Multi-Channel Spectrum Analysis of Surface Waves

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Abstract— **Spectrum Analysis of surface waves (SASW) is one of the most effective non-invasive methods for soil characterization. Surface waves travel in the medium along a free boundary and can be easily detected by using a transducer placed on the free surface of the boundary. Traditional methods of SASW are two-station methods that use the phase information at two receivers to determine phase velocity as a function of frequency. Multi-station methods have also been developed by using a two-dimensional Fourier transform approach, but these methods exhibit poor resolution. We propose a new method based on vector processing of data obtained from an array of tri-axial sensors to produce a high resolution, multi-modal spectrum of the surface waves. These different modes can be identified and reconstructed in time domain, and then inverted to obtain the shear velocity profile of the subsurface.**

I. Introduction

Waves that propagate in a medium can be roughly divided into two main categories: body waves and surface waves. Surface waves are generated only at a free boundary and can be essentially of two types: Love waves and Rayleigh waves. Rayleigh waves are always generated when a free surface exists in a continuous body. In a vertically heterogenous medium the phase velocity of the Rayleigh wave is a function of frequency and this dependence is strictly related to the mechanical parameters of the medium [1]. Hence, if we can determine the dispersion curve (i.e., phase velocity vs. frequency), it is possible in principle to calculate the mechanical parameters of the medium. This technique of determining the dispersion curves is the basis of the SASW methods. Traditional methods are based on data collected at two receivers from which the phase of the Average Cross-Power Spectrum is used to calculate the phase velocity [1]. One crucial step in this process is unwrapping the cross power spectrum phase, because additive noise can produce fictitious jumps in the wrapped phase. Some array techniques have also been developed based on frequency-wavenumber analysis, using the 2-D Fourier transform, but these suffer from poor resolution [1].

Our technique is based on the combination of a temporal Fourier transform across time t followed by a pole-zero model across the spatial domain x. Using the amplitude and root estimates from pole-zero modelling, it is possible not only to construct dispersion curves, but also to obtain insight into several other important parameters. One such property by which different types of surface waves can be identified is polarization. A surface wave consists of particle motion along a specific path, e.g., a Rayleigh wave involves particle motion along a retrograde elliptical path [2]. Hence, we can use polarization to identify these waves, because we have extended our algorithm to the two-channel case. The array data is collected by means of tri-axial sensors, from which we use two channels that measure the horizontal and vertical particle motion. Sensors actually measure acceleration of the particles in these directions. Polarization ellipses can be constructed by estimating the complex amplitudes of the measurements in these two channels. In addition to the complex amplitude, we can also estimate wave-number and attenuation, which can be used to extract individual modes and reconstruct them in the space-time domain. The following sections will describe the parametric modelling method and also the processing that we have implemented for numerical data and field data.

II. Parametric Model for Surface waves-Vector Sensor Approach

The parametric model is based on technique developed in [3–4] for sonic logging applications. For the single channel case, the collected data $s(\mathbf{x},t)$ is a function of space and time. We can represents this in the (k-ω) domain as

$$s(\mathbf{x},t) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(k,\omega)e^{j(k\mathbf{x}-\omega t)}dk\,d\omega \qquad (1)$$

where $\mathbf{x}$ is the spatial position, k is the spatial wave-number, and ω is the temporal frequency. By taking a temporal Fourier transform across t, we have

$$S(\mathbf{x},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(k,\omega)e^{j(k\mathbf{x})}dk \qquad (2)$$

At each temporal frequency ω, pole-zero modelling is done across the spatial dimension to get a model consisting of a sum of exponentials that represents propagating waves. Thus, we can approximate the integral in (2) with

$$S(\mathbf{x},\omega) \approx \sum_{p=1}^{P} a_p(\omega)e^{jk_p(\omega)\mathbf{x}} \qquad (3)$$

where P is the model order.

In the two-channel case the collected data $s(\mathbf{x},t)$ is a vector with two channels, i.e.,

$$s(\mathbf{x},t) = \begin{bmatrix} s_x(\mathbf{x},t) \\ s_z(\mathbf{x},t) \end{bmatrix} \qquad (4)$$

where $s_x(\mathbf{x},t)$ is the horizontal displacement channel and $s_z(\mathbf{x},t)$ is the vertical displacement channel. If we do the processing as explained above, and estimate the poles and zeros separately for each channel, then we must match the information in the (k-ω) domain to find the vector of complex amplitudes for the x and z channels. For a plane wave impinging on m two-channel sensors, we can represent the collected data at a specific frequency ω (after taking the Fourier transform) as:

$$\underline{\mathbf{S}}(\omega) = [\underline{\mathbf{S}}_1(\omega), \underline{\mathbf{S}}_2(\omega) \ldots, \underline{\mathbf{S}}_m(\omega)] \quad (5)$$

where

$$\underline{\mathbf{S}}_i(\omega) = \begin{bmatrix} S_x(\mathbf{x}_i,\omega) \\ S_z(\mathbf{x}_i,\omega) \end{bmatrix} \quad (6)$$

