An Investigation of Time Reversal Techniques in Seismic Landmine Detection

Pelham D. Norville[a], Waymond R. Scott, Jr.[a], and Gregg D. Larson[b]
[a]School of Electrical and Computer Engineering
[b]School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0250

ABSTRACT

A system is under development at the Georgia Institute of Technology that utilizes a seismic source to propagate Rayleigh waves through a medium such as soil. Non-surface-contacting electromagnetic sensors are used to detect the displacement of the medium created by interaction of the Rayleigh waves with a target, such as a landmine. The system has been tested in a relatively uncluttered medium and has yielded encouraging results, demonstrating that the system is effective for the detection of targets buried just below the surface.

The system performs well in an uncluttered medium. However, when the medium is filled with a large number of scattering objects, the Rayleigh wave will be broken up by the scatterers in the medium to the point that the wave front no longer interacts with the target as it would in an uncluttered medium. This causes detection of a target to be uncertain or impossible. In an effort to extend the application of this system to a highly cluttered medium, the time reversal method is applied to the seismic system, and evaluated for focusing Rayleigh wave fronts at a desired location. Numerical and experimental results are presented for a propagation medium with no scatterers present, and with multiple scatterers present. Time-reverse focusing results are also compared to uniform excitation and time-delay beamforming methods.

1. INTRODUCTION

The landmine detection system which is under development at the Georgia Institute of Technology functions by exciting elastic waves which propagate through the soil (Fig. 1). Several wave types are generated by the seismic source, but the one which is most important for detection of landmines is the Rayleigh surface wave that propagates along the air-soil interface of the ground. When the Rayleigh wave passes through an object buried in the sand, it excites that object into vibration. Objects such as rocks and sticks scatter wave fronts, but usually vibrate very little themselves (Fig. 2a). Landmines, due to their structure, exhibit a resonant vibration between the flexible top of the landmine and the soil layer above the mine (Fig. 2b). The resonating landmine excites surface displacements of the ground which are then measured using the non-contacting electromagnetic sensors.

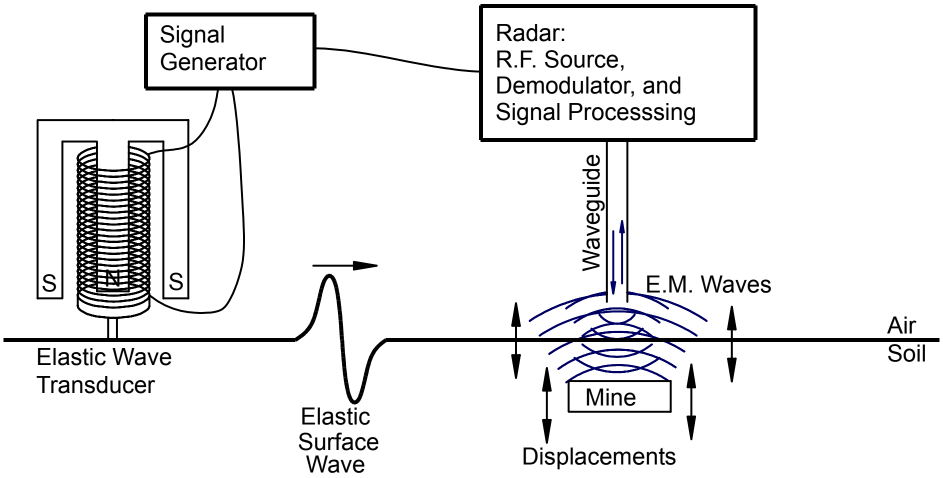


Fig. 1. Schematic of the elastic wave landmine detection system.

Detection and Remediation Technologies for Mines and Minelike Targets IX, edited by Russell S. Harmon, J. Thomas Broach, John H. Holloway, Jr., Proceedings of SPIE Vol. 5415
(SPIE, Bellingham, WA, 2004) 0277-786X/04/$15 · doi: 10.1117/12.542659

The effectiveness of the detection system depends on relatively large displacements of the ground at the location of a landmine in order to excite this flexural resonance. In a highly cluttered medium, most of the energy in the transmitted wave is scattered off objects in the medium. The result is that very little coherent energy reaches the target, and any small resonance that is excited is difficult to detect due to the numerous waves continuously bouncing off the scattering objects in the medium. Time-reverse focusing methods allow for energy to be focused at any location in a highly cluttered medium, without any knowledge of the characteristics of the medium.

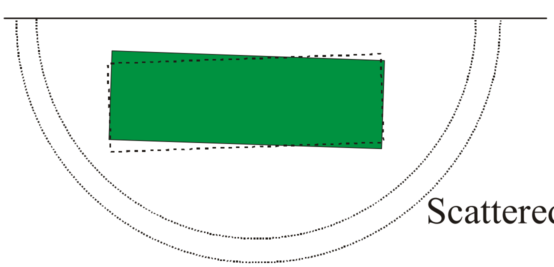


Fig. 2a. A non-resonant object vibrates, but does not transmit significant seismic energy to the surface.

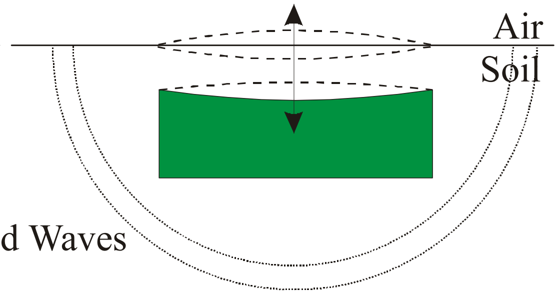


Fig. 2b. A resonant object causes a significant displacement on the surface.

A numerical model, employing the finite-difference time-domain method has been constructed and willl be used to compare time-reversal to other methods of excitation. Time reversal is compared to standard uniform excitation techniques and time-delay focusing methods. The numerical simulations show that time-reverse focusing gives improved results in the presence of clutter over other excitation methods. Experimental results obtained in the landmine detection laboratory at the Georgia Institute of Technology will be presented that confirm the results of the numerical model. Further experiments will test the efficacy of time-reversal focusing by introducing landmines into the cluttered environment and comparing the detection effectiveness of time-reverse focusing to other focusing methods.

2. BASIC TIME-REVERSAL THEORY

The principles of time-reversal for acoustic waves in liquids and solids have been treated fairly extensively in the literature [1 – 4]. This section will provide basic information on the theory behind time reversal. For more thorough treatments, the listed references should be consulted.

