this requires the antenna to be immersed in a homogeneous medium of infinite extent (for a monopole antenna, a half-space of material adjacent to an image plane of infinite area). A material boundary, if present, would reflect energy radiated by the antenna and, in turn, affect the impedance of the antenna. The impedance would then depend on the electrical constitutive parameters of the materials on both sides of the boundary.[1]

In practical applications, there is often dissipation in the medium being measured, which attenuates the reflections from the boundaries. The impedance measured in a finite volume can then be essentially the same as that in a medium of infinite extent. However, in situations where the dissipation and volume are not large enough, the reflections from boundaries can significantly affect the measured constitutive parameters. This is illustrated by the results presented in Fig. 2. Here the constitutive parameters σ_e, ϵ_{er} of a saline solution measured with a monopole antenna (h = 6.00 cm, h_i = 2.00 mm, $2a$ = 3.04 mm, and $2b$ = 7.00 mm) are compared with the values expected from previous measurements [2]. The monopole was centered on a plastic hemispherical tank of radius 21 cm filled with the solution (Fig. 3). A rational function of order three was used to represent the normalized impedance, and results were obtained for frequencies in the range 2 MHz $\leqslant f \leqslant$ 130 MHz ($|kh| = \beta_r h$). The errors introduced by the reflections from the boundaries of the tank are evident in these results; obviously, the spherical symmetry of the tank enhances the errors.

Sometimes the effects of unwanted reflections from material boundaries can be removed by signal processing [3]. For example, the measured reflection coefficient (frequency domain) of the antenna can be Fourier transformed and used to obtain the response (time domain) of the antenna to a pulse waveform. Reflections from boundaries can be identified in the time domain and windowed out. After transformation back to the frequency domain, the reflection coefficient (impedance) can be used to obtain the undistorted constitutive parameters of the medium. For this method to be useful, the measurements must be made over a wide range of frequencies, and the reflections from boundaries (time domain) must be separated from the response of the antenna. The later requirement means that the dimensions of the volume of material being measured must be much larger than the length of the antenna.

This paper will describe a method which uses parasitic elements to reduce the radiation from the antenna, thus reducing the effect of boundaries on the measurement of the constitutive parameters.

[1] Conductors that can be assumed perfect are excluded, since they would not introduce additional material parameters.

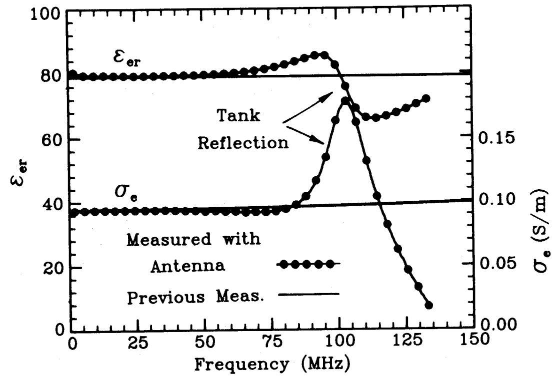

Fig. 2. The effective relative permittivity and effective conductivity of a saline solution measured with the monopole antenna in the hemispherical tank, T = 23°C.

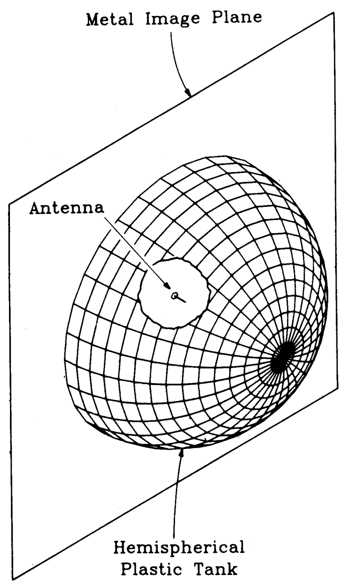

Fig. 3. Hemispherical tank used to measure constitutive parameters of liquids, radius = 21 cm.

II. Monopole with Parasitic Elements

It is largely the radiation from the antenna that causes the boundaries of the material to affect the impedance of the antenna. In Fig. 4, the measured normalized resistance for a monopole antenna in air is shown as a function of $|kh| = \beta_o h$. These data are for the same monopole discussed in Section I. Note from (1) that the normalized resistance R_n and the resistance R are the same when the antenna is in free space (air).

