GPR IMAGING USING COMPRESSED MEASUREMENTS

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ABSTRACT

A new data acquisition and imaging method exploiting the sparsity of the target space is presented for ground penetrating radar (GPR) imaging. Sparsity is enforced by solving a convex ℓ_1 minimization problem which uses a very small number of random measurements. The method can greatly reduce the data acquisition time while producing sparse target space images. Simulation and experimental data results are provided to show that the method has excellent resolution and is robust to noise and random spatial sampling.

Index Terms— GPR, subsurface imaging, compressive sensing, ℓ_1 minimization, sparsity

1. INTRODUCTION

Ground Penetrating Radar (GPR) is an important remote sensing tool used in a wide variety of areas such as civil engineering, landmine detection and environmental applications [1]. A traditional time-domain GPR probes the subsurface by transmitting electromagnetic (EM) pulses and processing the reflections due to permittivity discontinuities in the ground. A stepped-frequency GPR (SFGPR) probes the environment with a discrete set of frequencies [2]. Standard time- [3] and frequency- [4, 5] domain imaging algorithms basically perform matched filtering with the impulse response of the data acquisition process to form the images. Existing imaging algorithms require fine spatial sampling and high-rate time/frequency measurements. This leads to high data acquisition times which can become the bottleneck of the GPR subsurface imaging process. Any possible prior information about the target space, e.g., sparsity, is not used to enhance the quality of subsurface images.

Results from the theory of compressive sensing (CS) [6, 7] show that a sparse signal $\boldsymbol{x} = \boldsymbol{\Psi s}$ can be reconstructed from a small number of linear measurements $\boldsymbol{y} = \boldsymbol{\Phi x}$, in the form of randomized projections by solving a constrained ℓ_1 minimization problem which can be solved efficiently with linear programming.

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In many cases, potential targets cover a small part of the total subsurface volume to be imaged. In other words, we have *a priori* information that the target space is spatially sparse. To represent the data at the GPR receiver as a linear function of target space, a dictionary of all possible GPR data is generated. For the purposes of this paper, we assume that the target space consists of point like reflectors. The dictionary entries are produced by discretizing the target space, and synthesizing the GPR time/frequency model data for each discrete spatial position. The point-like target assumption is not crucial; other models can be used as long as the received data can be calculated for the assumed target model. The sparsity assumption is also not limited to point targets; other models can be used as long as the targets are sparse or compressible in some transform domain.

Using the CS framework, instead of taking traditional time or frequency GPR samples, only a very small number of informative measurements in the form of random linear projections are acquired. The imaging is done by solving a convex ℓ_1 minimization problem. While taking less number of measurements decreases the data acquisition load significantly, solving a convex optimization problem for imaging increases the load for signal processing. The algorithm can also handle random spatial sampling. It is robust with respect to noise and model mismatches. Another important property of the proposed method is its ability to resolve closely spaced targets that cannot be resolved by standard imaging methods.

2. COMPRESSIVE SUBSURFACE IMAGING

While standard imaging algorithms [3–5] generate images by applying a matched filter of the measured data with the impulse response of the data acquisition process, the CS method attempts to represent the measured data as a linear combination of dictionary elements. The advantage of this approach is that we can incorporate prior information directly into the estimation of the unknown spatial image and use the CS framework.

Creating a dictionary for GPR data: To generate a dictionary for the GPR data, we need a target model for which we can calculate the expected GPR return and sparsely forms the target space. A point-target model is used in this purpose because it is a simple GPR data model and it is widely used in standard imaging algorithms. The dictionary entries are produced by discretizing the target space, and synthesizing the GPR time/frequency model data for each discrete point target. In this paper, we will develop the method for a frequency domain GPR system. A similar development can be done for a time domain system; see [8,9] for extended analysis for both time and frequency domains.

