

# Time-Reversal Focusing of Elastic Waves in Inhomogeneous Media: An Application to an Elastic-Wave Landmine Detection System

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## ABSTRACT

Time-reversal focusing is studied in the context of an elastic-wave landmine-detection system. Time-reversal focusing has been previously applied to the system and proven to be useful in focusing energy to targets in inhomogeneous media with discrete non-uniform scattering objects. In earlier experiments, these scattering objects took the form of multiple rocks buried throughout the region of interest. In this study, the performance of time-reversal focusing is evaluated for the case of uniform discrete scattering objects. Cylindrical and spherical scatterers are buried below the surface to provide uniform scattering. In other media, high concentrations of uniform scatterers have been observed to produce super-resolution of the time-reversal focus point. In this paper, the super-resolution phenomenon is examined in the context of the elastic wave landmine detection system operating in a soil medium.

**Keywords:** time-reversal, focusing, elastic waves, seismic

## 1. INTRODUCTION

A landmine detection system (Fig. 1) is currently being developed at the Georgia Institute of Technology that uses elastic surface waves to detect the presence of buried landmines. The system, which is well documented in the literature,<sup>1</sup> depends on the interaction of elastic surface waves with a landmine in order to excite a resonance in the layer of soil between the flexible top of the landmine and the surface of the ground. This resonance in turn produces characteristic surface displacements which are measured by the system in order to detect a landmine.

The uniform propagation of elastic surface waves may be significantly disturbed if scattering objects are present in the soil or propagation medium. Time-reversal focusing has been previously examined as a method to focus energy to a particular location in the presence of numerous scattering objects. Time-reversal focusing has demonstrated significant improvements in surface displacements at a focus point when compared to other excitation methods.<sup>2</sup> The previous study by Norville *et al.* observed no evidence of the super-resolution phenomenon which has been observed in other examinations of time-reversal focusing<sup>3</sup> and is predicted by theoretical examinations of super-resolution.<sup>4,5</sup>

Results are generated using a three dimensional finite-difference time-domain model of elastic wave propagation. An examination and comparison of the scattering and mode conversion from both cylindrical and spherical scattering objects is presented. Next, focusing results for different configurations of scattering objects are presented. Particular emphasis is placed on a comparison of the focusing spot size for each configuration of scattering objects. Conclusions will be drawn about the relationship between super-resolution and scattering levels, including the effects of mode conversion.

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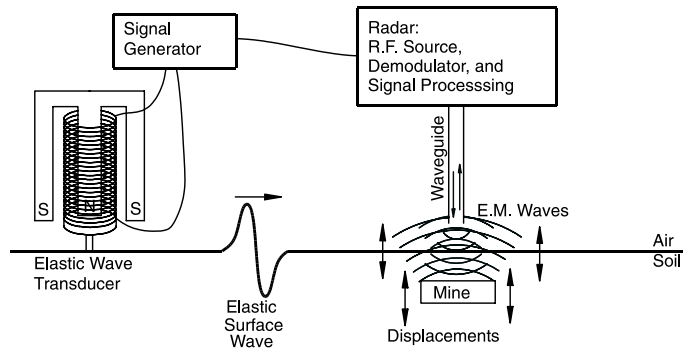


Figure 1. The elastic-wave landmine detection system.

## 2. AN INTRODUCTION TO TIME-REVERSAL FOCUSING

Time-reversal focusing is a technique to focus propagating waves to some desired location. The most significant advantage of time-reversal focusing over other focusing techniques is that it requires no knowledge of the properties of the propagation medium in order to successfully focus propagating waves to a desired location. This means that time-reversal focusing is insensitive to clutter and changes in wave propagation speed throughout the medium. The theory of time-reversal focusing and its implementation have been well documented in the literature.<sup>2, 6-8</sup> For this reason, only a brief introduction is offered here and the reader is encouraged to consult the references for further detail.

The basis for time-reversal focusing in solid media begins with the elastic wave equation of motion,

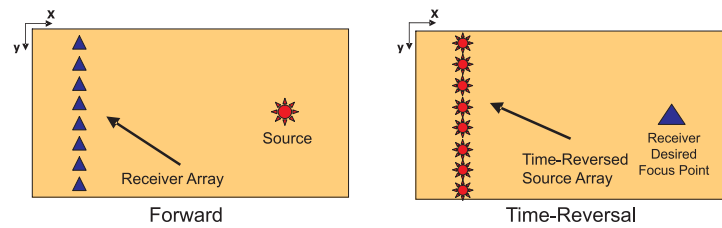
$$\rho_s \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu) (\nabla (\nabla \cdot \vec{u})) - \mu (\nabla \times (\nabla \times \vec{u})) \quad (1)$$

where  $\vec{u}$  is displacement,  $\lambda$  and  $\mu$  are the Lamé constants of the medium and  $\rho_s$  is the density.