The radiation from the monopole can be reduced significantly by placing parasitic elements close to the monopole. The symmetrical arrangement of two parasitic elements shown in Fig. 1(b) is particularly effective in this regard. The measured normalized resistance for the monopole with two parasitic elements (h_p = 7.00 cm, $2a_p$ = 3.04 mm, and $2d$ = 1.63 cm) is also shown in Fig. 4. The effect of the parasitic elements is clearly shown: the normalized resistance is $\lesssim 1\ \Omega$ for frequencies below resonance, and at resonance it is less than 2 percent of the resistance of the single monopole. The

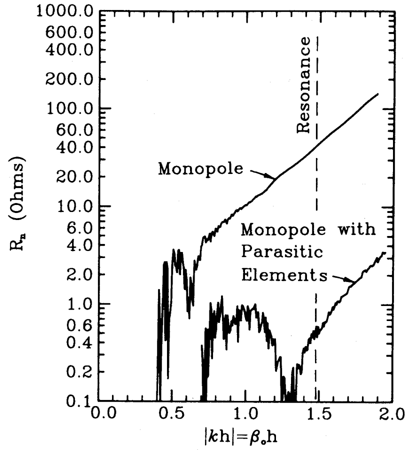

Fig. 4. Measured normalized resistance as a function of $\beta_0 h$ for monopole antenna and monopole antenna with two parasitic elements in air.

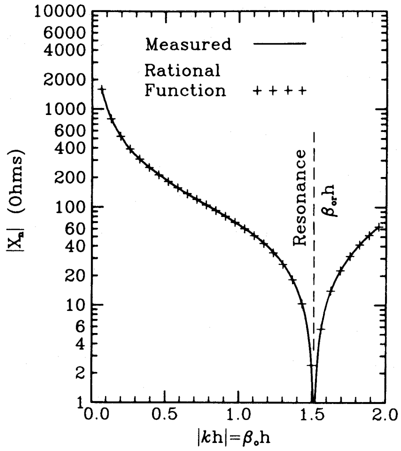

Fig. 5. Comparison of normalized reactance measured in air with rational function representation for monopole antenna with two parasitic elements.

reduction in radiation does not occur at all frequencies; at frequencies well above resonance (not shown in Fig. 4), particularly near antiresonance $|kh| \approx 3.0$, the resistances of the two structures can be comparable.

The procedure outlined in Section I for measuring the constitutive parameters of a medium using a single monopole can also be used when parasitic elements are included. In fact, the application of the procedure to the latter structure is simpler. The discussion here will be restricted to the case where $|kh| \lesssim \beta_r h$ (operation below resonance), where the parasitic elements reduce the radiation, and the rational function of order three is a very good representation for the normalized impedance.

The resistance (radiation resistance) of the monopole with two parasitic elements in a lossless medium will be assumed to be zero for $|kh| \lesssim \beta_r h$; the consequences of this simplifying assumption will be discussed later. The normalized impedance (3) then becomes

$$Z_n(kh) \approx jK \frac{1}{kh} \left[\frac{1 + b_2(kh)^2}{1 + a_2(kh)^2} \right] \tag{6}$$

which is a pure reactance when the medium is lossless, $kh = \beta h$.

The three coefficients (K, b_2, and a_2) can be determined from simple measurements made with the antenna in a standard medium (usually free space) which will be assumed lossless, $kh = \beta_s h$. The coefficient K is determined from the low-frequency behavior of the impedance:

$$K = \lim_{|\beta_s h| \to 0} \left[\frac{-j(\beta_s h)^2}{k_0 h} Z(\omega, \tilde{\epsilon}_{rs}) \right] = -(h/cC_0) \tag{7}$$

where c is the speed of light in free space, and C_0 is the static capacitance of the antenna in free space. The coefficient b_2 is found from the resonant frequency ω_r of the antenna ($X_n(\beta_s h) = 0$, $\beta_{sr} h = \beta_r h$):

$$b_2 = -1/(\beta_{sr} h)^2 \tag{8}$$

and the coefficient a_2 is determined using the reactance of the antenna $X(\omega_p, \tilde{\epsilon}_{rs})$ measured at a specified intermediate frequency ω_p, for which $\beta_{sp} h < \beta_r h$:

$$a_2 = -\frac{1}{(\beta_{sp} h)^2} \cdot \left[1 + \frac{K}{\sqrt{\tilde{\epsilon}_{rs}} X(\omega_p, \tilde{\epsilon}_{rs})} \frac{(\beta_{sp} h)^2 - (\beta_{sr} h)^2}{\beta_{sp} h (\beta_{sr} h)^2} \right]. \tag{9}$$