To begin understanding time-reversal, first consider the wave equation governing elastic wave motion in solids,

$$\rho_s \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu)(\nabla(\nabla \cdot \vec{u})) - \mu(\nabla \times (\nabla \times \vec{u}))$$

This equation assumes no body forces are present on the material and that the medium in which the waves are propagating is lossless. However, if any actual loss is small, the terms added to the above equation can be neglected. Taking note that the equation contains only second order time derivatives, it becomes clear that if there is a solution to this equation, $\vec{u}(\vec{r}, t)$, then $\vec{u}(\vec{r}, -t)$ must also be a solution to this equation. In order to work with the reverse time solution, but in a causal fashion, we will choose to work with the formulation $\vec{u}(\vec{r}, T-t)$ over the interval (T,0) such that we are only dealing with a finite time duration.

In the purest form, a time-reversal cavity may be constructed which records wave forms for time T, on a 3-D surface, surrounding a source. If these signals are transmitted in reverse (t → -t) simultaneously then they will focus back to the original source location. This implementation would be difficult to realize due to the inherent challenges of observing propagating waves on the surface of a 3-D volume in a solid medium.

Next, take note that Rayleigh waves are the primary mode of signal excitation in the landmine detection system, and that these waves decay exponentially as one moves deeper into the medium. Some energy is lost to mode conversion, and scattering from objects in the soil, but most of the energy remains near the surface. Because of this, the detection problem now is actually a quasi-2-D problem. This means that instead of forming a time-reversal cavity surrounding a volume, a closed contour on the surface surrounding the source should give a solution very close to that of the time-reversal cavity.

Even construction of the 2-D closed contour would be difficult and impractical to implement in most situations. As a simplification, the 2-D contour is further reduced to a simple array of receivers, referred to in the literature as a time reversal mirror (TRM). This formulation is a simplification and does not take into account all the outward propagating waves, but it is more feasible to implement. While more practical to implement, the array is subject to the common limitations of array techniques involving diffraction, spot size, introduction of grating lobes etc [2].

The TRM concept is the one which will be implemented in the numerical and experimental results presented in this paper. While the TRM does not provide optimal focusing when compared to a 2-D or 3-D time-reversal cavity, it produces good focusing results from a sensible implementation of the time-reversal concept.

3. THE TIME-REVERSAL FOCUSING METHOD

In its simplest form, time-reversal focusing is accomplished by first determining a location that should be the focal point. An ideal source is placed at the focal point and an elastic wave is excited. This wave propagates away from the source and the signal is received and recorded at an array of sensors far from the focus point (Fig. 3). Next, the receiver array is replaced with an array of sources. Then the signals received at each of the receiver points are reversed in time. This means that if a signal $f_N(t)$ is recorded at receiver N, the corresponding time-reversed signal is $r_N(t) = f_N(-t)$. All N signals, $r_N(t)$, are transmitted simultaneously. The result will be that the waves add constructively to cause a large excitation in the region surrounding the desired focal point. As can be inferred from the procedure, no knowledge of the characteristics of the medium is necessary to take advantage of time-reversal focusing.

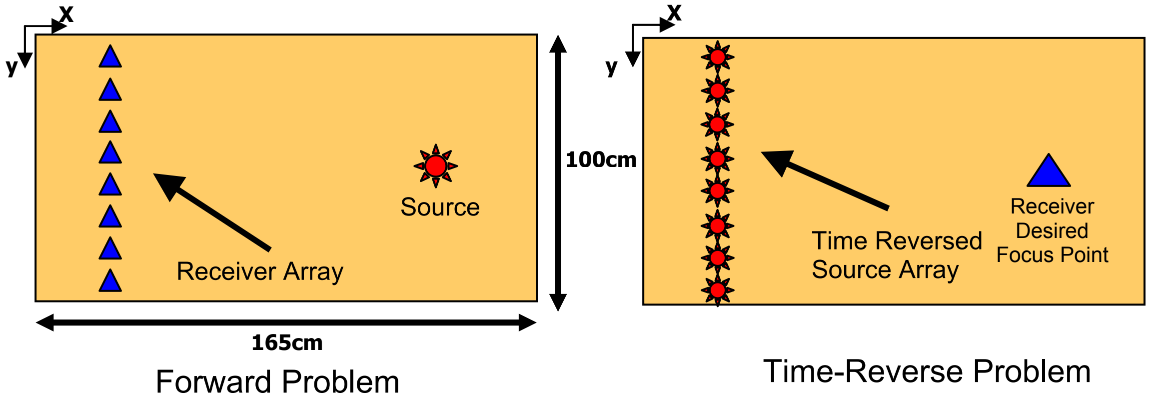


Fig. 3. Experimental / numerical configuration for the time-reversal focusing method.

While this simple method of time-reversal focusing is straightforward to apply and is applicable to numerical simulations, it would not be very practical in the case of an experiment or a field application. It would certainly be un-advisable to place a seismic source on top of a location where a landmine may be buried! Instead, the principle of reciprocity may be applied to the first part of the problem. Reciprocity states that the transfer function imparted to a signal by traveling from *point A* to *point B* is the same as the transfer function imparted to the same signal if traveling from *point B* to *point A*. Employing reciprocity allows a signal to be sent from each receiver location and then recorded at the desired focal point by a non-contact sensor. This means instead of only 1 forward signal transmission recorded on N receivers, N separate signals must be sent and recorded individually at the desired focal point. The result is N time-reverse signals which are identical to the ones created using the simple time-reversal formulation.

4. THE NUMERICAL MODEL

The numerical technique used to create the model is the three-dimensional finite-difference time-domain (FDTD) method. A first-order particle-velocity and mechanical-stress formation for the elastic wave fields is developed and then discretized from its continuous differential form. The solution space is then divided into a grid composed of individual cubes known as unit cells. Each of the elastic wave stresses and velocities are located at discrete spatial points in the three dimensional grid. Using the finite differences between field quantities in adjacent unit cells, the field quantities are computed at each discrete time step. The solution space is surrounded on all 4 sides and the bottom by a perfectly matched layer (PML) which absorbs all outgoing waves. This prevents reflections from the edges of the grid, which makes the solution space appear as an infinite half-space. The top surface of the grid is terminated using a free surface boundary condition, which approximates the air-soil interface. A detailed description of the model can be found in the literature [5].

