Georgia Institute of Technology
School of Electrical and Computer Engineering
ECE 4435 Op Amp Design Laboratory Fall 2005

Design Project 1, Part 2
A White Noise and Pink Noise Generator

The following explains how to design a circuit that has an impedance magnitude that exhibits an approximate slope versus frequency of $-n$ decades/decade. For this design project, you are to use the circuit for Case B for which $N = 6$ with $n = 0.5$, $f_1 = 20\,\mathrm{Hz}$, and $f_6 = 20\,\mathrm{kHz}$. To design the circuit, you must specify the desired impedance at f_1. For $f > f_1$, the impedance varies approximately as $1/\sqrt{f}$. Thus if you specify the impedance at 20 Hz to be $10\,\mathrm{k}\Omega$, the impedance at 20 kHz will be approximately $316\,\Omega$. You may find that the impedance approximation is better if f_1 is slightly larger than 20 Hz and f_2 is slightly smaller than 20 kHz.

For the second part of the design lab, you are to design the network for the desired impedance. Use SPICE, Mathcad, or any other program to plot its impedance as shown in the example in Fig. 2 below along with an impedance that varies as the desired impedance at 20 Hz divided by $\sqrt{f/20}$. Select a resistor to put in parallel with the network that will cause the impedance to shelve at a constant value below 20 Hz. If you look at Fig. 2, you will see that the approximating impedance at f_1 is larger than the desired impedance. Therefore, the addition of the parallel resistor can be used to adjust the approximating impedance at 20 Hz so that it better approximates the desired impedance.

One way to convert the white noise into pink noise would to be to drive the impedance with a current source. The Howland current pump is a possible circuit. However, the the input impedance of this circuit varies with the load impedance. If the load impedance is large at dc, the Howland current pump has an input impedance approaches zero. The textbook never mentions this very undesirable feature. The only current source circuit in the book which will drive a grounded load that doesn't have the undesirable property of the Howland current pump is the circuit in Problem 2.16. This circuit acts as a current source with the value $i_L = v_I/R_3$ provided the condition $R_1 = R_2 + R_3$ is satisfied. For example, if $R_3 = 1\,\mathrm{k}\Omega$, the transconductance gain is $1\,\mathrm{mA/V}$. In this case $R_1 = 11\,\mathrm{k}\Omega$ and $R_2 = 10\,\mathrm{k}\Omega$ could be used. Decreasing the resistors by a factor of 10 would multiply the transconductance gain by 10. The input resistance of the circuit is R_1.

Another possible way to convert the the white noise into pink noise is to use the impedance as the feedback element in an inverting op-amp amplifier. This is the simplest solution, but it would be interesting to try the current source.

You should design an output stage with a potentiometer so that the amplitude of the output pink noise can be varied from to 0 to 5 V rms over the band from 20 Hz to 20 kHz. You should document the performance of the circuit by using the oscilloscope to measure its waveforms and a spectrum analyzer to measure its spectrum.

The Approximating Impedance

Figure 1 shows the Bode magnitude plot of an impedance which exhibits a slope of $-n$ dec/dec between the frequencies f_1 and f_6. Also shown are the asymptotes of an approximating impedance which exhibit alternating slopes of -1 and 0. Four frequencies are labeled between f_1 and f_6 at which the slopes of the asymptotes change. In the general case, let there be N frequencies, where N is even and $N \geq 4$. In this case, the number of asymptotes having a slope of 0 is $(N-2)/2$. Let k be the ratio of the asymptotic approximating impedance to the desired impedance at $f = f_1$.

The desired impedance at f_1 is labeled $|Z_1(f_1)|$.The approximating impedance at f_1 is labeled $k|Z_1(f_1)|$ where the value of k can be calculated from the equations given.

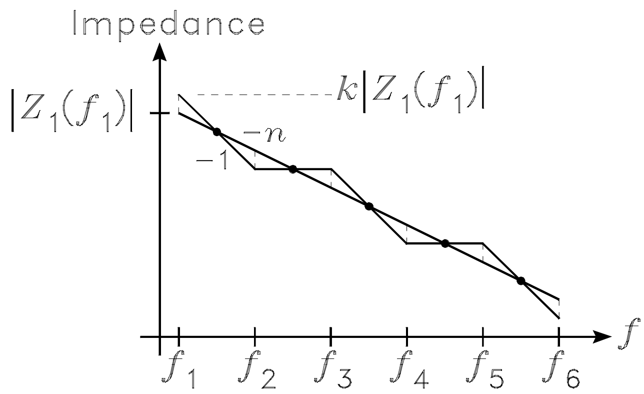

Figure 1: Magnitude plots of the desired impedance and the asymptotes of the approximating impedance versus frequency.