The accuracy of this representation (6) is shown in Fig. 5 where the measured normalized reactance of the previously described monopole with two parasitic elements is compared with the rational function approximation. For the range of values displayed $1.0\ \Omega \leqslant |X_n| \leqslant 1.5 \times 10^3\ \Omega$ and $|kh| \leqslant \beta_r h$, the root mean square (rms) difference is less than 1 percent.

Recall that when the monopole antenna is used to determine the constitutive parameters of a medium, and the normalized impedance is represented by a rational function of order three, the roots of the quartic equation in kh (4) must be found. For the monopole with parasitic elements, the quartic equation can be rewritten as a quadratic equation in $\tilde{\epsilon}_r = \epsilon_{er} - j\sigma_e/\omega\epsilon_0$:

$$\tilde{\epsilon}_r^2 + b\tilde{\epsilon}_r + c = 0 \tag{10}$$

with

$$b = \frac{1}{(k_0 h)^2 a_2} [1 - jb_2 K k_0 h / Z(\omega, \tilde{\epsilon}_r)] \tag{11a}$$

and

$$c = -jK \left[\frac{1}{(k_0 h)^3 a_2 Z(\omega, \tilde{\epsilon}_r)} \right]. \tag{11b}$$

Only one of the two roots to this equation is the correct one for determining the constitutive parameters of the medium. At low frequencies $|kh| \ll 1$, the correct root is the one closest to the value

$$\tilde{\epsilon}_r = jK/k_0 h Z(\omega, \tilde{\epsilon}_r). \tag{12}$$

As the frequency is increased, the correct root can be determined by a tracking procedure that picks the root closest to the correct root for the preceding lower frequency.

The monopole with two parasitic elements was used to measure the constitutive parameters of the previously described saline solution in a hemispherical tank. The antenna was calibrated in air with the reactance measured at the intermediate point $\beta_{sp}h = \beta_0 h = 1$. The results from these measurements are shown in Fig. 6. The agreement with the results of previous measurements is seen to be very good. A comparison of Figs. 2 and 6 shows that the reflections from the boundaries of the tank, which greatly affected the measurements made with the single monopole, have a much smaller effect on the measurements made with the monopole with two parasitic elements. In fact, there is no perceptable effect on ϵ_{er}, and only a very small effect on σ_{er} in the frequency range 75 MHz $\lesssim f \lesssim$ 135 MHz.

As previously mentioned, the assumption of zero resistance (radiation resistance) when the monopole with parasitic elements is in a lossless medium imposes a limitation on the measurement procedure. Specifically, in a medium with low loss, a small resistance due to radiation may be interpreted as a loss due to dissipation and an erroneous value for the effective conductivity determined.[2] To quantify this effect, consider the following argument.

Let the measured resistance of the antenna be almost entirely due to radiation; that is, due to terms (powers of kh) that were ignored in the rational function (6). However, this resistance is attributed to dissipation in the medium; that is, attributed to the wavenumber being complex $kh = \beta h - j\alpha h$ in (6). Now for a low-loss medium $\alpha h/\beta h \ll 1$, and frequencies near resonance $|kh| \approx \beta_r h$, the normalized resistance (6) is approximately

$$R_n(\beta_r h) \approx \frac{-2K}{\beta_r h[1 + a_2(\beta_r h)^2]} \left(\frac{\alpha h}{\beta_r h}\right). \tag{13}$$

For low-loss media, the loss tangent is approximately

$$p_e \equiv \frac{\sigma_e}{\omega \epsilon_{er} \epsilon_0} \approx 2(\alpha h/\beta h). \tag{14}$$

Thus (13) can be rewritten as

$$p_e \approx \frac{-\beta_r h R_n(\beta_r h)[1 + a_2(\beta_r h)^2]}{K}. \tag{15}$$

Now, since $\beta_r h \approx \pi/2$, and K is negative

$$p_e \approx \frac{\pi R_n(\beta_r h)[1 + \pi^2 a_2/4]}{2|K|}. \tag{16}$$

Equation (16) can be used to estimate the error Δp_e in the measured loss tangent that will arise due to the ignored radiation; with R_n the resistance due to radiation, this error is

$$\Delta p_e \equiv \frac{\pi R_n(\beta_r h)[1 + \pi^2 a_2/4]}{2|K|}. \tag{17}$$

[2] Here the terms "resistance due to radiation" and "resistance due to dissipation" are used loosely. Of course, in a lossy medium all of the net power supplied to the antenna must eventually be dissipated in the medium.

