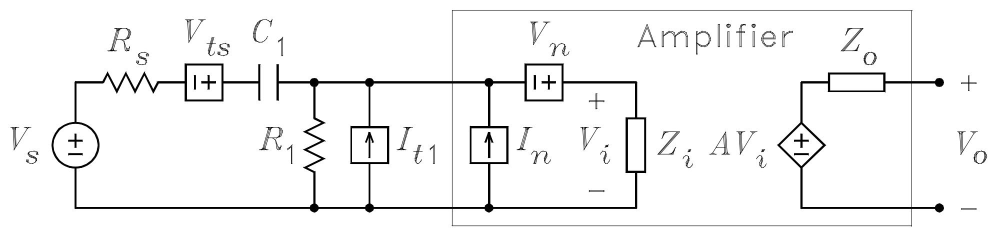
ECE 6416 Assignment 2

1. If V_n and I_n in the noise model of an amplifier are not correlated, show that the addition of a series resistor R_1 at the amplifier input causes the correlation coefficient on the source side of R_1 to be
$$\rho = \frac{i_n R_1}{\sqrt{4kTR_1\Delta f + v_n^2 + i_n^2 R_1^2}}$$

2. If V_n and I_n in the noise model of an amplifier are not correlated, show that the addition of a shunt conductance G_2 at the amplifier input causes the correlation coefficient on the source side of G_2 to be
$$\rho = \frac{v_n G_2}{\sqrt{4kTG_2\Delta f + v_n^2 G_2^2 + i_n^2}}$$

3. The figure shows the noise model of an amplifier. The frequency is $f = 1\,\text{kHz}$. It is given that $R_s = 1\,\text{k}\Omega$, $R_1 = 2\,\text{k}\Omega$, $C_1 = 0.1\,\mu\text{F}$, $v_n/\sqrt{\Delta f} = 4\,\text{nV}/\sqrt{\text{Hz}}$, $i_n/\sqrt{\Delta f} = 8\,\text{pA}/\sqrt{\text{Hz}}$, $\gamma = 0$, $Z_i = 2800\angle -45\,°$, $A = 20\angle 45\,°$, $Z_o = 1200\angle 30\,°$. The open-circuit input voltage with $Z_i = \infty$ can be written $V_{i(oc)} = KV_s + V'_{ni} = K\left(V_s + V'_{ni}/K\right)$, where $K = V_{i(oc)}/V_s$ when all noise sources are neglected and V'_{ni} is the open-circuit input voltage due to all noise sources. The latter is given by $V'_{ni} = V_{teq} + V_n + I_n Z_{eq}$, where V_{teq} is the thermal noise generated by the real part of the impedance $Z_{eq} = R_1 \| \left(R_s + 1/j\omega C_1\right)$. It follows that the equivalent noise voltage in series with V_s is $V_{ni} = V'_{ni}/K$.



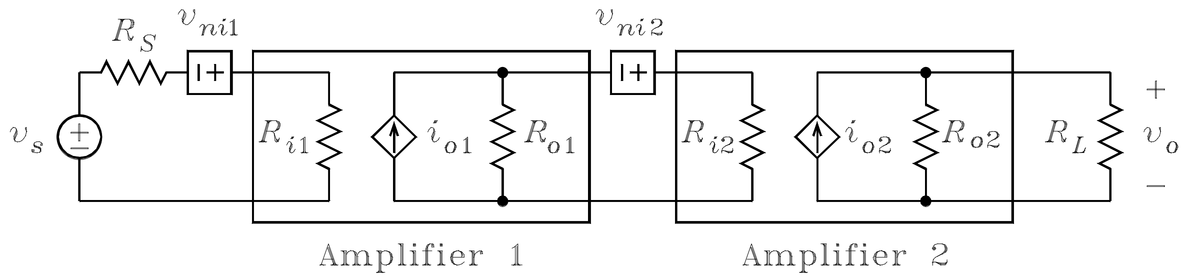
(a) Show that $|K| = 0.589$ and $v'_{ni}/\sqrt{\Delta f} = 10.5\,\text{nV}$.