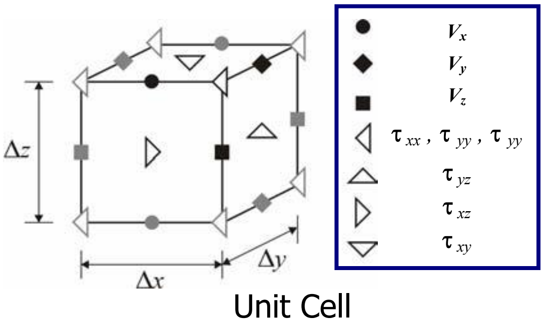


Fig. 4a. The unit cell for the FDTD model [5].

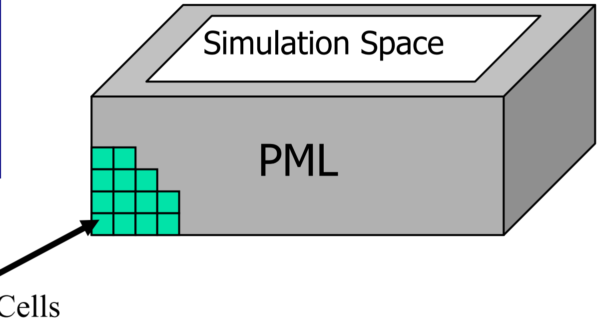


Fig. 4b. The solution space for the FDTD simulation: composed of unit cells. The center is the area of the simulation while the edges are terminated by a perfectly matched layer (PML).

5. MODEL PARAMETERS

5.1 General FDTD Parameters

Several parameters were used to define the solution space for the model. The physical size of the space is 165 cm x 100 cm x 50 cm. This space was discretized into equally sized unit cells of 0.5 cm on a side for a total computational model size of 330 x 200 x 100 cells. The solution space was surrounded on four sides and the bottom with a 10 cell thick perfectly matched layer (PML) to absorb all outgoing waves. The surface was terminated in a free surface boundary to simulate the air-soil interface. The time step (Δt) between successive calculations was set to 0.75 μs and the simulation was run for approximately 35 ms for a total of 46359 time steps. The computation of results was performed on a 64 node Beowulf Cluster of Athlon XP 2200+ processors and required approximately 4.5 hours for computation of the results for each simulation.

5.2 Soil Parameters

Soil is generally a layered medium with each layer having its own set of physical characteristics. To accurately simulate a physical situation and to encourage better agreement between numerical and experimental results, a soil profile was selected that approximates the measured characteristics of damp compacted sand [5]. The profile matches the characteristics measured in the experimental facility used to create the measurements presented in the second half of this paper (Fig. 5).

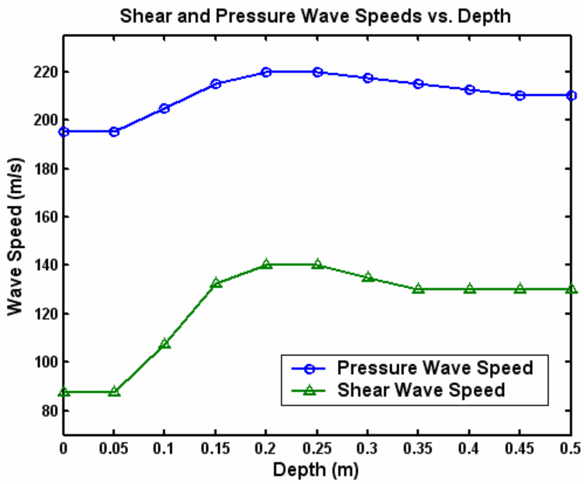


Fig. 5. The shear and pressure wave profiles used in the numerical simulations. The profiles are based off the measured characteristics of the sand in the experimental facility.

5.3 Scattering Objects

Spheres are used as scattering objects in the numerical simulations to break up the wave fronts. Though they are uniform in shape, spheres are sufficient to break up wave fronts if the medium is sufficiently filled with randomly distributed spheres of multiple sizes. The spheres are modeled with material parameters similar to those of many types of stone or other dense scattering media commonly found in soil: Vpressure = 3824 m/s, Vshear = 2342 m/s, Density (ρ) = 2280 kg/m^3. A random distribution of spheres of 3 cm, 5 cm and 7 cm diameters was created (Fig. 6). All spheres are placed with their tops 2 cm below the surface. Since the Rayleigh wave used in the landmine detection system is a surface wave, deeply buried scattering objects would have negligible effects on displacements measured at the surface.

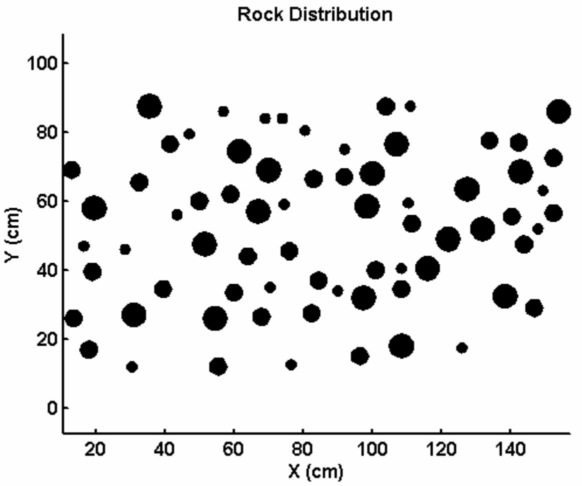


Fig. 6. The distribution of rocks introduced in the numerical simulation as scattering objects. All rocks are located 2 cm below the surface.

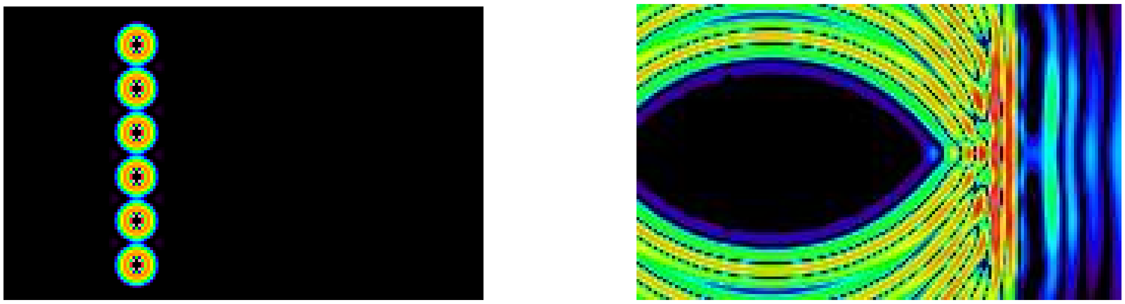
5.4 The Source – Receiver Array

The simple case of time reversal, as described in Sec. 3, is used in the numerical model (Fig. 3). 6 receivers are arranged in a line and are spaced 15 cm apart. A point source is placed at the desired focal location. In the reverse problem, the array of receivers is replaced with an array of independently controlled point sources.