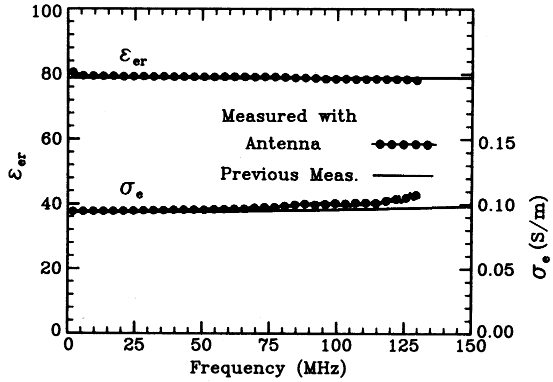

Fig. 6. The effective relative permittivity and effective conductivity of a saline solution measured with the monopole antenna with two parasitic elements in the hemispherical tank, T = 23°C.

The equivalent error for the effective conductivity is

$$\Delta \sigma_e \equiv \frac{\pi \omega \epsilon_{er} \epsilon_0 R_n(\beta_r h)[1 + \pi^2 a_2/4]}{2|K|}. \tag{18}$$

The monopole with two parasitic elements described earlier, Fig. 4, has $R_n(\beta_r h) \approx 0.5\ \Omega$, $|K| \approx 100$, and $a_2 \approx -0.15$. Thus the estimated error when measuring low-loss materials with this antenna is $\Delta p_e = 5.1 \times 10^{-3}$, or $\Delta \sigma_e = 2.8 \times 10^{-4}$ f(GHz)ϵ_{er}. To verify this estimate of error, the antenna was used to measure the constitutive parameters of air ($\sigma_e = 0$, $\epsilon_{er} = 1.0$) over the frequency range 50 MHz $\leqslant f \leqslant$ 1.17 GHz. The results are shown in Fig. 7. The relative permittivity is close to unity, while the conductivity is small but not zero. This small measured conductivity is the error associated with ignoring the radiation from the antenna. At the frequency f = 1.17 GHz, the estimated error from (18) is $\Delta \sigma_e = 3.3 \times 10^{-4}$ S/m, which agrees with the results on the graph.

III. A Probe for *In Situ* Measurements

Monopole probes are particularly useful for making *in situ* measurements, since they can be easily inserted in materials such as geophysical and biological media. However, when the dissipation in the medium is low requiring a large volume of material and a large image plane their usefulness is restricted. The addition of parasitic elements to the probe, as illustrated in the last section, can greatly reduce the requirements on the volume of material and the size of the image plane.

Fig. 8 is a photograph of a practical probe that was constructed for measuring *in situ* the constitutive parameters of soil. The monopole and two parasitic elements are mounted on a circular disc of diameter 30.5 cm.[3] The dimensions for the elements are h = 12.0 cm, $2a$ = 6.35 mm, $2b$ = 1.40 cm, h_p = 14.0 cm, $2a_p$ = 6.35 mm, and $2d$ = 3.30 cm. A thin plastic bead supports the center conductor of the coaxial line, and is flush with the disk, h_i = 0. For a measurement, this probe is pressed into the soil until the disk contacts the surface. When the soil is particularly hard, a sturdy tool with the same geometry as the probe is used to make holes in the

[3] Measurements indicate that the impedance (reactance) measured in air is essentially independent of the size of the disk when the radius of the disk is greater than or equal to the length of the monopole.

