Electroacoustic-Analogous Circuit Models for Filled Enclosures*

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The thermodynamic and mechanical effects of filling in a closed-box loudspeaker enclosure are studied. It is shown that the acoustical-analogous circuit that models the increase in compliance due to the thermodynamic effects is a capacitor in parallel with a series resistor and capacitor, while the acoustical-analogous circuit that models the mechanical parameters of the filling material consists of an inductor in series with the parallel combination of a resistor and a series inductor, resistor, and capacitor.

0 INTRODUCTION

The effects of a filling material such as fiberglass in a closed-box loudspeaker baffle are well known [1]–[3]. The major ones are to increase the compliance of the enclosed air as well as the losses and the acoustic mass loading of the air. In the analogous circuits for closed-box systems, these effects are normally modeled by adjusting the elements in the circuits for the unfilled box by "rules of thumb" to account for the effects of the filling [3]. In this way, the need to model the effects of the filling analytically is avoided. Although this approach normally yields acceptable design results, it makes it difficult to make theoretical comparisons.

This paper presents an analytical study of the low-frequency effects of filling materials in an enclosure. The approach is to review first the development of the electroacoustic-analogous circuit model for an unfilled enclosure. The circuit is then modified to account for the effects of a filling. The increase in compliance is modeled by investigating the thermodynamic effects of the filling on the air compression and rarefaction in the enclosure. The increases in mass loading and enclosure losses are modeled by investigating the mechanical properties of the filling fibers.

1 THE ANALOGOUS CIRCUIT MODEL OF AN UNFILLED BOX

Let Z_{AB} denote the acoustic impedance of the air load on the rear of a loudspeaker diaphragm mounted in one wall of a sealed box. The diaphragm is modeled as a flat piston of area S_D. The box wall area is denoted by S_B. Fig. 1(a) illustrates the case where the piston occupies the entire area of the box wall, that is, $S_D = S_B$. It is assumed that a plane wave is radiated into the box when the piston vibrates. The acoustic impedance of the air in the box is that of a piston in a closed tube. It is given by [1]

$$Z_{AB} = -j\frac{\rho_0 c}{S_B}\cot(kd) \tag{1}$$

where $k = \omega/c$ and d is the depth of the box. At low frequencies, such that $d < \lambda/7$, a two-term approximation to the cotangent function can be made to obtain the approximate relation

$$Z_{AB} \simeq \frac{1}{j\omega C_{AB}} + j\omega M_{AB} \tag{2}$$

where C_{AB} and M_{AB} are given by

$$C_{AB} = \frac{V_B}{\rho_0 c^2} \tag{3}$$

$$M_{AB} = \frac{\rho_0 d}{3S_B} \tag{4}$$

V_B being the internal box volume, $V_B = S_B d$.

The analogous circuit for Z_{AB} consists of an acoustic compliance (capacitor) C_{AB} in series with an acoustic mass (inductor) M_{AB}. The compliance C_{AB} is a function of the volume of the box and is independent of the box geometry. It follows, therefore, that if the piston area

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of the driver is changed so that it is less than the area of the box wall in which it is installed, the equation for the acoustic compliance C_{AB} will remain unchanged [1].

The acoustic mass M_{AB} is a function of the ratio of the box depth d to the box wall area S_B. This is not independent of box geometry. In addition, the expression for M_{AB} is correct only when the diaphragm occupies the full area of the box wall. A modification of the expression is sought so that it will be valid when $S_D < S_B$. Consider Fig. 1(b), where the piston diaphragm area S_D is shown to be very much smaller than the box wall area S_B. The figure illustrates a spherical wave radiated into the box. If S_D is made small enough compared to S_B, the acoustic mass M_{AB} must approach that for a piston in an infinite baffle. Thus two "calibration points" are known. If $S_D = S_B$, M_{AB} is given by Eq. (4). If $S_D << S_B$, M_{AB} must approach the infinite baffle value [1] given by $M_{A1} = 8\rho_0/(3\pi^2\sqrt{S_D/\pi})$.