Discretizing the target space $\boldsymbol{\pi}_T$ which lies in the product space $[x_0, x_f] \times [y_0, y_f] \times [z_0, z_f]$ generates a finite set of target points $\mathcal{B} = \{\boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \ldots, \boldsymbol{\pi}_N\}$, where N determines the resolution and each $\boldsymbol{\pi}_j$ is a 3D vector $[x_j; y_j; z_j]$. Here (x_0, y_0, z_0) and (x_f, y_f, z_f) denote the initial and final positions of the target space to be imaged along each axis. When the GPR is at the i^{th} scan point, the j^{th} column of $\boldsymbol{\Psi}_i$ which corresponds to a target at $\boldsymbol{\pi}_j$ can be written as

$$[\boldsymbol{\Psi}_i]_j = \exp\left[-j\omega(t - \tau_i(\boldsymbol{\pi}_j))\right] \tag{1}$$

Repeating (1) for each discrete possible target position creates the dictionary $\boldsymbol{\Psi}_i$ when the GPR is at the i^{th} aperture point. Size of $\boldsymbol{\Psi}_i$ will be $L \times N$ where L is the number of frequency steps for SFGPR.

The received signal at the i^{th} aperture point for multiple targets can be written as

$$\boldsymbol{\zeta}_i = \sum_{k=1}^{P} \boldsymbol{b}(k) \exp\left[-j\omega(t - \tau_i(\boldsymbol{\pi}_k))\right] \tag{2}$$

assuming that the targets do not interact and superposition is valid. The received signal from multiple targets can be written in terms of the dictionary as

$$\boldsymbol{\zeta}_i(f) = \boldsymbol{\Psi}_i \boldsymbol{b} \tag{3}$$

where $\boldsymbol{b}$ is a weighted indicator vector defining the target space, i.e., if there is a target at $\boldsymbol{\pi}_j$, the j^{th} index of $\boldsymbol{b}$ should be nonzero. Our goal is to find $\boldsymbol{b}$ which is actually an "image" of the medium.

Random Frequency Sampling: Standard SFGPRs measure a regularly spaced set of L frequencies at each aperture point, hence the dimension of $\boldsymbol{\zeta}_i$ is $L \times 1$. We propose a new data acquisition model based on compressive sampling (CS) [6,7,10] which would uses a very small number of "random" measurements to construct the target space image $\boldsymbol{b}$, if the target space is sparse. We measure a random subset of M frequencies, where $M < L$ at each aperture point. In matrix form, the new measurements $\boldsymbol{\beta}_i$ can be written as

$$\beta_i = \boldsymbol{\Phi}_i \boldsymbol{\zeta}_i = \boldsymbol{\Phi}_i \boldsymbol{\Psi}_i \boldsymbol{b}. \tag{4}$$

where $\boldsymbol{\Phi}_i$ is an $M \times L$ measurement matrix constructed by randomly selecting M rows of an $L \times L$ identity matrix, which amounts to measuring random frequency points at the i^{th} aperture. This reduces the data acquisition time by a factor of L/M for SFGPRs.

Subsurface Imaging: For imaging we use K aperture points and form a "super problem" with the combined matrices $\boldsymbol{\Psi} = [\boldsymbol{\Psi}_1^T, \ldots, \boldsymbol{\Psi}_K^T]^T$, and $\boldsymbol{\Phi} = \text{diag}\{\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_K\}$, and the measurements $\boldsymbol{\beta} = [\boldsymbol{\beta}_1^T, \ldots, \boldsymbol{\beta}_K^T]^T$. The result of the CS theory is that the target space indicator vector $\boldsymbol{b}$ can be recovered exactly from $M = C\left(\mu^2(\boldsymbol{\Phi}, \boldsymbol{\Psi}) \log N\right) K$ measurements $\boldsymbol{\beta}$ with overwhelming probability [7], by solving the ℓ_1 minimization problem

$$\hat{\boldsymbol{b}} = \text{argmin} \, \|\boldsymbol{b}\|_1 \qquad s.t. \quad \boldsymbol{\beta} = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{b} \tag{5}$$

where $\mu(\boldsymbol{\Phi}, \boldsymbol{\Psi})$ is the coherence between $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ [11].