5.5 Excitation Methods

When producing elastic waves in the soil, there are several different ways to excite the array of transducers. Each of these methods has advantages and disadvantages for use in any particular detection scenario. This section presents the three array excitation methods that will be compared in the numerical model results and the experimental results. The primary advantages and disadvantages of each method will also be noted. The results will be presented as pseudo color graphs of the magnitude of the vertical component of the particle velocity at the surface. The pseudo color scale is a 40 dB logarithmic scale from white (0 dB) to black (-40 dB).

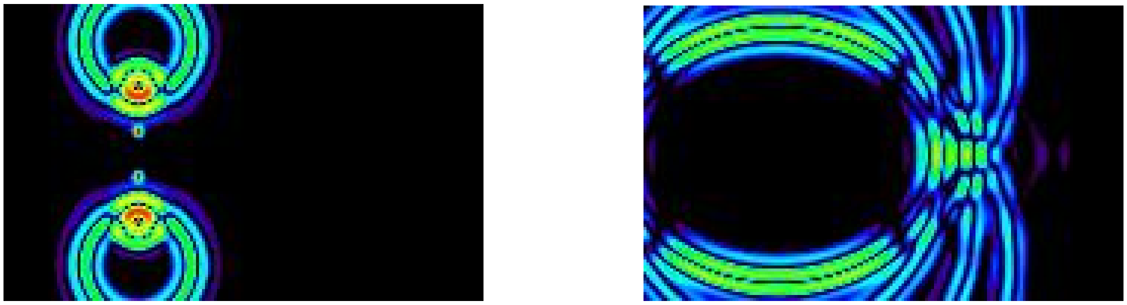
The simplest case excites all 6 point sources using identical excitation. In this case, all sources are excited with identical differentiated Gaussian pulses. Without scatterers present, a nearly uniform wave front propagates across the surface of the medium (Fig. 7). This excitation method is simple to create, and requires no a priori knowledge of the physical characteristics of the medium. It also excites each location along the wave front uniformly as long as no scattering objects are present. In the presence of clutter, the wave front may be scattered, reducing the uniformity of the excitation.



(a) (b)

Fig. 7. Identical Excitation: All sources are excited with identical differentiated Gaussian pulses. (a) All 6 pulses are being transmitted from the source array simultaneously. (b) The wave front created by the uniform excitation propagates along the surface.

The second excitation type uses Gaussian pulses that are time-delayed such that all pulses arrive at a focus location at the same time. Since the propagation paths from the sources to the focal point differ in length, the pulses can be time delayed such that all the pulses arrive at the focal point at the same time (Fig. 8). Because this method focuses energy to a specific point, it will create a larger excitation at the focal point but it will do so at the expense of exciting all points in the medium uniformly. In addition, the calculation of the time-delays for each pulse requires the knowledge of the propagation speed throughout the entire medium.



(a) (b)

Fig. 8. Time Delayed Excitation: Sources are excited with differentiated Gaussian pulses time delayed to cause focusing at a desired location. (a) Pulses furthest from the focus point begin first. (b) All pulses arrive simultaneously at the focus point adding constructively.

The third excitation type implements time-reversal focusing. For time-reversal, six separate simulations are run in which a pulse is propagated from each of the sources, recorded at the focal point and then reversed in time (played back in reverse, t → -t). The time-reversed signals are then transmitted from their corresponding source locations (Fig. 9). Time-reversal has the same disadvantage of time-delayed focusing: the larger excitation created at the focus point is at the expense of exciting all points in the medium uniformly. A particular advantage of time-reversal over time-delayed focusing techniques is that it requires no knowledge of the propagation speed in the medium.

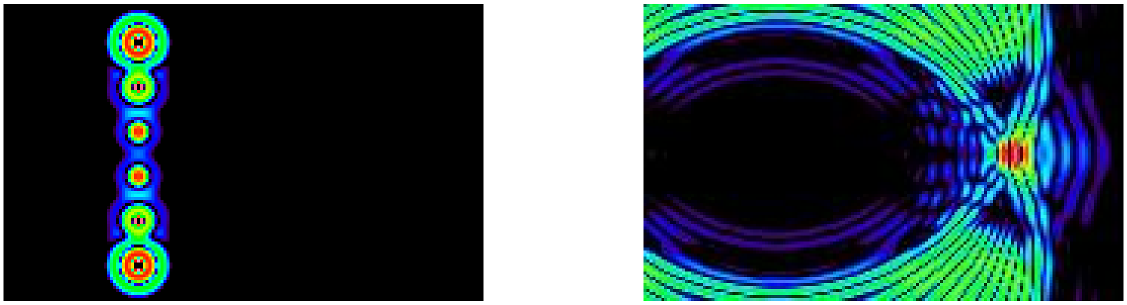

(a) (b)

Fig. 9. Time-reversal excitation: Sources are excited using the signal recorded at their respective locations in the forward problem. (a) The time reversed signals being propagating away from the sources. Since the signals furthest from the source would have arrived last in forward time, they are transmitted earlier in reverse time. (b) Time reversal causes all waves to add constructively at the focal point.

6. NUMERICAL MODEL RESULTS

Depictions of excitation types were shown in Fig. 7 - 9 without scatterers present in the medium. However, the purpose of this study is to investigate the effectiveness of time reversal in comparison to other excitation methods in the presence of scatterers. Therefore, further results will only be presented for cases in the presence of scattering objects.

6.1 Identical Source Excitation

All the sources are excited uniformly, and the flat wave front interacts with the various scattering objects (Fig. 6) present in the medium (Fig. 10). As the wave propagates through the scattering objects, it is broken up such that no coherent wave arrives at the target location. This case demonstrates that in the presence of high clutter, uniform excitation of the sources provides low signal levels to the target location and very little chance of exciting a resonance in a landmine at such a location. Identical excitation works well if little clutter is present, providing a uniform wave front which excites equal signal levels throughout most of the solution space.