This equation assumes that there are no external forces present on the medium and that the waves are propagating in a lossless environment. With these assumptions in place, the equation appears as in Eq. 1 and lacks any odd order time derivatives. The result of this is that if there is some solution to the equation,  $\vec{u}(\vec{r}, t)$ , then  $\vec{u}(\vec{r}, -t)$  must also be a solution to this equation.

A common implementation of this concept is using a time-reversal mirror (TRM).<sup>8</sup> Time-reversal focusing using a TRM is a two-step process. First, a source is located at a desired focus point, and an array of receivers is situated some distance away from the desired focus point (Fig. 2).<sup>2</sup> The source is excited and waves propagate from the focus point to the TRM receiver array. The individual signals,  $f_n(t)$ , for each of the  $n$  receivers are recorded and then time-reversed to yield  $f_n(-t)$ . In the second step of time-reversal focusing, the receiver array is replaced with a source array and the time-reversed signals are transmitted from their respective sources. The time-reversal excitation propagates through the medium and focuses both temporally and spatially at the location of the desired focus point.

While the TRM method of time-reversal focusing has been shown to be a robust form of time-reversal focusing, it does present some notable limitations. The focusing spot size of the TRM is limited by diffraction. This means the smallest possible spot size is limited by the smallest wavelength present in the excitation signal. The TRM also has a finite aperture, which limits focusing resolution.



**Figure 2.** Implementation of time-reversal focusing using a TRM.

### 3. APPLICATION OF TIME-REVERSAL FOCUSING TO LANDMINE DETECTION

The landmine detection system (Fig. 1) that is currently being developed at the Georgia Institute of Technology utilizes the interaction of elastic waves with objects in the soil to detect a target. The system excites elastic waves, which propagate through the soil and interact with anything that may be present in the medium including scattering objects such as rocks and debris, or changes in the material properties propagation medium, such as the presence of a road-bed.

The wave type of particular utility in the detection of buried landmines is the Rayleigh surface wave. The Rayleigh wave is a guided wave that propagates along the boundary between the surface of the soil and the air. In the case of interaction with a landmine, a strong Rayleigh wave will excite a resonance in the layer of soil between the flexible top of the landmine and the surface. This resonance serves as a strong indicator of the presence of a landmine.

The landmine detection system depends on the interaction of elastic surface waves with the buried landmine in order to generate the resonance used to detect the target. The surface wave characteristics may be altered by the presence of scattering objects or material property changes within the medium. The presence of such variations from the background material can cause scattering which may weaken the excitation pulse the arrives at the location of target.

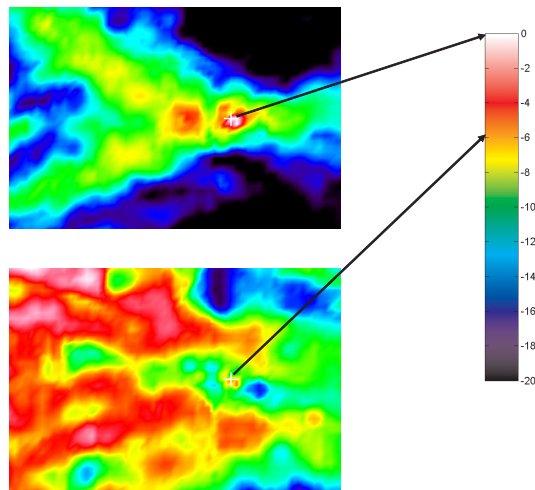
To overcome the effects of scattering objects and material property changes, a method must be devised for focusing energy to the location of a suspected target. If one is able to focus energy to a desired location, this aids in the excitation of a resonance, should a target be present at that spot. Time-reversal focusing has been previously investigated and shown to have great potential as a method for delivering an appropriate excitation signal to a focus location<sup>2</sup> (Fig. 3).