The optimization problem in (5) is valid only for the noiseless case because it uses an equality constraint. If the GPR signal is noisy, i.e., $\zeta_i^N(t) = \zeta_i + n_i(t)$, then the compressive measurements $\boldsymbol{\beta}_i$ at the i^{th} aperture position have the following form:

$$\boldsymbol{\beta}_i = \boldsymbol{\Phi}_i \boldsymbol{\zeta}_i^N = \boldsymbol{\Phi}_i \boldsymbol{\Psi}_i \boldsymbol{b} + \boldsymbol{u}_i \tag{6}$$

where $\boldsymbol{u}_i = \boldsymbol{\Phi}_i \boldsymbol{n}_i \sim \mathcal{N}(0, \sigma^2)$ and $\boldsymbol{n}_i$ is the concatenation of the noise samples at aperture point i which is assumed to be $\mathcal{N}(0, \sigma_n^2)$. It is shown in [12–15] that a stable recovery of the sparsity pattern vector $\boldsymbol{b}$ is possible by solving a modified convex optimization problem, called the Dantzig Selector [13],

$$\hat{\boldsymbol{b}} = \arg\min \|\boldsymbol{b}\|_1 \qquad \text{s.t.} \quad \|\boldsymbol{A}^T(\boldsymbol{\beta} - \boldsymbol{A}\boldsymbol{b})\|_\infty < \epsilon \tag{7}$$

where $\boldsymbol{A} = \boldsymbol{\Phi}\boldsymbol{\Psi}$. For proper selection of ϵ, the noise statistics of the measured data can be estimated or cross-validation [16] can be implemented.

3. RESULTS

To illustrate the CS method for SFGPRs, assume a 2D homegeneous target space of 30 cm $\times$ 30 cm containing three randomly placed point targets as shown in Fig. 1(a). The target space is scanned by a GPR at a height of 10 cm and collects frequency domain measurements at frequencies 100 MHz to 10 GHz with 100 MHz frequency steps. The simulated data for the scenario at an SNR of 0 dB is shown in Fig. 1(b).

Instead of measuring all 100 frequencies we only use a random subset of 20 frequencies at each aperture. The randomly measured frequencies are indicated by black points in Fig. 1(c). If we had measured all the space-frequency domain data and applied backprojection (BP) we would obtain the image show in Fig. 1(d). The BP result using the randomly selected data is severely degraded as shown in Fig. 1(e). For the proposed CS method solving (7) from the randomly selected data generates the target space image shown in Fig. 1(f). For the numerical solution of (7) a convex optimization package called `CVX` [17] was used. The CS method finds the correct target positions with less clutter since the ℓ_1 minimization forces sparse solutions. All the target space images in Fig. 1(d,e,f) are normalized to their own maxima and are shown on the same scale.

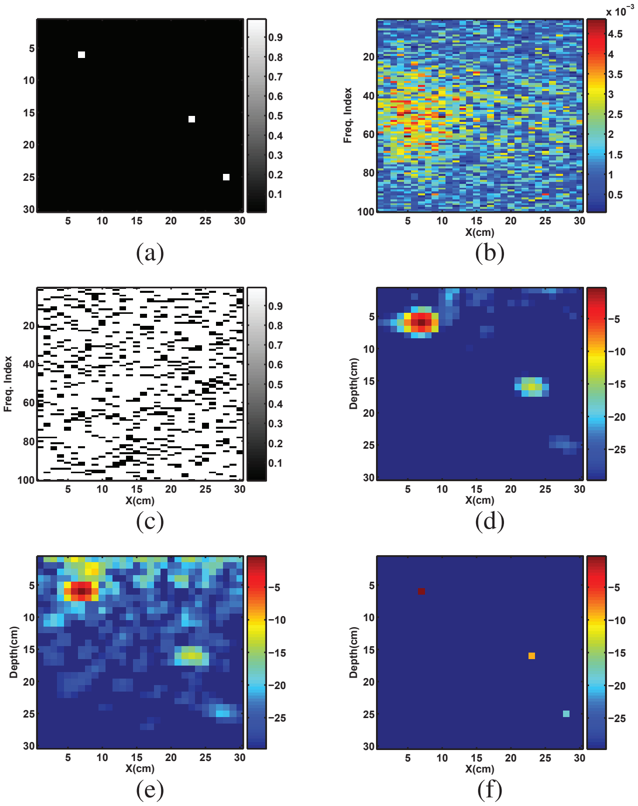

Fig. 1. (a) Target space, (b) Noisy space-frequency domain target space response at SNR $= 0$ dB for 30 aperture points, (c) Black points indicate the randomly measured frequencies at each aperture position, when 20% of the total frequencies are used, (d) Frequency domain BP image using all the space-frequency data from (b), (e) Frequency domain BP method using only the randomly selected 20% , (f) Solution obtained with the CS method using (7). $\epsilon = 0.1339$ is used for the results shown here.