With n, f_1, and f_N specified, the object is to specify k and f_2 through f_{N-1} such that the ratios between each even subscripted frequency to the odd subscripted frequency to its left are equal and the intersection points (indicated by dots on the plot) occur at the geometric mean of the adjacent frequencies. In this case, the length of the six dashed vertical lines in Fig. 1 are equal and the asymptotes of the approximating impedance approximate the desired impedance in an equal ripple sense between f_1 and f_N. It is straightforward to show that the following conditions must hold:

$$k=\left(\frac{f_2}{f_1}\right)^{\frac{1-n}{2}}=\left(\frac{f_4}{f_3}\right)^{\frac{1-n}{2}}=\cdots=\left(\frac{f_N}{f_{N-1}}\right)^{\frac{1-n}{2}} \tag{1}$$

$$\begin{aligned} f_2 &= f_1^{1-n}f_3^n \\ f_4 &= f_3^{1-n}f_5^n \\ &\vdots \\ f_{N-2} &= f_{N-3}^{1-n}f_{N-1}^n \end{aligned} \tag{2}$$

$$\begin{aligned} f_3 &= f_2^n f_4^{1-n} \\ f_5 &= f_4^n f_6^{1-n} \\ &\vdots \\ f_{N-1} &= f_{N-2}^n f_N^{1-n} \end{aligned} \tag{3}$$

Solutions to these equations are given below for the cases $N=4$ and $N=6$.

Case A: $N=4$

Let f_1 and f_4 be specified. For $N=4$, Eqs. (1) – (3) can be solved to obtain

$$k=\left(\frac{f_4}{f_1}\right)^{\frac{n(1-n)}{2(1+n)}} \tag{4}$$

$$f_2 = f_1^{\frac{1}{1+n}} f_4^{\frac{n}{1+n}} \tag{5}$$

$$f_3 = f_1^{\frac{n}{1+n}} f_4^{\frac{1}{1+n}} \tag{6}$$

Let $Z_1(f)$ be the approximating impedance function. It is given by

$$Z_1(f) = \frac{k\,|Z_1(f_1)|}{j(f/f_1)} \times \frac{1 + j(f/f_2)}{1 + j(f/f_3)} \tag{7}$$

Case B: $N = 6$

Let f_1 and f_6 be specified. For $N = 6$, Eqs. (1) - (3) can be solved to obtain

$$k = \left(\frac{f_6}{f_1}\right)^{\frac{n(1-n)}{2(2+n)}} \tag{8}$$

$$f_2 = f_1^{\frac{2}{2+n}} f_6^{\frac{n}{2+n}} \tag{9}$$

$$f_3 = f_1^{\frac{1+n}{2+n}} f_6^{\frac{1}{2+n}} \tag{10}$$

$$f_4 = f_1^{\frac{1}{2+n}} f_6^{\frac{1+n}{2+n}} \tag{11}$$

$$f_5 = f_1^{\frac{n}{2+n}} f_6^{\frac{2}{2+n}} \tag{12}$$

The approximating impedance as a function of frequency for this case is given by

$$Z_1(f) = \frac{k\,|Z_1(f_1)|}{j(f/f_1)} \times \frac{1 + j(f/f_2)}{1 + j(f/f_3)} \times \frac{1 + j(f/f_4)}{1 + j(f/f_5)} \tag{13}$$

Example Plots

To illustrate the accuracy of the approximating functions, let the impedance given by Eq. (**??**) be approximated over a three-decade band for the case $n = 0.5$. The smaller the value of n, the poorer the approximation. In the author's experience, the value of n for most loudspeaker drivers is in the range from 0.6 to 0.7. Thus the value $n = 0.5$ results in an approximation that is worse than what can be expected with the typical driver.

Figure. 2 shows the calculated Bode magnitude plots. Curve a is the desired impedance. Curve b is the approximating impedance for $N = 4$. Curve c is the approximating impedance for $N = 6$. It can be seen that the approximating impedance functions ripple about the desired function over the band of interest with a maximum deviation occurring at the two frequency extremes. Between the two extremes, the maximum deviation is less than it is at the extremes because the design equations are derived from the asymptotes of the approximating function.

The Compensating Circuits

Network A

Figure 3(a) shows a circuit consisting of two capacitors and one resistor which can be used to realize the impedance of Eq. (7). The impedance is given by

$$Z_1(s) = \frac{1}{s(C_1 + C_2)} \times \frac{1 + s/\omega_2}{1 + s/\omega_3} \tag{14}$$