Each individual channel can be modelled by (3), which gives a model consisting of P parameters. Hence, the x channel would be

$$S_x(\mathbf{x},\omega) \approx \sum_{p=1}^{P} A_{xp}(\omega) e^{jk_p(\omega)\mathbf{x}} \quad (7)$$

and likewise for the z channel. The disadvantage of this approach is that we must hope that $k_p(\omega)$ will be the same in both models in order to get complex amplitude estimates that can be used for vertical and horizontal particle motion.

Vector IQML

A better approach is to determine one model simultaneously for the two channels of array data. The pole-zero modelling technique used in this paper is based on the IQML (Iterative Quadratic Maximum Likelihood) algorithm which is also called the Steiglitz-Mcbride extension of Prony's method [5]. We have reformulated the IQML algorithm for the multi-channel case. The input data is the vector in (5) consisting of the complex amplitudes from both channels at a specific frequency. The algorithm output is P estimates for the poles which are the same for both channels, and also the complex amplitudes for each channel which are different. Hence, we would obtain

$$\begin{aligned} \underline{\mathbf{S}}(\omega) &= [\underline{\mathbf{S}}_1(\omega), \underline{\mathbf{S}}_2(\omega) \ldots, \underline{\mathbf{S}}_m(\omega)] \\ &\approx \begin{bmatrix} \sum_{p=1}^{P} A_{xp}(\omega) e^{jk_p(\omega)\mathbf{x}} \\ \sum_{p=1}^{P} A_{zp}(\omega) e^{jk_p(\omega)\mathbf{x}} \end{bmatrix} \end{aligned}$$

From the poles we can determine the wave-number $k(\omega)$ and the attenuation $\alpha(\omega)$ from which we obtain the dispersion curves of velocity vs. ω. The complex amplitude estimates are used to determine the strength of different wave components and also to obtain the parameters for the polarization ellipses. Thus, different types of waves can be identified by using velocity and polarization, or the waves can be reconstructed again in the time domain by using (for the z channel):

$$s_z(\mathbf{x},t) = \sum_i A_z(\omega_i) e^{(\alpha(\omega_i)\mathbf{x} + j(\omega_i t + k(\omega_i)\mathbf{x}))} \quad (8)$$

and likewise for the x channel.

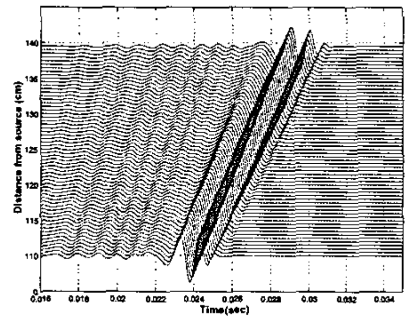

Fig. 1. Vertical channel space-time data.

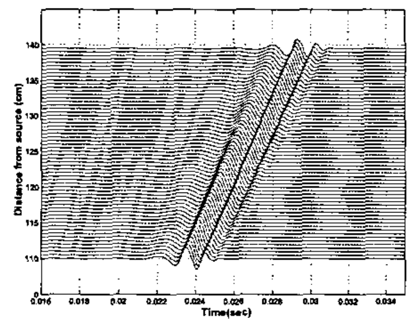

Fig. 2. Horizontal channel space-time data

III. Processing of the Data

Testing of this new algorithm has been carried out on both synthetic data and field data.

Synthetic Data

Numerical data generated from a 3-D FDTD model can accurately model elastic wave propagation in a stratified medium [2]. The data simulate what the sensors would have measured on the surface with a known stratified medium specified in the model. Examples of synthetic data for the horizontal and vertical channels is shown in Figs. 1 and 2, where the horizontal axis is time and the vertical axis is the sensor position (distance from the source). The first sensor lies 110 cm from the source with a distance of 0.5 cm between consecutive sensors. The total number of sensors used is 60, covering an aperture of 30 cm.