For lack of a better model, it is assumed that M_{AB} varies linearly with the ratio S_D/S_B between the two calibration points. Therefore the equation for M_{AB} is taken to be

$$M_{AB} = \frac{\rho_0 d}{3S_B}\left(\frac{S_D}{S_B}\right) + \frac{8\rho_0}{3\pi^2\sqrt{S_D/\pi}} \times \left(1 - \frac{S_D}{S_B}\right) = B\,\frac{\rho_0}{\sqrt{\pi S_D}} . \qquad (5)$$

This equation defines the mass-loading factor B for the rear of the diaphragm.[1] It is straightforward to solve for B to obtain

$$B = \frac{d}{\sqrt{S_B}}\,\frac{\sqrt{\pi}}{3}\left(\frac{S_D}{S_B}\right)^{3/2} + \frac{8}{3\pi}\left(1 - \frac{S_D}{S_B}\right) . \qquad (6)$$

It follows that B is a function of the box geometry. There are two sets of ratios for box dimensions that are used to minimize the effects of standing waves inside the box at the higher frequencies where electroacoustic approximations break down [4]. These ratios are $0.8 \times 1.0 \times 1.25$ and $0.6 \times 1.0 \times 1.6$. It is assumed that the driver is mounted in the box panel having the larger dimensions so that the first number in each set of ratios represents the relative depth of the box. In this case, the quotient $d/\sqrt{S_B}$ in Eq. (6) for B has the value $0.8/\sqrt{1.0 \times 1.25} = 0.716$ for the first set of ratios and $0.6/\sqrt{1.0 \times 1.6} = 0.474$ for the second set. Fig. 2 gives the variation of B with S_D/S_B for these two ratios.

[1] In [1, p. 217] Beranek defines the mass loading factor B (sometimes called the Beranek B factor), but he never gives an analytical expression for it. His Fig. 8.6 gives a graph for B with no explanation of how it was obtained. It is believed that the present approach is the one used by Beranek to generate the graph.

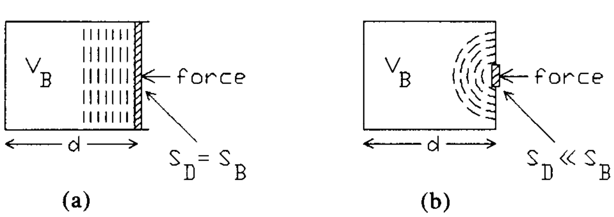

Fig. 1. Piston radiator installed in one wall of closed-box baffle. (a) Piston area is equal to box wall area. Wave radiated into box is plane wave. (b) Piston area is much smaller than box wall area. Wave radiated into box is spherical wave.

2 MODELING THE THERMODYNAMIC EFFECTS OF FILLING

When filling material is added to an enclosure, the effective acoustic volume of the box is increased. This can be explained by comparing the velocity of sound in filling material such as fiberglass to that in air. The filling slows down the wave propagation. If the filling is in an enclosure, this would cause the time required for a sound wave that emanates from the rear of a loudspeaker diaphragm to be reflected from an internal surface of the box to be greater in a filled box than in an unfilled box. Therefore the box "looks" bigger to the driver.

In free air a sound wave is modeled as an adiabatic process [1] for which the velocity of sound is given by $c = \sqrt{\gamma P_0/\rho_0}$, where $\gamma = 1.4$ is the ratio of the specific heat of air at constant pressure to the specific heat at constant volume, ρ_0 is the air density, and P_0 is the static air pressure. To a first approximation, an acoustic wave in a filled medium is modeled as an isothermal process for which $c = \sqrt{P_0/\rho_0}$. This equation can be predicted from the one for an adiabatic process by considering γ to be variable; it has the value $\gamma = 1$ for an