### 4. SUPER-RESOLUTION IN TIME-REVERSAL FOCUSING

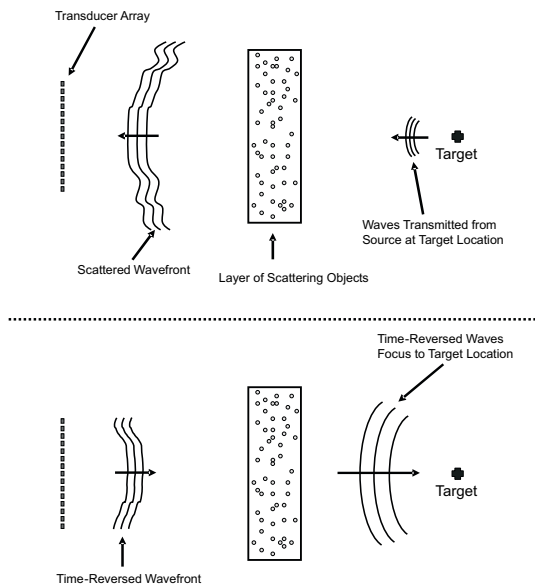
Applications of time-reversal have their origins in fluid media.<sup>6,7,9</sup> The result of time-reversal focusing behavior in one study is of particular interest.<sup>3</sup> In this study, Derode *et al.* examined the effects of time-reversal focusing in a fluid media with a layer of discrete scattering objects between the focus point and the TRM (Fig. 4). The authors observed not only that time-reversal focusing was effective in focusing to the desired focus point, but that the focusing resolution exceeded the focusing limits expected due to aperture size constraints.

Derode *et al.* reasoned that the observed super-resolution was due to the high-order scattering present in the time-reversal experiment. The multiple reflections created by the scattering objects induced a longer mean path for the arrival of the excitation signal at the focus point. These observations lead to the concept of an effective aperture. The effective aperture is determined by observing the focusing resolution, and determining what equivalent physical aperture would be required for focusing at this resolution if no scattering objects were present in the medium. The effective aperture is therefore one way of measuring the level of inhomogeneity in a medium.

A theoretical examination of super-resolution in elastic media<sup>4</sup> demonstrates mathematically that super-resolution should occur in the case of an elastic inhomogeneous medium. This examination of super-resolution differs from the experiments performed by Derode *et al.*<sup>3</sup> in that the inhomogeneity assumed by Blomgren *et al.*<sup>4</sup> is a statistical random distribution of inhomogeneity instead of a collection of discrete scattering objects in



**Figure 3.** A comparison of the maximum displacement at the location of a buried TS-50 for time-reversal (top) and uniform excitation (bottom) cases.



**Figure 4.** The experimental setup used by Derode *et al.* incorporating a layer of discrete scattering objects. In the first half of the experiment a source is placed at the target location. The received signals are time-reversed and re-transmitted by the TRM. After passing through the scattering layer, the waves focus to the target location.

the medium. The theoretical investigation requires some simplifying assumptions in order to generate a closed form solution. It also requires knowledge of a 3-D inhomogeneous Green's function for propagation within the medium.

Super-resolution is a phenomenon that if present in elastic inhomogeneous media, could be exploited to improve focusing resolution. Further, an examination of actual focusing resolution and a comparison to the expected resolution based on the physical aperture may lead to additional information about the properties and extent of inhomogeneity within a medium. Previous studies examining the application of time-reversal focusing to the landmine detection problem<sup>2</sup> noted no evidence of super resolution. In an effort to better characterize the behavior of time-reversal focusing and super-resolution, a study of the effects of different scattering objects

is presented here.

## 5. THE NUMERICAL MODEL

The results are generated using a three-dimensional finite-difference time-domain model of elastic wave propagation. The model is constructed by developing a first-order velocity-stress formulation of the elastic wave equation and constitutive equations. These equations are then discretized, replacing continuous derivatives with discrete spatial and time steps. The physical space to be simulated is divided into small cubes, called unit cells (Fig. 5a). The field quantities for the discretized elastic wave model are stored at offset locations in time and space. The entire solution space is surrounded by a perfectly matched layer (PML) which absorbs all waves that propagate past the edge of the solution space (Fig. 5b). In this way, the solution space appears to be the observed portion of an infinite half-space. This model has been used to generate previous time-reversal focusing results<sup>2</sup> and is well documented in the literature.<sup>10</sup>