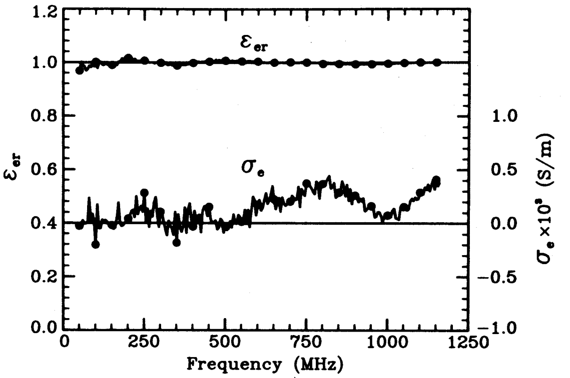

Fig. 7. The effective relative permittivity and effective conductivity of air measured with the monopole antenna with two parasitic elements.

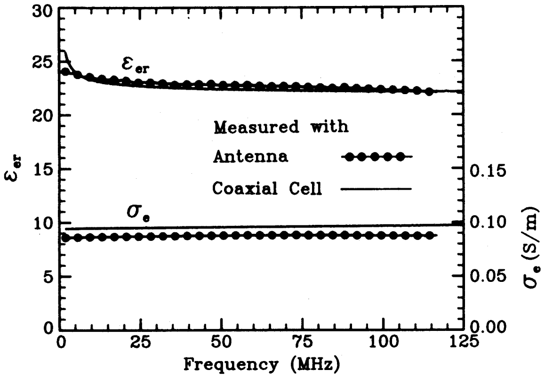

Fig. 9. A comparison of the constitutive parameters of sand saturated with a saline solution measured with the monopole antenna with two parasitic elements and measured with the open-circuited coaxial sample cell.

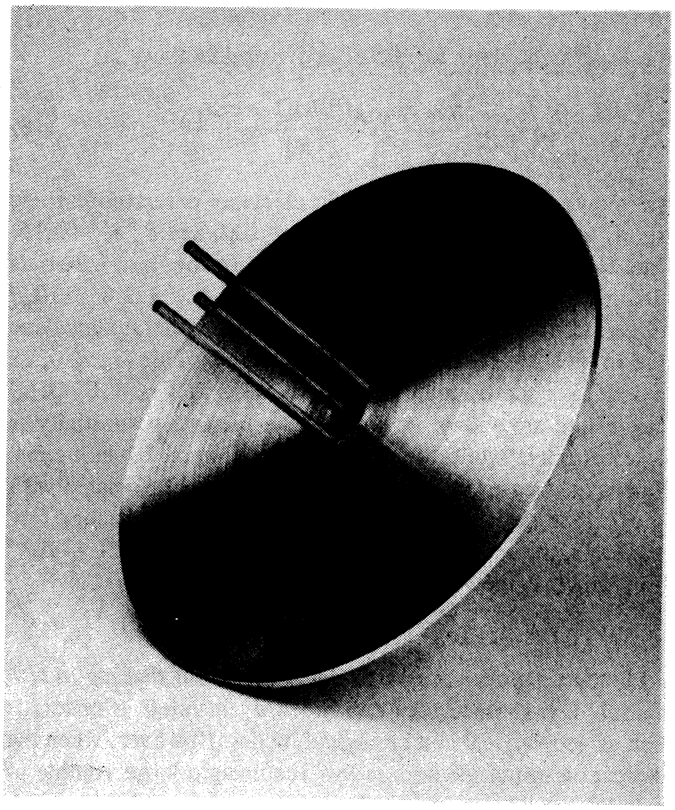

Fig. 8. Probe for measuring the *in situ* constitutive parameters of soils.

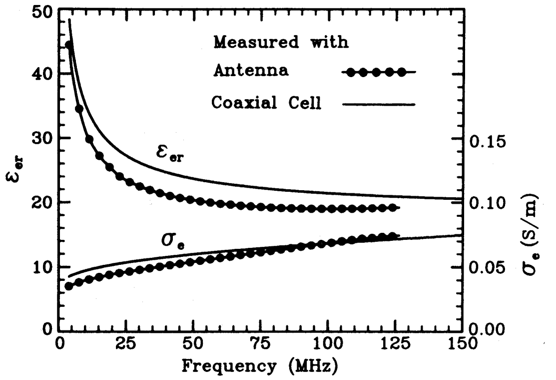

Fig. 10. A comparison of the constitutive parameters of red clay measured with the monopole antenna with two parasitic elements and measured with the open-circuited coaxial sample cell.

soil before the probe is inserted. The probe was calibrated in air with the reactance measured at the intermediate point $\beta_{sp}h = \beta_0 h = 1.0$. The estimated error when measuring low-loss materials is $\Delta p_e = 6.5 \times 10^{-3}$, or $\Delta\sigma_e = 3.6 \times 10^{-4} f(\text{GHz})\epsilon_{er}$.