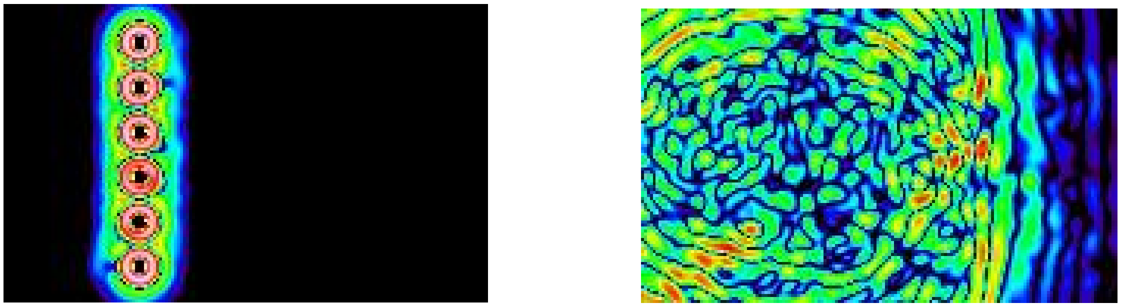

(a) (b)

Fig. 10. Two snapshots in time. (a) Identical waves are launched from the 6 sources. (b) The wave front is significantly altered upon arrival at the desired focal point due to scattering objects in the medium.

6.2 Time Delayed Source Excitation

If it is desired to focus energy to a specific location, the array of sources may be excited with time-delayed differentiated Gaussian pulses such that the pulse from each source arrives at the same time as all the others. If little clutter is present, this creates a focal point at the location of the target. In the presence of significant clutter, waves produced by each of the sources are subject to scattering off multiple objects, causing the waves to be broken up and arrive incoherently at the target location. Though scattering objects significantly reduce the effectiveness of the time-delayed differentiated Gaussian excitation, this method shows improvement over identical excitation with respect to wave front coherence and excitation level at the target location (Fig. 11). There are 2 primary are disadvantages to this method of focusing. Energy is only focused to one point, reducing the effectiveness of the excitation of targets at locations other than the focus point. Also, in order to accurately calculate the propagation delays between each source and the focus point, one must know the propagation speed in the medium between the source and the focus point. In the case of a medium randomly filled with scattering objects, any assumption of this propagation speed would be an approximation at best.

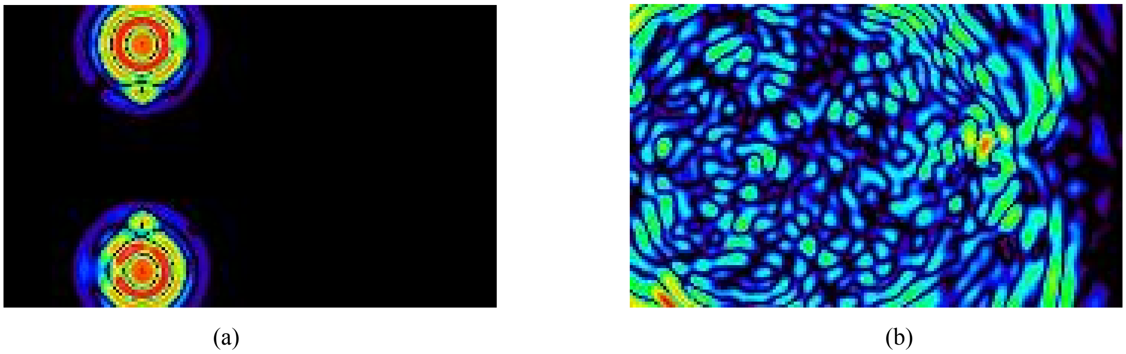

(a) (b)

Fig. 11. Two snapshots in time. (a) The time delayed pulses are launched from the 6 sources. (b) The focused wave fronts arrive at the desired focal point.

6.3 Time-Reversal Excitation

Time-reversed signals are transmitted simultaneously from their respective sources, as described in Sec. 5.5. Though no coherent wave front is produced, the waves sum constructively at the focus point, producing a large excitation only at that location.

The simulations show that in the presence of the chosen background, time reversal allows for effective excitation at the target location. The disadvantages of time reversal are that it allows for effective excitation only at a single target location, and if reciprocity is used, multiple simulations or experiments must be performed. In addition, because the waves that are transmitted in the reverse problem have already propagated through a dense field of scattering objects, they are already significantly altered from the original differentiated Gaussian pulses that were originally transmitted. This means that even though time-reversal causes a large excitation at the focus point, the scattered energy is generally higher than in the time-delayed focusing case.

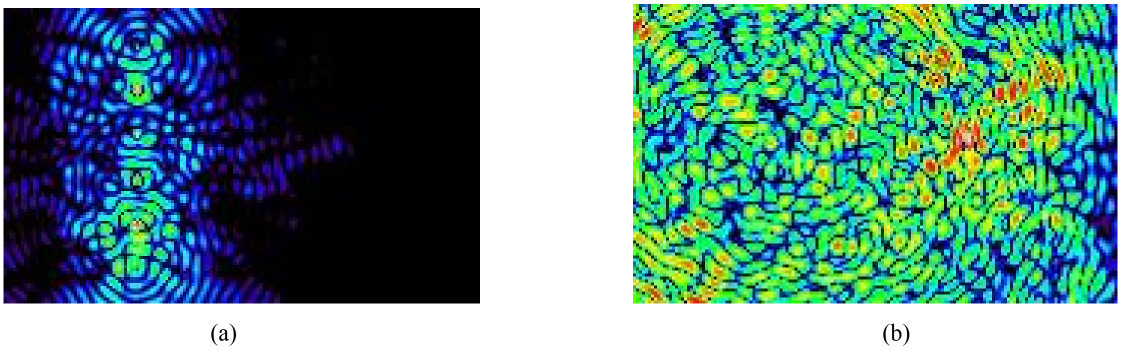
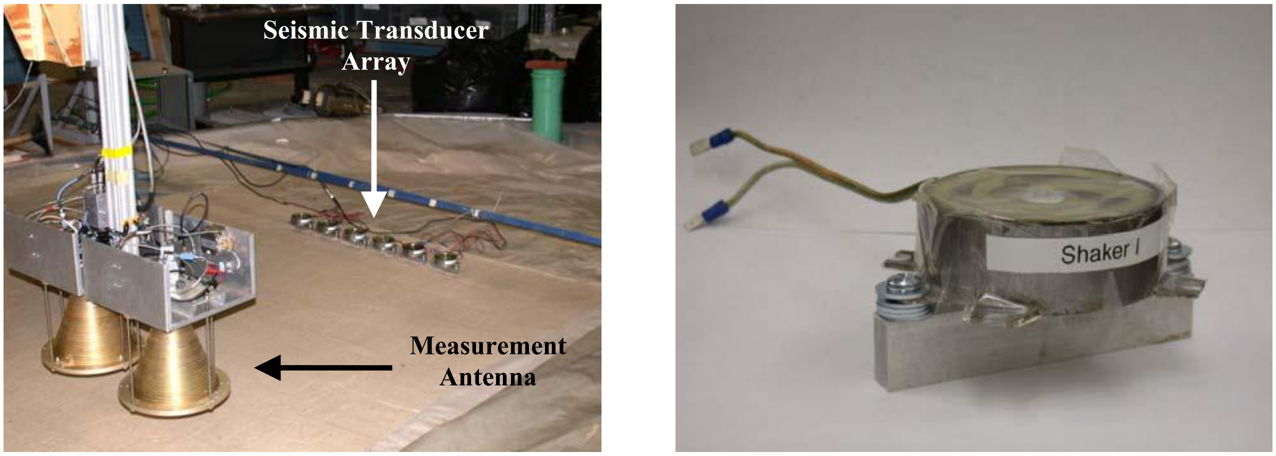