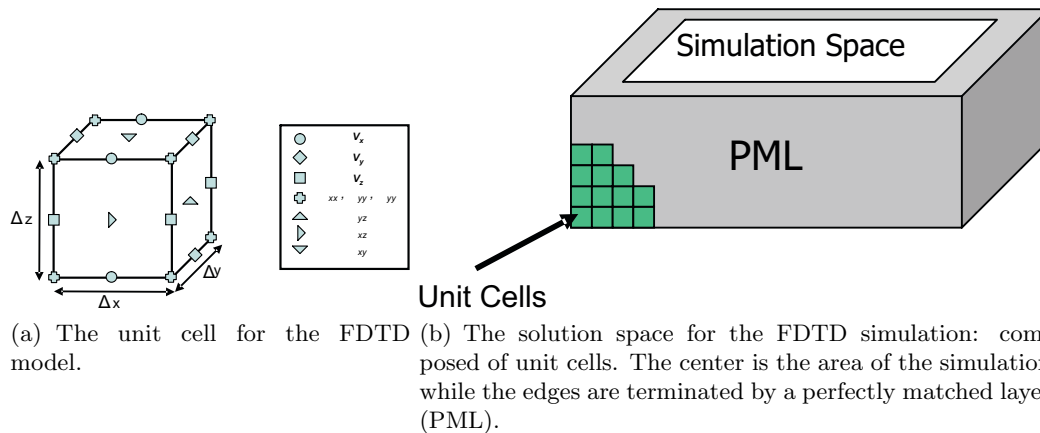


Figure 5. Finite-difference time-domain model construction

In order to generate data using the model which can be compared to experimental data, it is useful to simulate conditions similar to those that can be found in experimental data. The model of the soil used in the simulations is similar to the characteristics of damp compacted sand. The layered profile used in the numerical experiments is based off measurements taken in the experimental facility at the Georgia Institute of Technology.

## 6. SCATTERING OBJECTS: EXAMINATION OF MODE CONVERSION

When a propagating Rayleigh wave encounters a scattering object, some of the energy stored in the incident Rayleigh wave is converted into other wave types. These other wave types may not be bound to the surface, causing energy to propagate down into the soil. This is significant with respect to super-resolution phenomena because of the multiple scattering that occurs with large numbers of scattering objects.

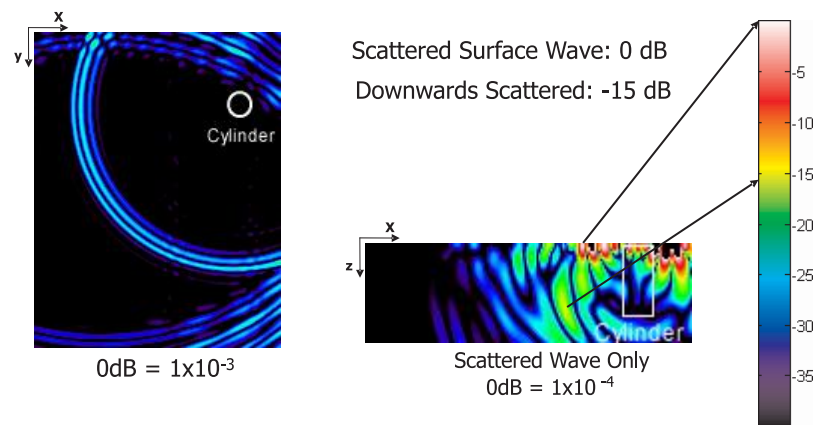
When large numbers of scatterers are present in the medium, the incident Rayleigh wave will be reflected multiple times. In a lossless environment with no mode conversion, the effect of this scattering would be longer mean path lengths for waves travelling from the source to the receiver, inducing super-resolution. In the case where mode conversion does occur, each subsequent interaction with a scattering object may cause more energy to be scattered downward. This results in less energy in the Rayleigh wave, and ultimately less energy available for time-reversal focusing.

Different types of objects scatter and mode-convert energy differently. A comparison of two types of scattering objects is shown in order to characterize and compare the scattering from a sphere and cylinder of the same radius. Both objects are made from concrete and are placed in layered soil. A uniformly excited array excites a differentiated Gaussian pulse which interacts with each of the objects.

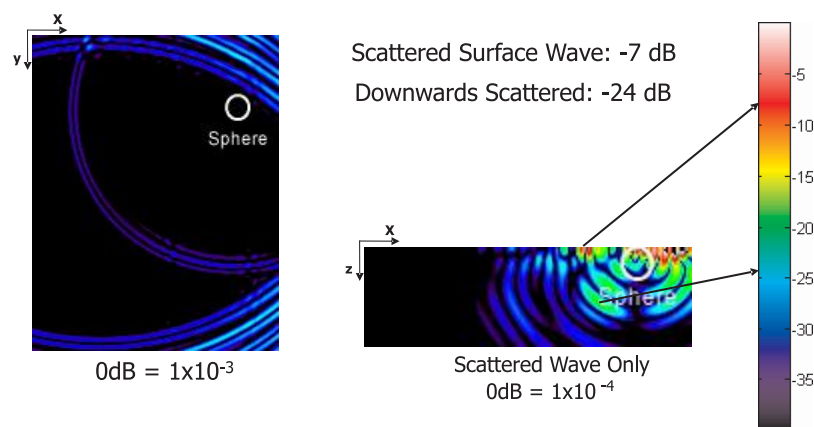
Figures 6 and 7 are time-snapshots of the propagation of the waves through the solution space. These images are presented on a 40 dB pseudo-color amplitude scale from white (0 dB) to black (-40 dB). A comparison of the intensity of the reflected waves on the surface (X-Y plane) for the cylinder and the sphere shows a significantly stronger surface reflection from the cylinder. This could indicate that the cylinder reflects energy more strongly, or it could indicate that for the sphere, more energy is being converted into waves no longer bound to the surface.