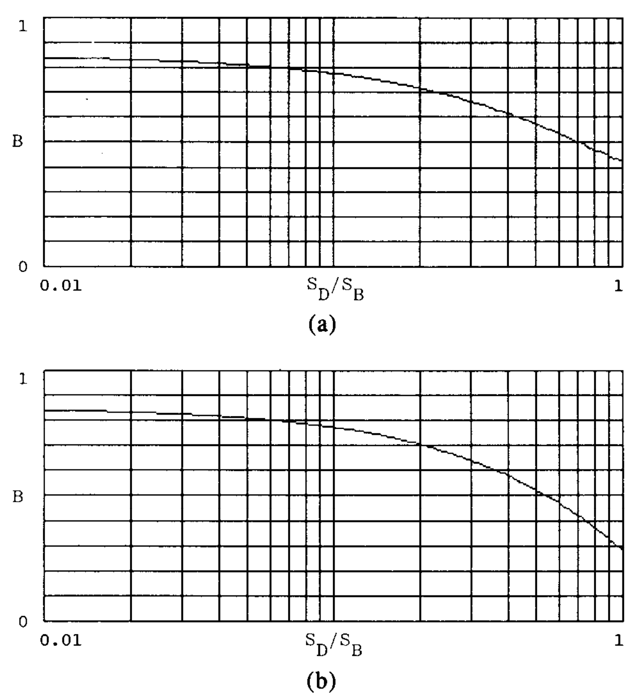

Fig. 2. Plot of mass-loading factor B as a function of ratio of diaphragm area to box wall area S_D/S_B. (a) Ratio of box dimensions $0.8 \times 1.0 \times 1.25$. (b) Ratio of box dimensions $0.6 \times 1.0 \times 1.6$. In each case, the diaphragm is mounted in the box wall having the largest area.

isothermal process. From Eq. (3) the box compliance can be written as a function of γ,

$$C_{AB} = \frac{V_B}{\rho_0 c^2} = \frac{V_B}{\gamma P_0} . \qquad (7)$$

This equation can be interpreted as saying that the effective volume of the box varies inversely with γ. It thus follows that a filled box can appear bigger than an unfilled box by the factor of 1.4, or by 40%.

In practice, the volume increase due to filling is never as large as 40%. To investigate this further, the thermodynamics of the air compression in a filled box must be studied. Consider the system illustrated in Fig. 3(a). Let the total volume V_B inside the vessel be written $V_B = V_a + V_f$, where V_a is that part of the volume that is occupied by air and V_f that occupied by filling. It is assumed initially that the air pressure is P_0 and that the air and the filling in the vessel are in thermal equilibrium at a temperature T_0. Let a unit step of force ΔF be applied to the piston in the figure. This will cause the volume of air to be compressed adiabatically to $V_a - \Delta V_1$, the air pressure to increase to $P_0 + \Delta P$, and the air temperature to increase to $T_0 + \Delta T_1$. Because the air is now at a higher temperature than the filling, the filling will be heated, which in turn will cause the air temperature to drop. As this occurs, the pressure in the vessel must remain constant because of the constant applied force. Thus as the temperature of the air drops, the piston will move in and the air volume will decrease at a constant pressure to a final value $V_a - \Delta V_1 - \Delta V_2$, while the air temperature drops to a final equilibrium value $T_0 + \Delta T_1 - \Delta T_2$. This must also be the final equilibrium temperature of the filling.

Fig. 3(b) illustrates the pressure versus volume for the air in the vessel during the processes described. The initial point is denoted by A on the diagram. When the force ΔF is applied, the adiabatic compression will cause a transition to point B and the piston will do work on the air ΔW_1. As the filling causes the air to cool, the constant-pressure compression will cause a transition to point C, and the piston will do work on the air ΔW_2. During the adiabatic compression, the work done on the air must equal the increase in internal energy. This gives the relation

$$\Delta W_1 = P_0 \Delta V_1 + \frac{1}{2} \Delta P \Delta V_1$$

$$\simeq P_0 \Delta V_1 = M_a c_v \Delta T_1 \qquad (8)$$

where M_a is the mass of the air and c_v is the specific heat of the air at constant volume. During the constant-pressure compression, the work done on the air must equal the increase in internal energy (which is negative) plus the heat lost to the filling. This gives the relation

$$\Delta W_2 = (P_0 + \Delta P)\Delta V_2 \simeq P_0 \Delta V_2$$

$$= -M_a c_v \Delta T_2 + M_f c_f (\Delta T_1 - \Delta T_2) \qquad (9)$$

where M_f is the total mass of the filling and c_f is the specific heat of the filling material.