3.1. Performance in Noise

To analyze the effects of noise on the target position, SNRs from -25 dB to 15 dB are tested. At each SNR level, 50 independent trials are performed. Figure 2 shows the variance of the estimated target locations versus SNR for the CS method and BP with varying number of frequency measurements at each scan point. It can be observed that the CS method using the same number of frequency measurements has smaller variances than the BP method. The variance of BP at moderate to high SNRs doesn't change too much due its resolution limit. The CS method provides much lower variances indicating increased resolution, which is also observed in other similar sparse signal reconstruction applications [18].

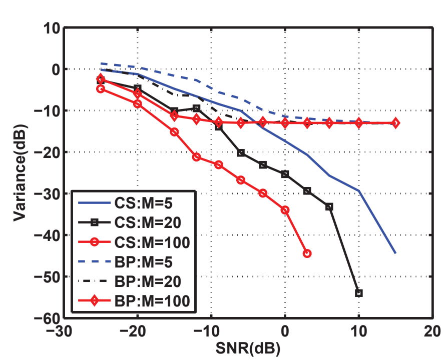

Fig. 2. (a) Target position variance vs. SNR. M is the number of frequencies used.

3.2. Buried Target Results

The performance of the proposed algorithm is tested on an experimental data taken over four buried targets: VS-1.6, PMF-1, VS-2.2 mines and a mine simulant. The burial positions of the targets are schematically shown in Fig. 3(a). The targets are collinear, so the GPR can measure one line scan over the targets. The phase centers of the antennas are elevated 27.8 cm above the surface, and the transmitter-receiver distance is 12 cm. The spatial step size is 2 cm.

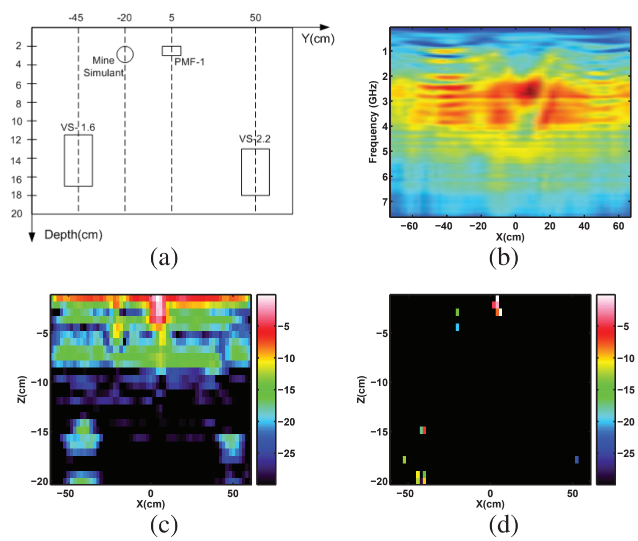

Fig. 3. (a) Experimental setup for buried target imaging, (b) Magnitude of the complete space-frequency measured GPR response of 3 buried targets, (c) target space image obtained with BP, and (d) with the CS method.

At each point 379 frequency points are measured. The magnitude of the measured raw frequency domain data is shown in Fig. 3(b). The frequency domain BP result using all the space-frequency data is shown in Fig. 3(c). All the objects can be seen in the migrated BP image. The CS method uses 100 random frequency points instead of 379 and yields the target space image in Fig. 3(d). Most of the energy in Fig. 3(b) is due to the reflection off the surface with only a minor part due to the mines. Both methods are pulling these small mine reflections out of the larger surface reflection. In the CS result all the targets can be seen with less clutter in the image. Both the CS and BP results are normalized to their own maxima and then shown on the same scale.

4. CONCLUSIONS

A novel data acquisition and subsurface imaging algorithm for GPRs is demonstrated. The new method exploits an a priori assumption of sparseness in the target space to reduce the data acquisition time significantly by decreasing the total number of measurements. An ℓ_1 minimization problem is solved to reconstruct the target space image from a small number of random measurements. Initial results show that extremely sparse images can be obtained with the proposed method compared to standard backprojection imaging algorithms.

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