In order to more closely examine the scattered waves, it is necessary to observe the wave fields as a function of depth. Time-snapshots of the solution space as a function of depth are prepared (X-Z plane) to enable a more clear assessment of the behavior of the scattered wave field. In these images, the incident wave field is subtracted and only the scattered waves are presented. In the snapshots of the scattered-only wave in Figures 6 and 7, the 0 dB level is adjusted once the incident field is removed to give better resolution to the scattered waves.

A comparison of Figures 6 and 7 confirms that the cylinder scatters significantly more energy than does the sphere. This is apparent in the 7 dB drop in displacement amplitude between the reflected Rayleigh wave for the cylinder (Fig. 6) and the sphere (Fig. 7). A comparison of the level of mode conversion for the two objects reveals that the difference in the amplitude of the reflected Rayleigh wave and the downward travelling wave is similar between the two objects: 17 dB of contrast for the cylinder and 15 dB for the sphere.



**Figure 6.** Snapshots of scattering and mode conversion from a single cylinder.



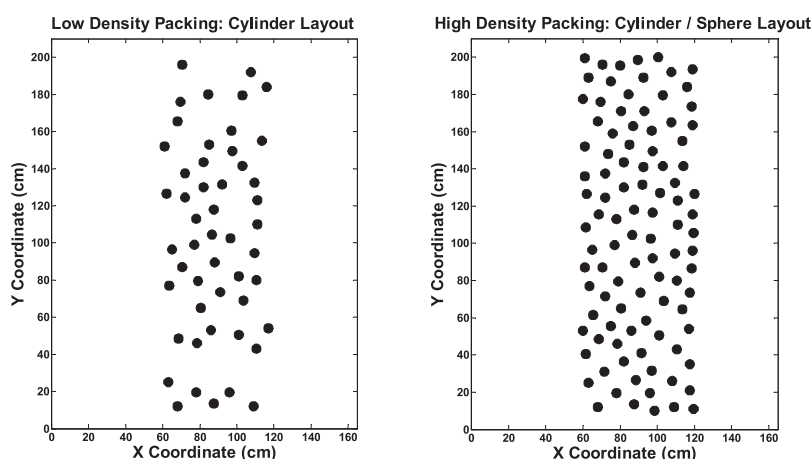
**Figure 7.** Snapshots of scattering and mode conversion from a single sphere.

The similar ratios of reflected Rayleigh to mode-converted waves indicate that for these two types of scatterers,

the relative levels of mode conversion are similar. This initial investigation of a single scattering object indicates that in a comparison of super-resolution for cylinders or spheres, super-resolution should be expected to be greater for cylinders since scattering is more effective, and with slightly less energy lost to mode conversion and downward scattering.

## 7. FOCUSING RESOLUTION ANALYSIS

In order to test the hypothesis that a set of scattering cylinders should produce greater super-resolution than a set of spheres, numerical simulations of time-reversal focusing are performed for different configurations of scattering objects. Four different time-reversal focusing simulations are performed: no scattering objects in the medium, a low density random distribution of 50 spheres, a low density random distribution of 50 cylinders, and a high density random distribution of 99 cylinders. The same low density distribution is used for the cylinders and the spheres. A bird's-eye view of the layout of the scattering objects for the high and low-density fields are presented in Figure 8. The spheres and the cylinders are made of concrete and have a radius of 2.5 cm. Both types of scatterers are buried at the surface of the sand. The cylinders are 30 cm long.