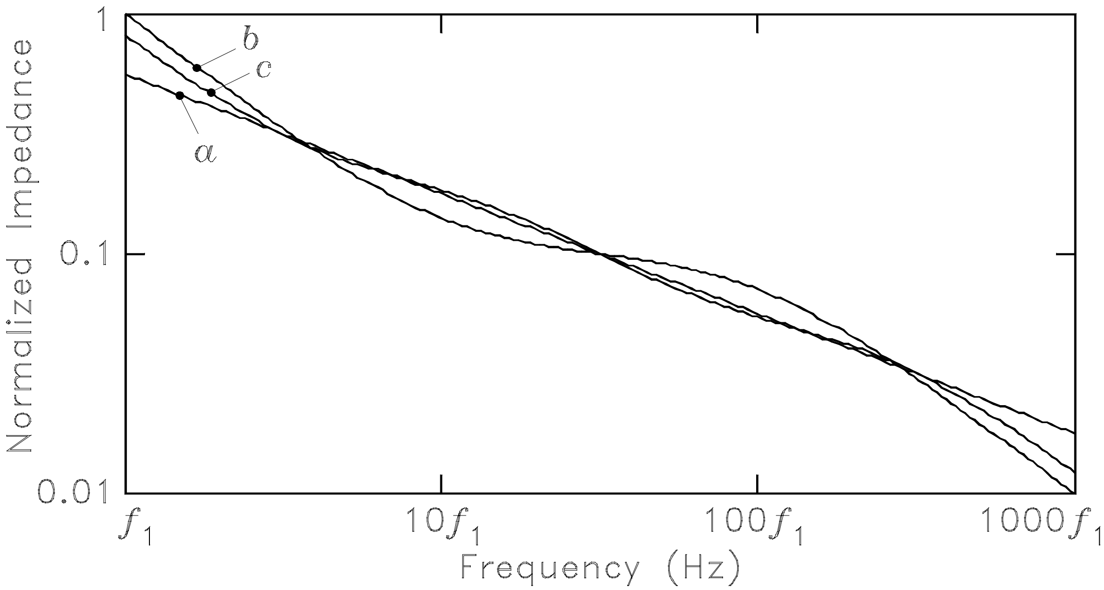

Figure 2: Example plots of the desired impedance (curve a) and the approximating impedances (curves b and c) versus frequency for the case $n = 0.5$.

where $s = j\omega = j2\pi f$ and

$$\omega_2 = 2\pi f_2 = \frac{1}{R_2 C_2} \tag{15}$$

$$\omega_3 = 2\pi f_3 = \frac{C_1 + C_2}{R_2 C_1 C_2} \tag{16}$$

The impedance of the circuit is equal to that of Eq. (7) if

$$C_1 = \frac{f_2}{2\pi f_1 f_3 k \left|Z_1(f_1)\right|} \tag{17}$$

$$C_2 = \frac{f_3 - f_2}{f_2} C_1 \tag{18}$$

$$R_2 = \frac{1}{2\pi f_2 C_2} \tag{19}$$

The circuit of Fig. 3(a) corresponds to Fincham's more general compensating network in [**?**].

Network B

Figure 3(b) shows a circuit consisting of three capacitors and two resistors which can be used to realize the impedance of Eq. (13). The impedance is given by

$$Z_1(s) = \frac{1}{s(C_1 + C_2 + C_3)} \times \frac{(1 + s/\omega_2)(1 + s/\omega_4)}{s^2/(\omega_3\omega_5) + s(1/\omega_3 + 1/\omega_5) + 1} \tag{20}$$

where

$$\omega_2 = 2\pi f_2 = \frac{1}{R_2 C_2} \tag{21}$$

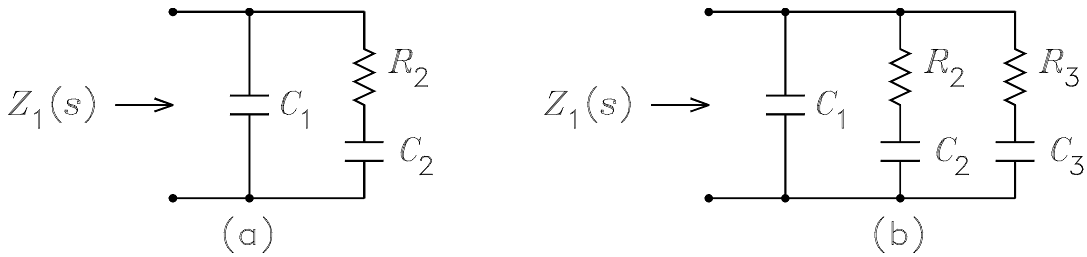

Figure 3: Circuits for approximating the impedance $Z_1(j\omega)$.

$$\omega_4 = 2\pi f_4 = \frac{1}{R_3 C_3} \tag{22}$$

$$\omega_3 \omega_5 = 2\pi f_3 \times 2\pi f_5 = \frac{C_1 + C_2 + C_3}{R_2 R_3 C_1 C_2 C_3} \tag{23}$$

$$\frac{1}{\omega_3} + \frac{1}{\omega_5} = \frac{1}{2\pi f_3} + \frac{1}{2\pi f_5} = \frac{R_2 C_2 (C_1 + C_3) + R_3 C_3 (C_1 + C_2)}{C_1 + C_2 + C_3} \tag{24}$$

The impedance of the circuit is equal to that of Eq. (13) if

$$C_1 = \frac{f_2 f_4}{2\pi f_1 f_3 f_5 k \left|Z_1(f_1)\right|} \tag{25}$$

$$C_2 = \frac{f_2 - f_3 - f_5 + f_3 f_5 / f_2}{f_4 - f_2} C_1 \tag{26}$$

$$R_2 = \frac{1}{2\pi f_2 C_2} \tag{27}$$

$$C_3 = \frac{f_3 - f_4 + f_5 - f_3 f_5 / f_4}{f_4 - f_2} C_1 \tag{28}$$

$$R_3 = \frac{1}{2\pi f_4 C_3} \tag{29}$$