Processing for this data set yields the dispersion curves shown in Fig. 3. These multi-modal dispersion curves are typical for surface waves [7]. Four different modes can be identified at the higher frequencies, with the strongest one being the Rayleigh wave. Traditional two station methods only produce the dominant mode which is usually the Rayleigh mode. The presence of the additional modes is related to the subsurface structure in the shallow region near the surface [7]. The predominant mode, identified as mode-0 or the Rayleigh wave, exhibits an elliptical polarization which has

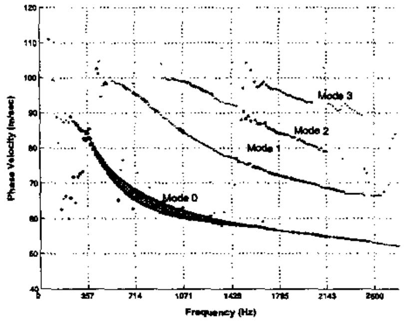

Fig. 3. Multi-Modal Dispersion Curves. The model order (P) used in this processing was $P = 4$.

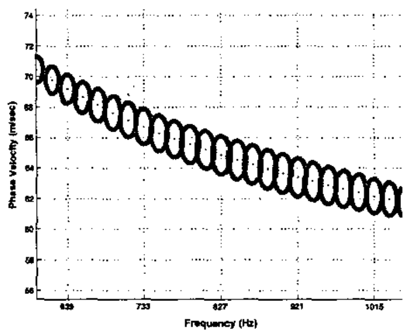

Fig. 4. Polarization Ellipses for Rayleigh wave (Mode-0).

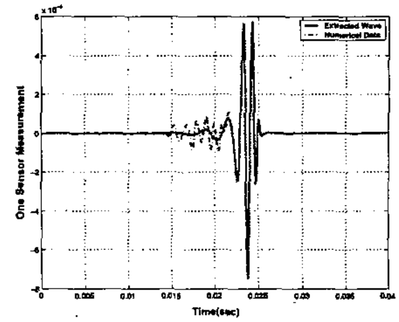

Fig. 5. Extraction of Mode-0, Vertical Channel.

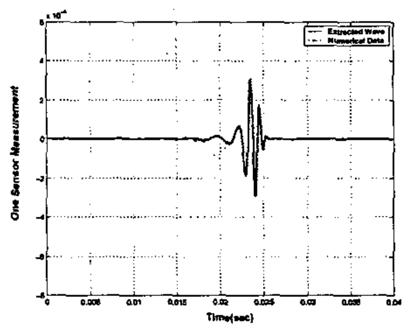

Fig. 6. Extraction of Mode-0, Horizontal Channel.

been extracted from the complex amplitude estimates and plotted in Fig. 4. At each frequency an ellipse is plotted at the corresponding phase velocity. The parameters for the ellipse are obtained by using the complex amplitude estimates for the horizontal and vertical particle motion. The parameters used are the major axis, minor axis, tilt angle and axial ratio for the ellipse. The sign of the axial ratio is used to indicate which direction the ellipse is rotating, either retrograde or prograde. The size of each ellipse is proportional to the complex amplitude values in the two channels. This is also encoded in the thickness of the line used when plotting the ellipse, with the thickness being proportional to $\sqrt{|A_x|^2 + |A_z|^2}$. Another parameter that we have encoded is the polarization direction with a dark blue color indicating retrograde motion (as in the Rayleigh wave), and a light red color for prograde. The vertical channel displacements are larger so the major axis of ellipse is tilted toward the vertical direction for the Rayleigh wave.

By extracting the individual modes from these dispersion curves, along with their parameters, we can reconstruct individual modes in the time domain using (8). This time-domain resynthesis was done for the fundamental mode, and is shown in Figs. 5 and 6 for the horizontal and vertical channels, respectively. The original numerical data is also shown for comparison. The reconstructed time-domain plot is in close agreement with the original data especially near the main pulse. The leading edge in the reconstructed plot does not follow the original, suggesting that it is related to other higher modes.

Processing for Field Data

The system used for data collection is described in [6,8]. The collected field data is shown in Figs. 7 and 8, for the horizontal and vertical channels, respectively. The first sensor is at a distance of 24 inches from the source with succeeding sensors one inch apart. Each sensor is a tri-axial accelerometer, but only the vertical and horizontal measurements are used. The total number of sensors used in the processing was 85, and the model order was $P = 3$. In Fig. 9, there are two dispersion curves visible with mode-0 being the stronger mode. The portion of spectrum in frequency range greater than 766 Hz and with velocities between 400 m/sec and 450 m/sec seems to be related to the pressure wave. The pressure wave is the fastest body wave, and it should appear at higher frequencies. In Fig. 10, the polarization ellipses for the Rayleigh wave (mode-0) are shown.