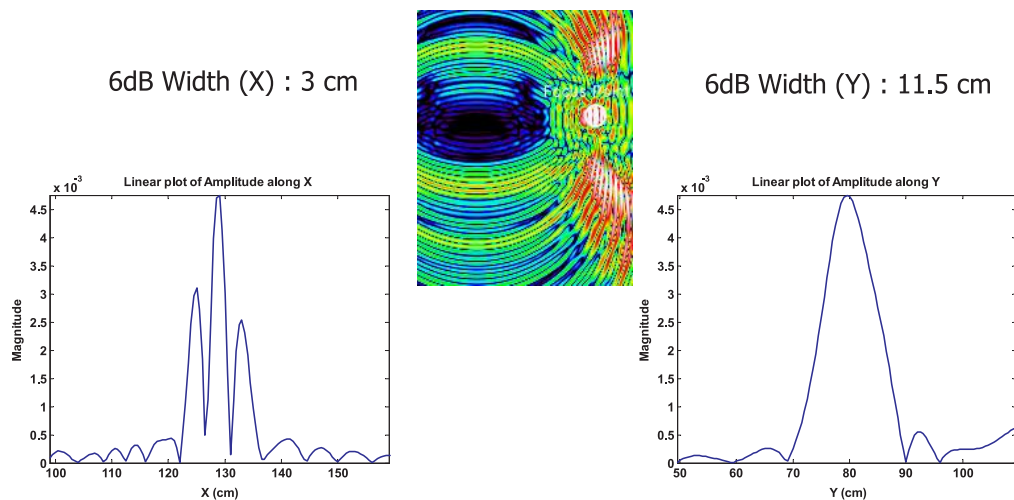
**Figure 8.** A bird's-eye view of the high and low density scattering object configurations.

The results for each focusing resolution analysis are presented in several forms. A pseudo-color image snapshot is presented, showing displacement on the surface at the time that focusing occurs. This image allows for a qualitative observation of the level of focusing, and an easy visual inspection of the focusing intensity and resolution. Two linear graphs of amplitude across the focus point are presented: one in the x-direction and one in the y-direction. The 6 dB width of the focusing spot is also measured in both the X and Y dimensions. All results are normalized such that equal energy is present in the time-reversed excitation signals for each different configuration of scattering objects.

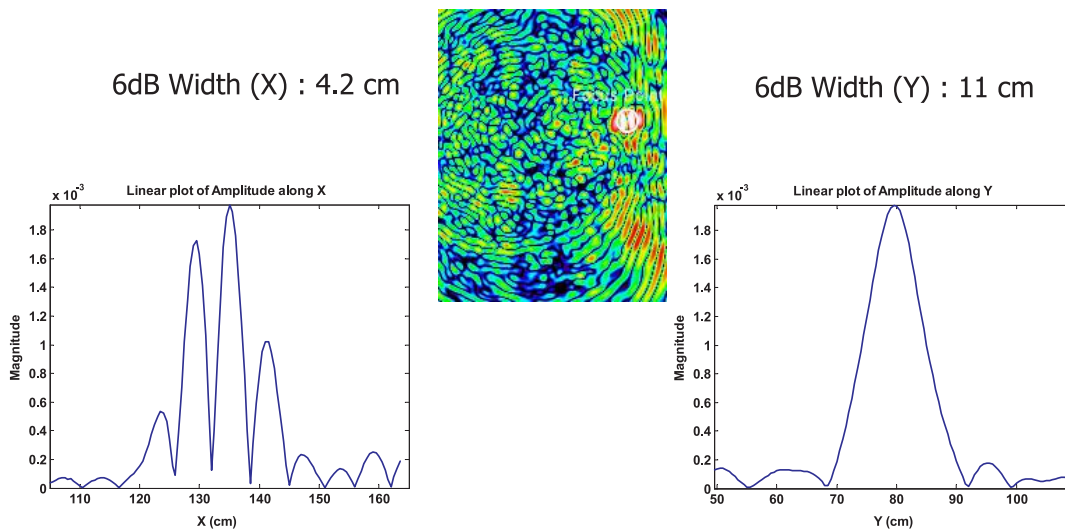
It was determined in Section 6 that for cylinders and spheres of the same radius and material composition, the cylinders scatter energy more effectively than the spheres. As the level of scattering increases (Figs. 9 - 11), the peak amplitude at the focus point clearly decreases since more energy is scattered and lost downward to mode conversion. The low-level scattering caused by the spheres (Fig. 10) is not significant enough to induce super-resolution, as can be noted by the actually worsened focusing resolution when compared to the case where no scattering objects are present (Fig. 9). The low-density configuration of cylinders (Fig. 11) scatters significantly more energy than the low-density configuration of spheres, and does induce super-resolution. The super-resolution of the focus point is only observed along the y-axis since this is the axis affected by aperture size. Focusing along the x-axis is limited by the diffraction effects described in Section 2.

If the results follow the trend demonstrated above, one would expect an increase in the number of cylinders to result in better focusing resolution due to increased higher order scattering. This is in fact, not what is observed.