Finally, the general gas law $PV = M(c_p - c_v)T = Mc_v(\gamma - 1)T$, where M is the gas mass, c_p is the specific heat at constant pressure, and $\gamma = c_p/c_v$, can be used to relate ΔV_2 to ΔT_2 to obtain

$$P_0 \Delta V_2 = M_a c_v (\gamma - 1) \Delta T_2 . \qquad (10)$$

The preceding three equations can be solved simultaneously to relate ΔV_2 to ΔV_1. The desired relation is

$$\frac{\Delta V_2}{\Delta V_1} = \frac{\gamma - 1}{1 + \gamma M_a c_v / M_f c_f} . \qquad (11)$$

This result can be used to solve for the apparent increase in the volume of the air caused by the filling. For an adiabatic compression, a small change in pressure is related to a small change in volume by the relation $\Delta P = \gamma P \Delta V / V$. Thus if the same change in pressure is to result from the compression of an unfilled volume V_a by ΔV_1 as for an unfilled volume V_{AB} by $\Delta V_1 + \Delta V_2$, ΔP must satisfy

$$\Delta P = \gamma P_0 \frac{\Delta V_1}{V_a} = \gamma P_0 \frac{\Delta V_1 + \Delta V_2}{V_{AB}} . \qquad (12)$$

This can be solved for V_{AB}/V_a to obtain

$$\frac{V_{AB}}{V_a} = 1 + \frac{\Delta V_2}{\Delta V_1} = 1 + \frac{\gamma - 1}{1 + \gamma M_a c_v / M_f c_f} . \qquad (13)$$

To put this equation into the final desired form, we write $V_a = V_B - V_f = V_B(1 - V_f/V_B)$, $M_a = \rho_0 V_a = \rho_0 V_B (1 - V_f/V_B)$, and $M_f = \rho_f V_f$, where ρ_0 is the density of the air and ρ_f is the bulk density of the filling material. We can solve for V_{AB} as a function of V_B to obtain

$$V_{AB} = V_B \left(1 - \frac{V_f}{V_B}\right) \times \left[1 + \frac{\gamma - 1}{1 + \gamma (V_B/V_f - 1) \rho_0 c_v / \rho_f c_f}\right] . \qquad (14)$$

This is the desired relation. It gives the apparent volume V_{AB} of air in a filled box as a function of the box volume V_B. We note that $V_{AB} = V_B$ if $V_f = 0$ and that $V_{AB} = 0$ if $V_f = V_B$. These limiting values certainly agree with intuitive reasoning. As a numerical example, the expression will be evaluated for uncompressed fiberglass of the type that is used in home construction for which $V_B/V_f \simeq 400$. We have $\rho_0 = 1.18$ kg/m^3, $\rho_f = 2400$ kg/m^3, $\gamma = 1.4$, $c_v = 717$ J/(kg · °C), and $c_f = 670$ J/(kg · °C). Evaluation of Eq. (14) yields

$V_{AB} = 1.31V_B$. Therefore we would expect that uncompressed fiberglass would increase the apparent volume of a box by about 31%.

The time required for the transition from point B to point C in Fig. 3(b) has been neglected. It is shown in the Appendix that the cooling effect of the filling can be modeled by an exponential time function. It follows that the transition from point B to point C can be modeled by an exponential time function so that the apparent volume of air in the box as a function of time can be written

$$V_{AB}(t) = V_B\left(1 - \frac{V_f}{V_B}\right) \times \left[1 + \frac{\gamma - 1}{1 + \gamma\,(V_B/V_f - 1)\,\rho_0 c_v/\rho_f c_f}\,(1 - e^{-t/\tau_f})\right] \tag{15}$$

where τ_f is the time constant and it is assumed that the applied unit step of force occurs at the time $t = 0$. It can be seen from this equation that the apparent volume of air has the initial value at $t = 0$ of $V_B(1 - V_f/V_B)$ and a final value at $t \rightarrow \infty$ given by Eq. (14). As shown by Eq. (7), the acoustic compliance of the air in the enclosure is proportional to the volume of air, where the constant of proportionality is $1/\gamma P_0 = 1/\rho_0 c^2$. It is desired to solve for the analogous circuit that Eq. (15) predicts. To do this, $V_{AB}(t)$ is Laplace transformed to obtain a function $V_{AB}(s)$, where s is the complex frequency. This is a transfer function representing the step response of the volume, which is converted to a transfer function representing the impulse response of the volume by multiplying by s. The transfer function representing the acoustical admittance $Y(s)$ of the volume is then obtained by multiplying by $s/\gamma P_0 = s/\rho_0 c^2$.