(b) Show that $v_{ni}/\sqrt{\Delta f} = 17.8\,\text{nV}$.

(c) If R_1 is replaced by an open circuit and C_1 is replaced by a short circuit, show that $v_{ni}/\sqrt{\Delta f} = 9.80\,\text{nV}$. This calculation illustrates how the addition of series and parallel elements increases the noise.

(d) Show that the addition of C_1 and R_1 increases the noise by 5.18 dB.

4. The figure shows the model of a two-stage amplifier. It is given that $R_S = 100\,\Omega$, $R_{i1} = R_{i2} = 1.2\,\text{k}\Omega$, $R_{o1} = R_{o2} = 5\,\text{k}\Omega$, $R_L = 2\,\text{k}\Omega$, $I_{o1} = g_{m1}V_{i1}$, $I_{o2} = g_{m2}V_{i2}$, $g_{m1} = g_{m2} = 13^{-1}\,\text{S}$, $v_{n1}/\sqrt{\Delta f} = v_{n2}/\sqrt{\Delta f} = 2\,\text{nV}/\sqrt{\text{Hz}}$, and $i_{n1}/\sqrt{\Delta f} = i_{n2}/\sqrt{\Delta f} = 100\,\text{pA}/\sqrt{\text{Hz}}$. The noise sources can be assumed to be uncorrelated. V_{ni1} models the noise generated both by R_S and the first amplifier and can be written $V_{ni1} = V_{ts} + V_{n1} + I_{n1}R_S$. V_{ni2} models the noise generated by the second stage and can be written $V_{ni2} = V_{n2} + I_{n2}R_{o1}$. The noise generated by R_L can be neglected. All other resistors are noiseless because their noise is contained in V_{ni1} and V_{ni2}.



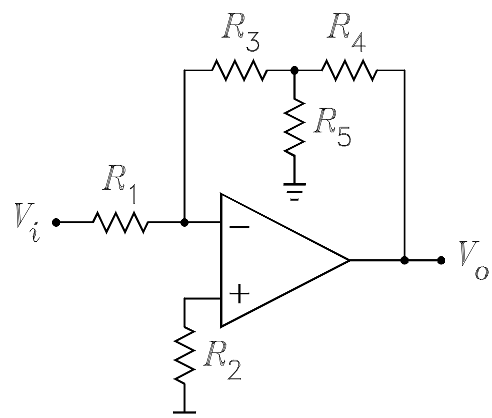
(a) Show that the voltage gain can be written as a product of terms as follows:

$$A = \frac{g_{m1}R_{i1}}{R_S + R_{i1}} \times R_{o1} \times \frac{g_{m2}R_{i2}}{R_{o1} + R_{i2}} \times (R_{o2}\|R_L) = 7551 \;\; (77.6\,\text{dB})$$

(b) Show that components of $v_{ni}/\sqrt{\Delta f}$ due to v_{ts}, v_{n1}, i_{n1}, v_{n2}, and i_{n2}, respectively, are $1.265\,\text{nV}/\sqrt{\Delta f}$, $2\,\text{nV}/\sqrt{\Delta f}$, $10\,\text{nV}/\sqrt{\Delta f}$, $5.633\,\text{pV}/\sqrt{\Delta f}$, and $1.408\,\text{nV}/\sqrt{\Delta f}$.

(c) Show that $v_{ni}/\sqrt{\Delta f} = 10.37\,\text{nV}$.

5. The figure shows an op amp circuit with a T feedback network. Except for noise, the op amp is ideal. It is given that $R_1 = 1\,\text{k}\Omega$, $R_2 = 500\,\Omega$, $R_3 = 901\,\text{k}\Omega$, $R_4 = 9.9\,\text{k}\Omega$, and $R_5 = 100\,\Omega$. The op amp noise is modeled by a noise voltage source V_n in series with the non-inverting input and two noise current sources I_{n1} and I_{n2} from each input to ground. It is given that $v_n/\sqrt{\Delta f} = 10\,\text{nV}/\sqrt{\text{Hz}}$ and $i_{n1}/\sqrt{\Delta f} = i_{n2}/\sqrt{\Delta f} = 1\,\text{pA}/\sqrt{\text{Hz}}$. All correlation effects can be neglected.