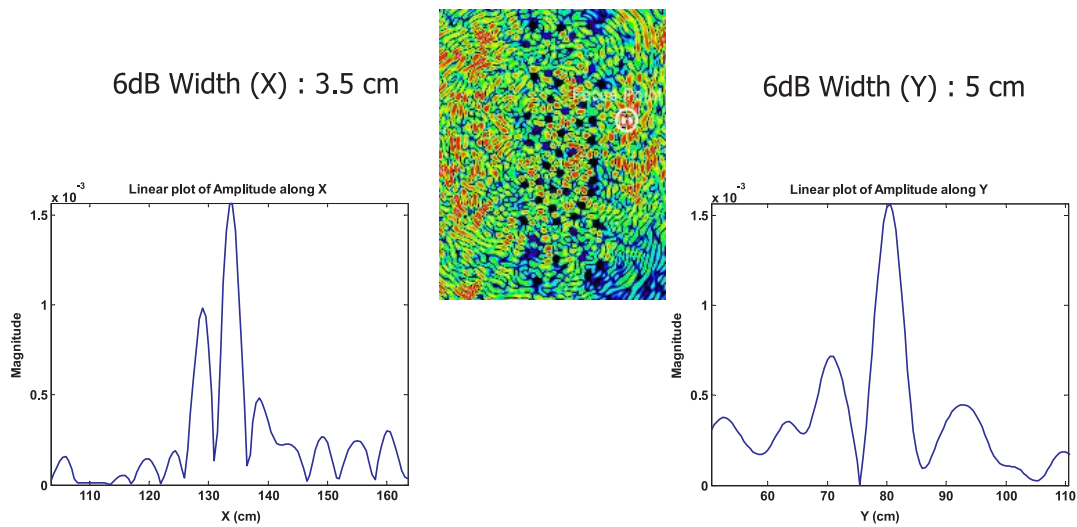
The peak amplitude at the focus point for the high density configuration of cylinders is more than an order of magnitude smaller than for the low density configuration of cylinders(Figs. 11, 12). Focusing resolution for the high density configuration of cylinders is slightly reduced from the low-density configuration of cylinders.



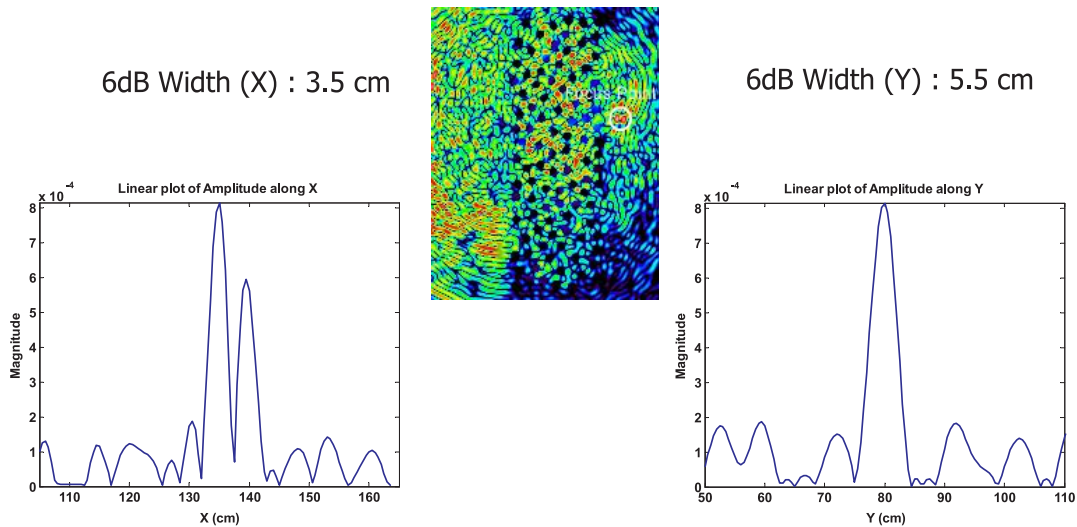
**Figure 9.** Focusing resolution analysis: no scattering objects.