When the procedure described for solving for $Y(s)$ is performed, it follows that the acoustical admittance of the volume is given by

$$Y(s) = \frac{V_B}{\rho_0 c^2}\left(1 - \frac{V_f}{V_B}\right) \times \left[s + \frac{\gamma - 1}{1 + \gamma\,(V_B/V_f - 1)\,\rho_0 c_v/\rho_f c_f}\,\frac{s}{1 + \tau_f s}\right] . \tag{16}$$

This equation represents the admittance of a compliance C_{AB1} in parallel with a series compliance C_{AB2} and a resistance R_{AB1}. The analogous circuit is shown in Fig. 4. The element values are given by

$$C_{AB1} = \frac{V_B}{\rho_0 c^2}\left(1 - \frac{V_f}{V_B}\right) \tag{17}$$

$$C_{AB2} = C_{AB1}\,\frac{\gamma - 1}{1 + \gamma\,(V_B/V_f - 1)\,\rho_0 c_v/\rho_f c_f} \tag{18}$$

$$R_{AB1} = \frac{\tau_f}{C_{AB2}} . \tag{19}$$

If the frequency is low enough, the reactance of the compliance C_{AB2} will be large compared to the resistance R_{AB1}, so that R_{AB1} can be considered a short circuit compared to C_{AB2}. The admittance $Y(s)$ then reduces to two parallel compliances. The sum of these is given by

$$C_{AB1} + C_{AB2} = \frac{V_B}{\rho_0 c^2}\left(1 - \frac{V_f}{V_B}\right) \times \left[1 + \frac{\gamma - 1}{1 + \gamma\,(V_B/V_f - 1)\,\rho_0 c_v/\rho_f c_f}\right] . \tag{20}$$

This expression for the total compliance is valid only in the frequency range for which $R_{AB1} << 1/2\pi f C_{AB2}$. This is the frequency range for which the air compression in the box can be modeled as an isothermal process. For uncompressed fiberglass filling of the type used in home construction, the analysis presented in the Appendix shows that the air compression in the box can be modeled by an isothermal process for frequencies below 6.4 kHz. This frequency increases if the fiberglass is compressed. For fiberglass filling, therefore, it can be concluded that the resistor R_{AB1} can be neglected in the design of woofer enclosures for most practical purposes.

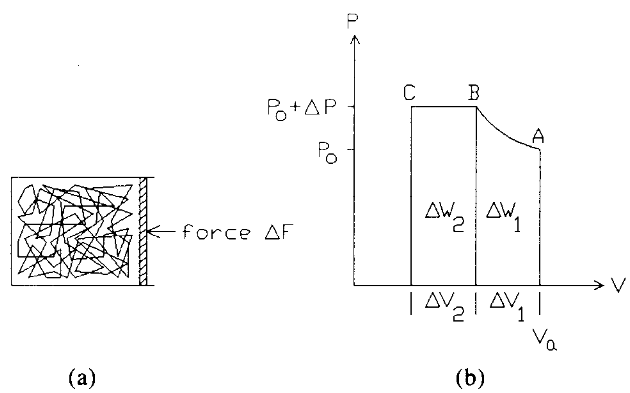

Fig. 3. Calculation of thermodynamic effects of filling. (a) Force applied to piston in one wall of closed vessel containing filling material. (b) Pressure versus volume for air in vessel.

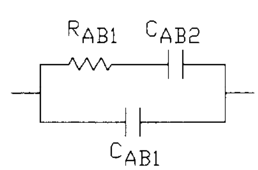

Fig. 4. Electroacoustic-analogous circuit modeling thermodynamic effects in filled closed box.

3 MODELING THE MECHANICAL PARAMETERS OF A FILLING

If a single fiber of the filling material is examined, it would be found to exhibit a mechanical mass m_f, compliance c_f, and damping resistance r_f. When air particles flow by the fiber, a force will be generated due to aerodynamic drag. Let the mechanical velocity of the fiber be denoted by u_f and the air particle velocity by u_a. If δ is the aerodynamic drag factor between the fiber and the air, the equation of motion for the fiber can be written [5]

$$m_f \frac{du_f}{dt} = \delta(u_a - u_f) + r_f u_f + \frac{1}{c_f}\int u_f \, dt \, . \qquad (21)$$