(a) (b)

Fig. 12. Two snapshots in time. (a) The time-reversed signals are launched from the 6 sources. (b) The concentration of time-reversed signals at the desired focal point.

7. EXPERIMENTAL SETUP

7.1 Experimental Facility

The experimental results were obtained in the landmine detection laboratory at the Georgia Institute of Technology (Fig. 13). The test facility consists of a large concrete wedge-shaped tank filled with approximately 50 tons of damp compacted sand. Sand was chosen as an effective soil medium since its properties are similar to many types of soil, and since it can be re-conditioned after objects are buried in it, to create consistent, reproducible results.



(a) (b)

Fig. 13. (a) The experimental facility. The seismic transducer array and the antenna are positioned over the sand tank.
(b) One of the seismic transducers mounted on an aluminum bar foot.

7.2 Seismic Sources

The seismic sources used to excite waves in the sand are small acoustic transducers mounted on 15 cm x 1.27 cm x 2.54 cm bar feet (Fig. 13). The feet are used to couple the seismic energy from the transducer into the ground. Unlike the numerical simulations, the actual excitation transmitted by the source is a chirp signal which propagates effectively in the sand and is easy to create using the seismic transducers. During post-processing, the received signal is de-convolved with the excitation chirp and convolved with a differentiated Gaussian pulse to create experimental results that are easier to interpret and can be directly compared to the numerical simulations.

7.3 Radar for Measurement of Surface Displacement

Displacement of the ground is measured using a non-contact electromagnetic sensor. The antenna used for the measurements is moved across the surface of the sand tank using a computer controlled positioner system. The surface is sampled at 2 cm increments over a 1 m x 1.5 m area. The antenna dwells over each location for approximately 10 seconds before taking data and moving to the next location. This ensures that the antenna is not vibrating during the measurement. A scan of the entire region requires approximately 10 hours to complete.

7.3 Introduction of Rocks

To introduce inhomogeneities into the ground, a large selection of rocks was buried below the surface. A first selection of medium-sized rocks of approximately 10 – 15 cm in diameter was buried in the tank (Fig. 14a) and then measurements were taken to determine the extent of the scattering created by the rocks. To create a more inhomogeneous environment, a second set of larger rocks of approximately 20 - 35 cm in diameter was then buried in the tank (Fig. 14b) along with the initial set.

Fig. 14. The layout of the rocks before being buried below the surface. Rocks in the left image were buried first. The rocks in the right image were added subsequently.

8. EXPERIMENTAL RESULTS

Several experimental cases similar to the numerical simulations will be presented. The three cases examine the effectiveness of identical source excitation, time delay focusing, and time-reverse focusing methods in a highly cluttered environment. The effective excitation of a resonance in a TS-50 landmine is also compared for the identical excitation (no focusing) case and the time-reverse focusing case.

8.1 Maximum Displacement and Time Snapshots

In the presentation of the numerical results, two snapshots in time were presented for each type of excitation. The first showed the amplitude of the displacement as the waves propagated away from the sources, and the second showed the amplitude of the displacement as the wave arrived at the focus point. This presentation allows one to easily visualize the propagation and focusing of the waves in time. However, it may not be the best metric to determine how well energy is focused to the focusing point. For comparison of the excitation types in the experimental cases, a plot showing maximum displacement at each measurement point over the entire time record is presented in addition to a time snapshot at the focusing time. For each location in the scan region, the entire time record is searched for the maximum value of the displacement and recorded in an array. This is then presented on a 40dB pseudo color plot (Fig. 15). In the two methods where focusing is used, the white cross indicates the desired focal point.

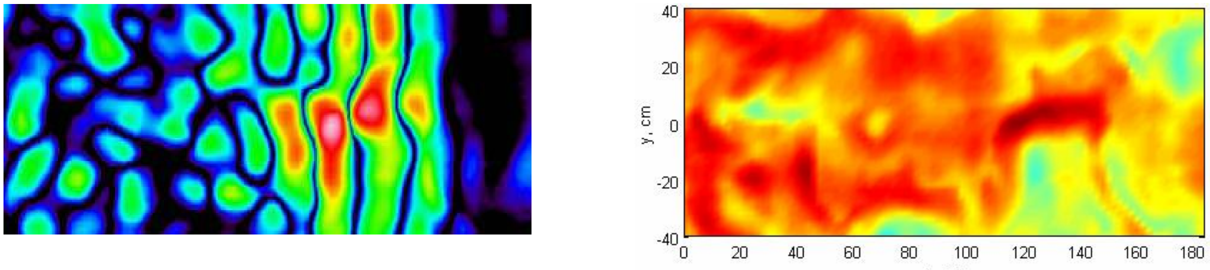


Fig. 15. No focusing: Snapshot and maximum displacement plot..

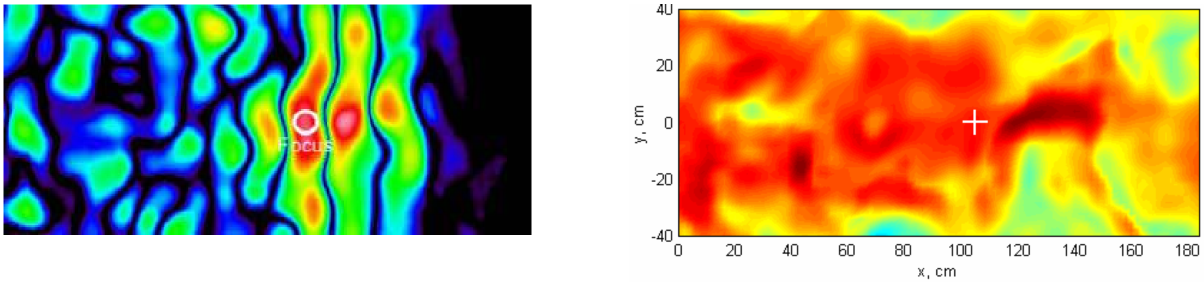


Fig. 16. Time-delayed focusing: Snapshot at focus point and maximum displacement plot.