**Figure 10.** Focusing resolution analysis: low density configuration of spheres.



**Figure 11.** Focusing resolution analysis: low density configuration of cylinders.



**Figure 12.** Focusing resolution analysis: high density configuration of cylinders

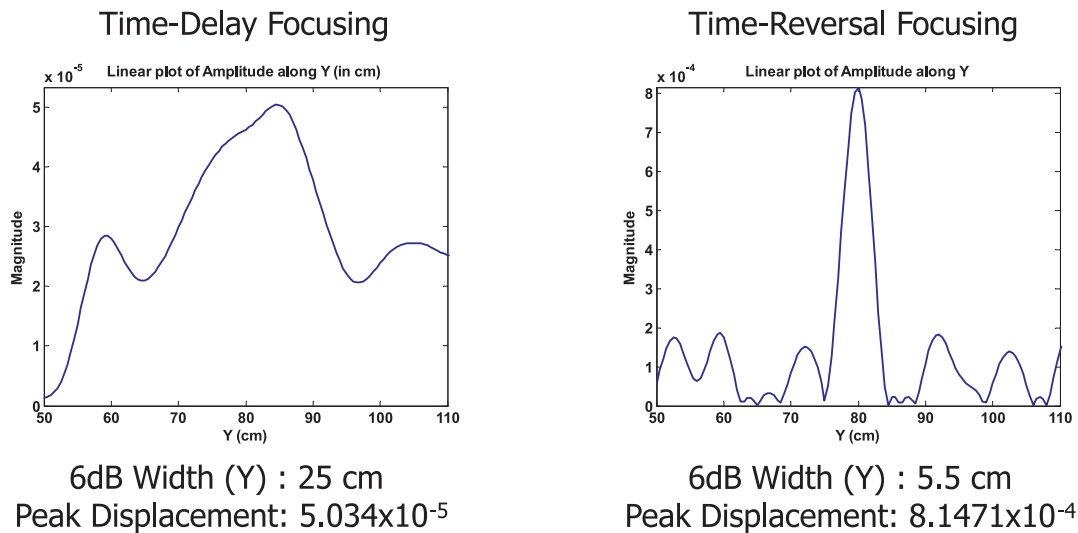
The observed lack of focusing improvement for greater scattering may seem to go against the previously demonstrated relationship between improved focusing resolution and the level of inhomogeneity. As the level of inhomogeneity increases, time-reversal focusing resolution improves until too much information is being lost to mode conversion. Because this study examines time-reversal focusing of the Rayleigh surface wave, any energy that is mode-converted and scattered downward appears as a loss of energy. Though total energy is physically conserved, energy not bound to the surface in the Rayleigh wave is not observed or available for time-reversal focusing. An understanding of when the relationship between focusing improvements due to high-order scattering and a focusing degeneration due to mode conversion is important for understanding the applicability of time-reversal focusing in the application of landmine detection. A summary of the results is presented in Figure 13.

The ultimate goal of time-reversal focusing is to achieve a greater level of excitation at some desired target location. Upon examination of the results presented in Figure 13, it may be tempting to dismiss the utility

## Focusing Resolution Analysis Summary

Type of Scattering Field	Coordinates of Focus (Xcm, Ycm)	6 dB Width (X)	6 dB Width (Y)	Peak Amplitude
No Scattering Objects	(129, 79.5)	3 cm	11.5 cm	$4.8 \times 10^{-3}$
Spheres	(135, 80)	4.2 cm	11 cm	$2 \times 10^{-3}$
Cylinders: Low Density Field	(135, 80)	3.5 cm	5 cm	$1.6 \times 10^{-3}$
Cylinders: High Density Field	(133.5, 80.5)	5.5 cm	5.5 cm	$8.14 \times 10^{-4}$

**Figure 13.** Focusing resolution analysis: summary of results



**Figure 14.** Focusing resolution analysis: a comparison of time-delay focusing and time-reversal focusing for the high density configuration of cylinders.

of super-resolution in time-reversal focusing since it appears that the peak displacement would be much higher when scattering objects are not present. In a realistic scenario, scattering objects will always be present in the landmine detection problem and cannot be removed as they can be in the numerical case. The challenge is to learn to use these scatterers as an advantage. To demonstrate the increase in effectiveness of time-reversal focusing when such a dense scattering field is present, a comparison between time-reversal focusing and time-delay focusing is presented in Figure 14. Time-reversal focusing offers a peak displacement greater than 1 order of magnitude larger and a 6 dB spot size almost 5 times narrower than time-delay focusing.

## 8. CONCLUSIONS

The effects of different types of discrete scattering objects on super-resolution in time-reversal focusing of elastic waves has been examined. The effects of the density of the field of scattering objects and the amount of energy lost to mode conversion were also studied. The results were generated using a three dimensional finite-difference time-domain numerical model of elastic wave propagation.

Focusing resolutions improvements were observed for cases where scattering was significant. The super-resolution effects were prominent only in the direction perpendicular to the aperture. Further, it was observed that super-resolution does begin to weaken as the level of inhomogeneity increases past a certain point. Beyond this level of inhomogeneity, large amounts of energy are lost to mode conversion such that a significant amount of energy in the excitation signal does not reach the receivers.

Future work should focus on quantifying a correlation between the energy lost to mode conversion and the level of super-resolution observed. An experimental verification of these results would also be prudent. Lastly, super-resolution effects in various scattering scenarios should be studied in the presence of a target, such as a landmine. These results will lead to a better understanding of the specific application of super-resolution phenomena in elastic-wave landmine detection.

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