When this equation is Laplace transformed, the fiber velocity u_f can be solved for as a function of u_a and the complex frequency s to obtain

$$u_f = u_a \frac{\delta}{m_f s + (r_f + \delta) + 1/c_f s} \, . \qquad (22)$$

It is assumed that the acoustic wave in the filling is a one-dimensional wave that propagates in the z direction. The total aerodynamic drag on the air particles over an area A in an interval dz due to the fibers is equal to the force on a single fiber due to the air multiplied by the total number of fibers N in the volume element $A\mathrm{d}z$. The air mass in this volume is $\rho_0(1 - V_f/V_B)A\mathrm{d}z$, where ρ_0 is the air density and V_f/V_B is the fraction of the box volume occupied by the filling. It follows that the momentum equation for the air in the volume element $A\mathrm{d}z$ is [5]

$$\rho_0\left(1 - \frac{V_f}{V_B}\right) A\mathrm{d}z \, \frac{\mathrm{d}u_a}{\mathrm{d}t} = [p(z) - p(z + \mathrm{d}z)]A - N\delta(u_a - u_f)$$

$$= -\frac{\partial p}{\partial z} A\mathrm{d}z - N\delta(u_a - u_f) \, . \qquad (23)$$

When this equation is Laplace transformed, Eq. (22) can be used to eliminate u_f to obtain the relation

$$\frac{\partial p}{\partial z} = -\rho_c \frac{\mathrm{d}u_a}{\mathrm{d}t} \qquad (24)$$

where ρ_c is defined to be the complex density of the medium. It is given by

$$\rho_c = \rho_0\left(1 - \frac{V_f}{V_B}\right) + \frac{n}{s}\left(\frac{1}{\delta} + \frac{1}{m_f s + r_f + 1/c_f s}\right)^{-1} \qquad (25)$$

where $n = N/A\mathrm{d}z$ is the number of fibers of filling per unit volume.

An analogous circuit that models the complex density ρ_c can be formed. To do this, ρ_c is used in place of ρ_0 in Eq. (5). It follows that the acoustic impedance Z associated with M_{AB} is then given by

$$Z = sB\frac{\rho_0}{\sqrt{\pi S_d}}\left(1 - \frac{V_f}{V_B}\right) + \frac{B}{\sqrt{\pi S_D}} n \left[\frac{1}{\delta} + \frac{1}{m_f s + r_f + 1/c_f s}\right]^{-1} . \qquad (26)$$

The first term in this equation represents the reactance of an acoustic mass similar to that given by Eq. (5) for an unfilled box. The other terms represent the impedance of an acoustic resistor in parallel with a series acoustic mass, resistance, and compliance. The equivalent circuit for Z is given in Fig. 5. The element values in this circuit are given by

$$M_{AB1} = B\frac{\rho_0}{\sqrt{\pi S_D}}\left(1 - \frac{V_f}{V_B}\right) \qquad (27)$$

$$M_{AB2} = \frac{B}{\sqrt{\pi S_D}} \rho_f \frac{V_f}{V_B} \qquad (28)$$

$$C_{AB3} = \frac{\sqrt{\pi S_D}}{Bn} c_f \qquad (29)$$

$$R_{AB2} = \frac{Bn}{\sqrt{\pi S_D}} r_f \qquad (30)$$

$$R_{AB3} = \frac{Bn}{\sqrt{\pi S_D}} \delta \qquad (31)$$

where the relation $\rho_f V_f/V_B = n m_f$ has been used to eliminate m_f from Eq. (28).

The limiting case for the circuit in Fig. 5 for the case $R_{AB2} \rightarrow 0$, $R_{AB3} \rightarrow \infty$, and $C_{AB3} \rightarrow \infty$ is of interest. Physically, this corresponds to the case where the motion of each fiber is controlled only by its mass and the aerodynamic drag between the air and the filling is so large that the fibers must vibrate with the same velocity

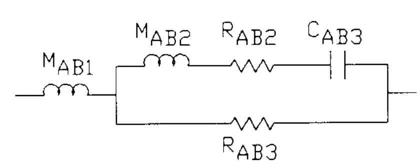

Fig. 5. Electroacoustic-analogous circuit modeling effects of mechanical parameters of filling in closed box.

as the air particles do. In this case, the circuit of Fig. 5 reduces to the series combintion of $M_{AB1} + M_{AB2}$, which is given by

$$M_{AB1} + M_{AB2} = \frac{B}{\sqrt{\pi S_D}} \left[\rho_0 \left(1 - \frac{V_f}{V_B} \right) + \rho_f \frac{V_f}{V_B} \right] . \quad (32)$$

The term in brackets in Eq. (32) can be thought of as being the average density of the air plus filling in the box. If the volume occupied by the filling is zero, that is, $V_f = 0$, the average density becomes ρ_0. If the filling occupies the entire box volume, that is, $V_f = V_B$, the average density becomes ρ_f. These results agree with intuitive reasoning.