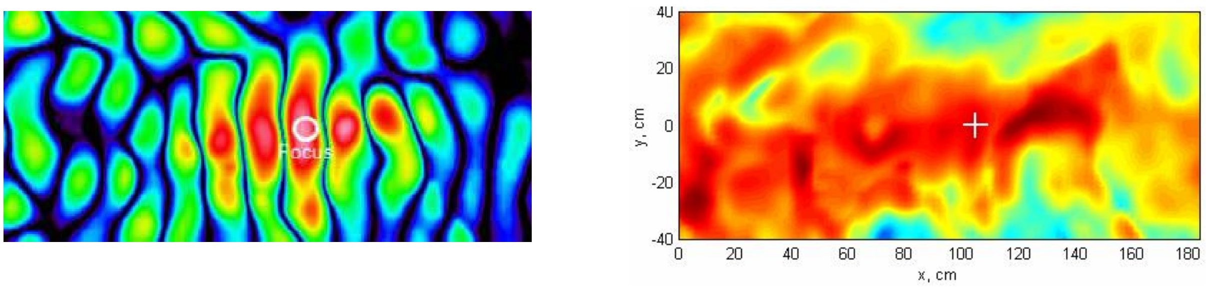


Fig. 17. Time-reverse focusing: Snapshot at focus point and maximum displacement plot.

As can be seen in Fig. 15 - 17, both the focusing methods show improvement over the case where focusing is not implemented. The results however, are not extremely dramatic. This leads to the conclusion that either focusing is not as effective as we anticipated, or that there is a physical reason for the lack in contrast. In this particular case, it was discovered that the layout of the rocks in the experimental cases led to a natural focusing of waves at a location just behind the desired focus point. This phenomenon, most evident in the maximum displacement plots shown above, is the reason for the low contrast between the focusing methods and uniform excitation.

8.2 Time-Reversal Focusing at Multiple Locations

To verify that time-reversal focusing works for multiple locations, the experiment was repeated using several different locations as the focus point. Since the layout of the scattering objects has not changed, the plots can be compared to the results for the case of no focusing (Fig. 15). The results are presented as a combination of a snapshot in time when the maximum excitation occurs at the focus point, and the plot of maximum displacement for the entire time record (Fig. 18 – 20). The desired focal point is indicated in each figure. The results for these three different focus points confirm that focusing can be achieved, irrespective of the focusing point chosen.

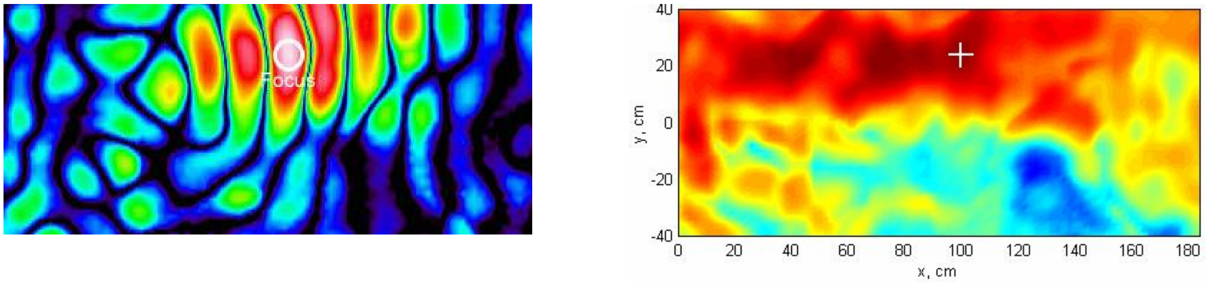


Fig. 18. Focus point: (85,24,0). Snapshot at focus point and maximum displacement plot.

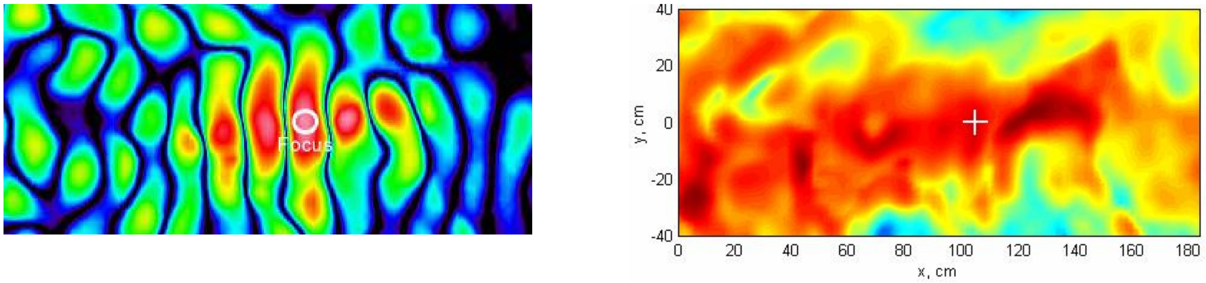


Fig. 19. Focus point: (90,0,0). Snapshot at focus point and maximum displacement plot.

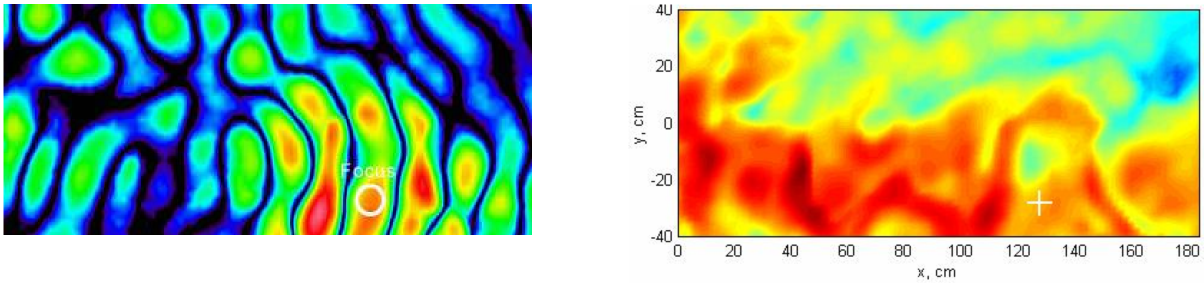


Fig. 20. Focus point: (113,-28,0). Snapshot at focus point and maximum displacement plot.