(a) Show that the voltage at the non-inverting input is given by

$$V_+ = V_n + (I_{n1} + I_{t2})\,R_2$$

(b) Use superposition to show that the voltage at the inverting input is given by

$$V_- = \left(\frac{V_i}{R_1} + \frac{V_o}{R_4 + R_3\|R_5}\frac{R_5}{R_3 + R_5} + I_{n2} + I_{teq}\right)[R_1\|\,(R_3 + R_4\|R_5)]$$

where I_{teq} is the noise current generated by the resistance $R_{eq} = R_1\|\,(R_3 + R_4\|R_5)$. Hint: Form Norton equivalent circuits looking from the V_- node into R_1 and into R_3.

The Norton current through R_3 can be solved for by applying current division to the current through R_4. The current through R_4 can be obtained by using Ohm's law. Use the Norton equivalents to solve for V_-.

(c) Set $V_+ = V_-$. With all noise terms zeroed, show that the voltage gain has the value $A = V_o/V_i = -90.1 \times 10^3$.

(d) With $V_i = 0$ and the thermal noise sources for the resistors activated, use the equation from parts 5a and 5b to show that the total spot noise output voltage due to the thermal noise of all resistors has the value $v_{no}/\sqrt{\Delta f} = 442\,\mu\text{V}/\sqrt{\text{Hz}}$.

(e) Use the equation from parts 5a and 5b to show that the spot noise output voltage due to the op-amp noise sources alone has the value $v_{no}/\sqrt{\Delta f} = 908\,\mu\text{V}/\sqrt{\text{Hz}}$.

(f) Use rms addition of the thermal noise and op-amp noise to show that the total spot noise output voltage is $v_{no}/\sqrt{\Delta f} = 1.01\,\text{mV}/\sqrt{\text{Hz}}$.

(g) Divide $v_{no}/\sqrt{\Delta f}$ by A to show that the equivalent spot noise voltage in series with v_i is $v_{ni}/\sqrt{\Delta f} = 11.2\,\text{nV}/\sqrt{\text{Hz}}$.

6. An amplifier has a voltage gain $V_o/V_i = 10$ (20 dB) and an input resistance $R_i = 10\,\text{k}\Omega$. It is driven from a voltage source which has an open-circuit rms output voltage $V_s = 1\,\text{V}$ and an output resistance $R_S = 1\,\text{k}\Omega$. The rms noise at the amplifier output in the band from 100 Hz to 100 kHz is found to be $v_{no} = 28.4\,\mu\text{V}$. When a $9\,\text{k}\Omega$ resistor is added in series with R_S, the rms output noise increases to $v'_{no} = 67.5\,\mu\text{V}$. If the V_n and I_n of the amplifier are not correlated, show that $v_n = 2.56\,\mu\text{V}$ and $i_n = 1.26\,\text{nA}$, where each are calculated over the band from 100 Hz to 100 kHz.

7. N identical amplifiers having $v_n/\sqrt{\Delta f} = 1.09\,\text{nV}/\sqrt{\text{Hz}}$ and $i_n/\sqrt{\Delta f} = 11.4\,\text{pA}/\sqrt{\text{Hz}}$ are driven from a source having the output resistance $R_s = 10\,\Omega$. Show that it would take 10 amplifiers in parallel to minimize the noise. For $N = 10$, show that $v_{ni}/\sqrt{\Delta f} = 639\,\text{pV}/\sqrt{\text{Hz}}$. Compared to a single stage, show that the dB improvement in the noise is 5.23 dB.

8. An input transformer is to be used to connect the source of problem 7 to the amplifier. If the winding resistances of the transformer can be neglected, show that the noise is a minimum if the transformer has the turns ratio $n = 3.1$. Show that $v_{ni}/\sqrt{\Delta f} = 639\,\text{pV}/\sqrt{\text{Hz}}$. Compared to the amplifier without the transformer, show that the dB improvement in the noise is 5.23 dB.