This probe was used to measure *in situ* the electrical constitutive parameters (σ_e, ϵ_{er}) of sand saturated with a saline solution and red clay (18 percent water by weight). The results from these measurements are shown in Figs. 9 and 10. In each case, the measurements start at a frequency of 2 MHz and end at the frequency where $|kh| = \beta_r h$ (117 MHz for the sand and 126 MHz for the clay). The measurements were made using a Hewlett Packard model 3577A Automated Network Analyzer with model 35677A S-Parameter Test Set.

Also shown in these figures are the constitutive parameters measured with an open-circuited coaxial sample cell [4]. This is not an *in situ* measurement; it requires a sample of the material to be packed into a coaxial line. The two sets of measured data are generally in good agreement. However, there is some noticeable shift in the results obtained by the two methods. This is probably due to the inability to pack the soil into the coaxial cell with the same density as it has *in situ*.

As a final test, the probe was used to measure the constitutive parameters of fresh water over the frequency range 500 kHz $\leqslant f \leqslant$ 66 MHz. This is an extreme test: a medium with high permittivity and low loss. The measured results, shown in Fig. 11, are generally in good agreement with those expected from previous measurements [2]. The rise in the measured conductivity at the higher frequencies is the aforementioned error associated with ignoring the radiation from the antenna. This error could have been predicted from the estimate (18). At the highest frequency, f = 66 MHz, the estimated error is $\Delta\sigma_e = 1.9 \times 10^{-3}$ S/m, which agrees with the error observed in Fig. 11.

IV. Summary and Conclusion

The previously developed technique for measuring the electrical constitutive parameters of materials using a single monopole antenna was extended to include a monopole with closely spaced parasitic elements. The addition of the parasitic elements reduces the radiation from the antenna and, in turn,

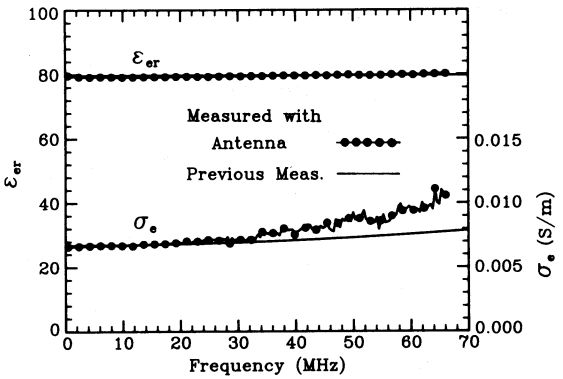

Fig. 11. The effective relative permittivity and effective conductivity of fresh water measured with the monopole antenna with two parasitic elements, $T = 21°C$.

reduces the effect of reflections from boundaries on the measurement. This allows smaller volumes of material and smaller image planes to be used when measuring materials with low dissipation.

A simple measurement procedure was presented for an antenna with two symmetrically located parasitic elements. The procedure is based on the representation of the normalized impedance by a rational function of order three. It is applicable for frequencies at which kh in the medium satisfies the inequality $|kh| \lesssim \beta_r h$. An estimate was made of the error in the procedure when measuring materials with low loss.

The design of a practical probe for the *in situ* measurement of soils was described and its use illustrated.

We would like to call the readers attention to a few minor typographical errors in the earlier paper [1]. In the third paragraph of the Introduction, "inosphere" should read "ionosphere." In Fig. 7(a), the dimensions 2.06 cm and 7.00 cm should be 2.06 mm and 7.00 mm, respectively. In [1, appendix, eq. (50)] the quantity B should have the term x^2R_r replaced by $x^2R_r^2$.

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Glenn S. Smith (S'65–M'72–SM'80–F'86), for a photograph and biography please see page 929 of the July 1986 issue of this TRANSACTIONS.

Waymond R. Scott, Jr. (S'81–M'85), for a photograph and biography please see page 929 of the July 1986 issue of this TRANSACTIONS.