8.3 Excitation of Resonance in a TS-50 Landmine: Comparison of No Focusing and Time-Reverse Focusing

The goal of developing the method of time-reversal focusing is to excite a greater displacement at a particular location. This in turn will better excite a resonance in a target if one is at the location in question. To test the effectiveness of time reversal in exciting such a resonance, a TS-50 landmine was buried in the medium.

The location chosen for the landmine was deliberate. In examining the maximum displacement plot for Fig. 15, it is apparent that the region surrounding the point (113, -28, 0) is a region which is shielded from large displacements due to the particular configuration of rocks. The best test of the time reversal method is to attempt to excite a large displacement at a location where one would not normally occur due to this physical layout of the rocks.

The landmine was buried, and time-reversal focusing was used to excite a resonance in the landmine. The results are compared to the case where uniform excitation is used to excite a resonance in the landmine. It can be seen (Fig. 21) that the level of excitation at the location of the TS-50 is 8 dB higher than for the unfocused case. While 8 dB is not a drastic increase in displacement, given the sheltered location of the landmine, this increase becomes significant. The plots in Fig. 21 are max displacement plots similar to those shown above.

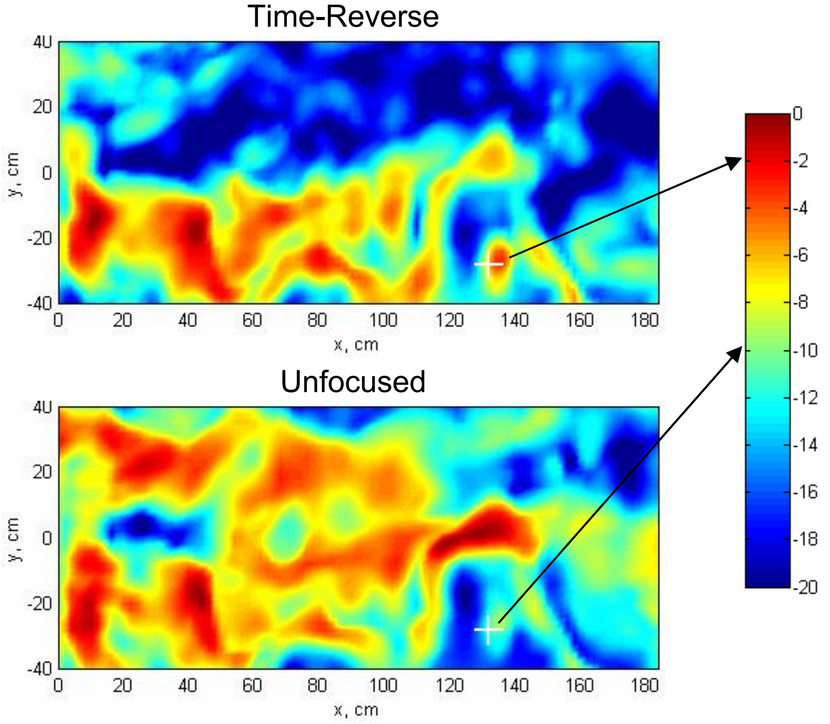


Fig. 21. Comparison of the maximum displacement for the unfocused and focused cases.
A TS-50 landmine is buried at the focal point.

9. CONCLUSIONS

In exciting resonances in a target, the bandwidth of the excitation pulse plays a role in the effectiveness of target detection. Broader bandwidth pulses excite resonances in a greater range of target sizes and materials since the excitation efficiency is related to the size of the object in comparison to a wavelength. In all cases, as the pulse travels from the source to the target, the soil acts as a filter which selectively attenuates certain frequencies. In the case of time reversal, this attenuation is made more significant since the signal passes through the medium twice: in the forward problem, and the reverse problem. To increase the effectiveness of time reversal, a logical improvement is to design a filter which pre-emphasizes those frequencies which will be attenuated by passage through the soil. This will improve the bandwidth of the excitation pulse, thereby exciting larger resonances, and improving target detection.

Time-reversal focusing has been shown to be an effective method of focusing energy to a single location. The specific advantage of time-reversal over other methods is that time-reversal requires no knowledge of the characteristics of the background medium. The main disadvantage of time reversal focusing is that it requires an additional experiment, which means it takes longer to perform than other focusing methods.

Given the disadvantages inherent in the time-reversal focusing method, it does not seem practical to scan an entire region using time-reversal focusing. A more effective method would be an adaptive detection scheme which employs time-reversal focusing for the “hard – to –reach” locations. A possible detection scheme would use identical excitation of all sources to first launch a wave used to determine the extent of the inhomogeneities and to look for possible targets. Each location is then assigned one of four conditions: “contains no target”, “contains a target”, “contains a possible target”, or “not enough information.” Time-reverse focusing can then be used to focus energy to the locations marked “contains possible target” and “not enough information.” By focusing additional energy to these locations, a resonant target should be more easily detected.

10. ACKNOWLEDGEMENTS

This work is supported in part by the U.S. Army Research Office under contract number DAAD19-02-1-0252.

11. REFERENCES

[1] M. Fink, C. Prada, F. Wu, and D. Cassereau, “Self focusing with time reversal mirror in inhomogeneous media,” in Proc. IEEE Ultrason. Symp. 1989, vol2, pp. 681 - 686.

[2] M. Fink, “Time reversal of ultrasonic fields – Part I: Basic principles,” IEEE Trans. Ultrason., Ferrorelec., Freq. Contr., vol 39, no. 5, Sept. 1992.

[3] N. Chakroun, M. Fink, and F. Wu, “Time reveral processing in ultrasonic nondestructive testing,” in IEEE Trans. Ultrason., Ferrorelec., Freq. Contr., vol 42, no. 9, Nov. 1995.

[5] C. Tsogka and G. C. Papanicolaou, “Time reversal through a solid-liquid interface and super-resolution,” Inverse Problems. Vol 18, pp. 1639-1657, Dec. 2002.

[5] C.T. Schröder,“On the interaction of elastic waves with buried land mines: an investigation using the finite-difference time domain method,” Ph.D. Thesis, Georgia Inst. of Technology, Atlanta, GA 2001.