4 THE COMPLETED ANALOGOUS CIRCUITS

The complete circuit that models the rear air load impedance on the diaphragm when the box contains filling can be formed. This is given by combining in series the circuits of Figs. 4 and 5. The completed circuit is shown in Fig. 6. For most applications, this circuit is probably too complicated for routine calculations. If the circuit can be approximated by a series mass, damping resistance, and compliance, it would greatly simplify calculations. The first step in doing this would be to consider R_{AB1} to be a short circuit, as discussed in Sec. 2. In this case, C_{AB1} and C_{AB2} could be combined into a single capacitor. There are two ways to reduce the rest of the circuit to a series circuit. One way would be to consider R_{AB3} to be an open circuit, as was discussed. A second way would be to consider M_{AB2} and C_{AB3} to have negligible reactance so that R_{AB2} and R_{AB3} could be combined into a single resistor. Either way, the circuit reduces to a series mass, damping resistance, and compliance. The accuracy of these approximations would depend on the type of filling.

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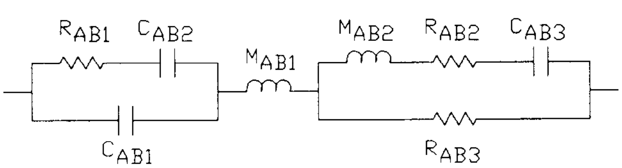


Fig. 6. Combined electroacoustic-analogous circuit modeling both thermodynamic effects and mechanical parameters of filling in filled closed box.

APPENDIX

The time variation of the thermodynamic cooling effect of filling in an enclosure has been studied in [6]. However, that paper contains errors that invalidate the conclusions stated. For this reason, a corrected and abbreviated analysis is presented here. Following a solution derived in [7], the analysis presented in [6] assumes that the temperature of the filling fibers remains constant as the air temperature varies. The same assumption is made here. Although this is not true in practice, it will be shown that the error introduced makes the conclusions obtained conservative.

Each filling fiber is modeled as a cylinder of radius r_1 around which there is a coaxial insulating cylinder of radius r_2. The air between the cylinders represents the average volume of air that must be cooled by each fiber. The ratio of r_2 to r_1 is denoted by m. Let ρ_m be the effective density of the filling in the enclosure and ρ_f be the bulk density of the fiber. These are related by the equation $m = r_2/r_1 = \sqrt{\rho_f/\rho_m}$. Let the temperature of the fiber be held constant at T_f. Denote the initial temperature of the air surrounding the fiber by T_i, where $T_i > T_f$. It can be shown [7] that the temperature distribution T as a function of time t and radius r, where $t \geqslant 0$ and $r_1 \leqslant r \leqslant r_2$ is given by

$$\frac{T(r, t) - T_f}{T_i - T_f} = -\pi \sum_{n=1}^{\infty} \frac{J_1^2(m\mu_n) V_0(\mu_n r/r_1)}{J_0^2(\mu_n) - J_1^2(m\mu_n)} e^{-\mu_n^2 at/r_1^2} \quad (33)$$

where J_0 and J_1 are Bessel functions of the first kind and a is the thermal diffusity of air ($a \simeq 1.87 \times 10^{-5}$ m^2/s). The function $V_0(\mu_n r/r_1)$ is given by

$$V_0\left(\frac{\mu_n r}{r_1}\right) = J_0\left(\frac{\mu_n r}{r_1}\right) Y_0(\mu_n) - J_0(\mu_n) Y_0\left(\frac{\mu_n r}{r_1}\right) \quad (34)$$

where Y_0 is a Bessel function of the second kind. The elements of the set $\{\mu_n | n = 1, 2, 3, \cdots\}$ satisfy the