The two modes were also extracted and reconstructed in the time domain (for the first 60 sensor positions) and this is shown in Figs. 11 and 12 for the horizontal channel. By comparing to Fig. 7 we can see which portions of the original sensor data correspond to these two different modes. Clearly we are able to separate these two modes, so it is easy separate the Rayleigh wave from the collected data in both channels.

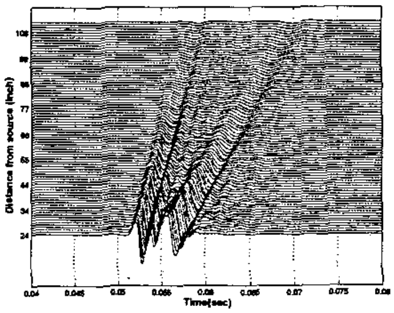

Fig. 7. Horizontal channel space-time data.

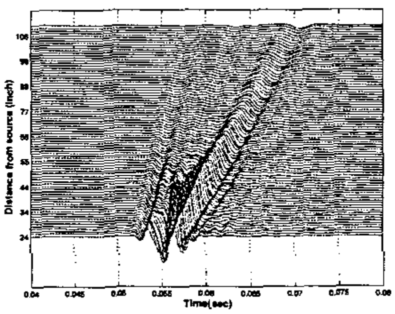

Fig. 8. Vertical channel space-time data.

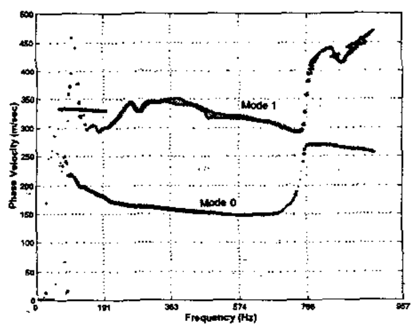

Fig. 9. Multi-Modal Dispersion Curves.

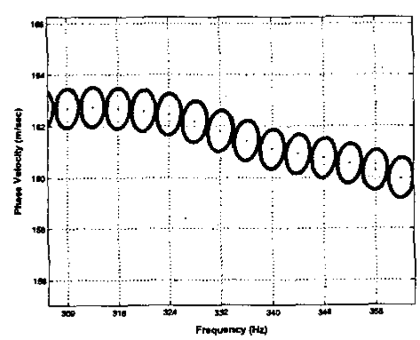

Fig. 10. Polarization Ellipses for Rayleigh wave (Mode-0).

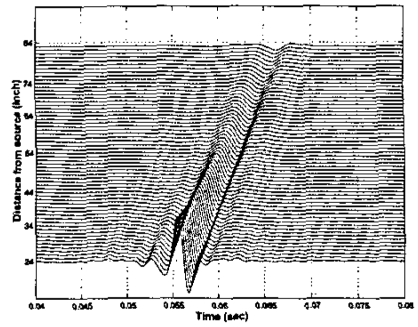

Fig. 11. Horizontal Channel Reconstruction (Mode-0) for the first 60 sensors positions.

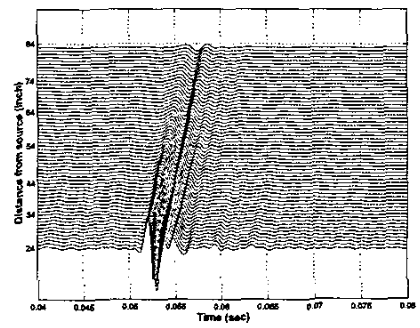

Fig. 12. Horizontal Channel Reconstruction (Mode-1) for the first 60 sensors positions.

IV. Conclusion

In this paper, a new method is proposed for multi-channel spectrum analysis for surface waves using a vector form of the IQML algorithm. Using this method we are able to separate not only the different modes and their polarization behavior, but also we can reconstruct these modes in the space-time domain. From collected field data we have succeeded in identifying and reconstructing the mode that is the Rayleigh wave. One application for this processing is to use the dispersion curve values for the Rayleigh wave as inputs to an inversion process that estimates the soil parameters [8]. Another ongoing investigation is to use the models of surface waves to detect land mines and underground tunnels [6].

Acknowledgment

This work is supported in part by the U.S. Army Research Office under contract number DAAD19-02-1-0252. The authors also like to thank Pelham Norville for generating numerical data, and Gregg D. Larson, James S. Martin, and George S. McCall for field data collection.

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