Correction

In the Appendix, the value $\mu_1 = 0.232$ in the numerical example is erroneously the value for μ_2. The correct value for μ_1 is

$$\mu_1 = 0.0465$$

With this value, the upper frequency limit for isothermal operation is

$$f = 257\,\mathrm{Hz}$$

not the $f = 6.4\,\mathrm{kHz}$ given in the paper.

equation

$$J_1(m\mu_n)Y_0(\mu_n) - J_0(\mu_n)Y_1(m\mu_n) = 0, \quad n = 1, 2, 3, \cdots \tag{35}$$

where $0 < \mu_1 < \mu_2 < \mu_3 < \cdots$.

The solution given by Eq. (33) represents a sum of damped exponentials. The term that damps out the slowest is the $n = 1$ term. Let τ_1 be the time constant associated with this term. It is given by

$$\tau_1 = \frac{r_1^2}{\mu_1^2 a} = \frac{r_2^2}{m^2\mu_1^2 a} . \tag{36}$$

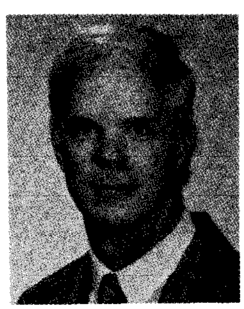
This is the dominant time constant for the temperature response. Following [6], it can be concluded that in the frequency band defined by $f < 1/2\pi\tau_1$, air compression in a filled enclosure can be modeled as an isothermal process provided the temperature of the filling is maintained constant.

An idea of the effect of allowing the temperature of the filling to increase as the surrounding air cools can be obtained from a simple analog. The heat capacity of either the air or the fiber can be expressed by the equation $Q = KT$, where Q is the heat capacity in joules, K is a constant depending on the material, and T is the absolute temperature. An electrical analog of this equation is the one relating the charge on a capacitor to the voltage, that is, $q = CV$, where q is the charge in coulombs, C is the capacitance, and V is the voltage. If Q is analogous to q and T is analogous to V, the cooling of the air can be modeled by the discharge of a capacitor C_1 through a resistor R. For the case that the fiber is held at a constant temperature, the capacitor discharges into a constant voltage that is analogous to the fiber temperature so that the time constant is RC_1. This corresponds to the case where the fiber has an infinite heat capacity so that it can be modeled as an infinite capacitor C_2 having a constant voltage. If C_2 is not infinite, its voltage will increase as the voltage on the C_1 decreases. It follows that the time constant in this case is $RC_1C_2/(C_1 + C_2)$, which is smaller than RC_1.

If the dominant time constant of the air–fiber system for the case where the fiber temperature is not constant is denoted by τ_f, it follows from the analog that $\tau_f < \tau_1$. Because $1/\tau_1 < 1/\tau_f$, it can be concluded that in the frequency band defined by $f < 1/2\pi\tau_1$, air compression in a filled enclosure can be modeled as an isothermal process in the case that the heat capacity of the filling is not infinite. Therefore a numerical evaluation of Eq. (36) for the typical parameters encountered in loudspeaker systems can be used to make a conservative calculation of the frequency band over which the air compression in the box can be modeled by an isothermal process.

In [6] it is given that a typical fiber of the fiberglass insulation used in home construction has a radius $r_1 \simeq 5 \times 10^{-6}$ m and a density $\rho_f = 2400$ kg/m^3. In its uncompressed state, the effective density of the fiberglass is $\rho_m \simeq 6$ kg/m^3. When it is compressed to its practical maximum, its effective density is $\rho_m \simeq 600$ kg/m^3. An evaluation of Eq. (35) for the uncompressed effective density of $\rho_m = 6$ kg/m^3 yields the value $\mu_1 = 0.232$. This gives an upper frequency limit for isothermal operation of $f = 6.4$ kHz. For any value of ρ_m larger than 6 kg/m^3, this frequency is increased. Because the resonance frequency of closed-box woofer systems is much lower than 6.4 kHz, it can be concluded that the transient thermal effects of fiberglass filling can be neglected in loudspeaker system design. (This result contradicts that given in [6], where it is stated that fiberglass must be compressed to four times its uncompressed state to obtain isothermal operation for frequencies less than 100 Hz.)

THE AUTHOR



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