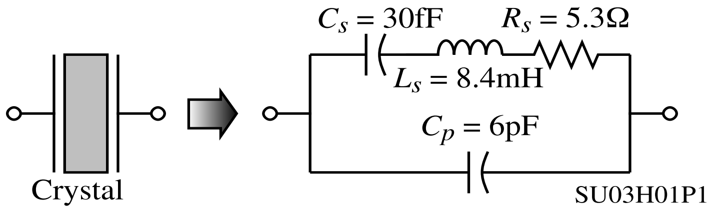
Homework Assignment No. 1 - Solution

Problem 1 - (10 points)

Solve for and evaluate the series and parallel resonance frequencies of the crystal whose model is shown. It is suggested to make appropriate assumptions as the exact frequencies are difficult to achieve.



Solution

Solving the exact frequencies for this problem is very challenging. It is better to assume that series resonance (minimum impedance) will occur approximately when the impedance of C_s cancels the impedance of L_s. This gives series resonance as

$$\omega_s{}^2 = \frac{1}{L_s C_s} \quad \rightarrow \quad f_s = \frac{1}{2\pi\sqrt{L_s C_s}} \approx \underline{\underline{10.026\text{MHz}}}$$

The parallel resonance can be approximated by assuming that it will occur close to the frequency when the impedance of the series branch equals the negative impedance of the parallel branch. This condition is given as,

$$\frac{1}{\omega C_p} = \omega L_s + \frac{1}{\omega C_s} \quad \rightarrow \quad \omega_p{}^2 = \frac{1}{L_s}\left(\frac{1}{C_s} + \frac{1}{C_p}\right) \quad \rightarrow f_p = \frac{1}{2\pi}\sqrt{\frac{1}{L_s}\left(\frac{1}{C_s} + \frac{1}{C_p}\right)}$$

$$f_p \approx \underline{\underline{10.051\text{MHz}}}$$

SPICE Simulation:

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Homework H01P1 - Crystal Impedance
IIN 0 1 AC 1.0
CP 1 0 6PF
CS 1 2 30FF
LS 2 3 8.4MH
RS 3 0 5.3OHM
RBIG 1 0 1GOHM
.AC LIN 101 9.5MEG 10.5MEG
.PRINT AC V(1)
.PROBE
.END
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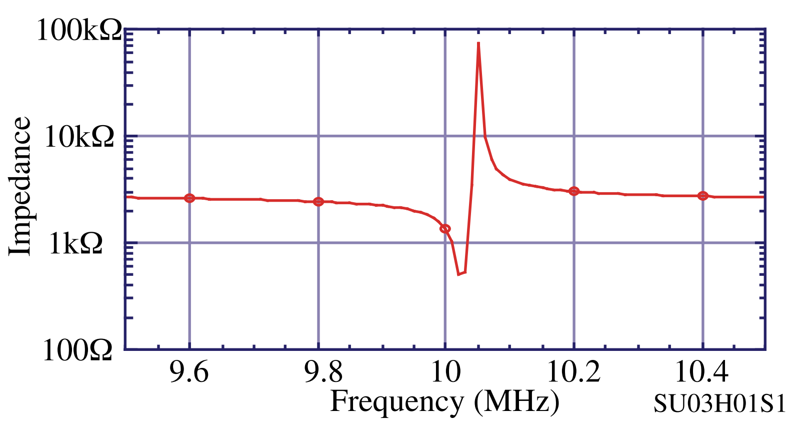
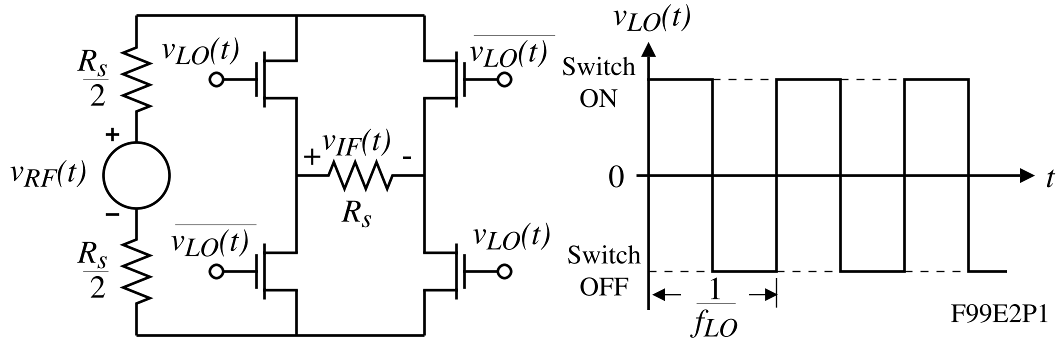


Problem 2 - (10 points)

A simple, doubly balanced passive CMOS mixer is shown along with the local oscillator waveform, $v_{OL}(t)$. Assume that $v_{RF}(t) = A_{RF}cos(\omega_{RF}t)$ and $v_{LO}(t)$ is the waveform shown below. (a.) Find the mixer gain, G_c, in dB if the switches are ideal. (b.) Find the mixer gain in dB if the switches have an ON resistance of $R_s/2$.



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Solution

Assume the switches have an ON resistance of R_{ON} and work both parts (a) and (b) simultaneously. Also, The equation for $v_{IF}(t)$ can be written as,

$$v_{IF}(t) = \left(\frac{R_s}{2R_s+2R_{ON}}\right) v_{RF}(t) \cdot \mathrm{sgn}[v_{LO}(t)]$$

$$V_{IF}(j\omega) = \left(\frac{R_s}{2R_s+2R_{ON}}\right) A_{RF}cos(\omega_{RF}t) \cdot \left[\frac{4}{\pi}cos(\omega_{LO}t) + \frac{4}{3\pi}cos\ (3\omega_{LO}t) + \cdots\right]$$

$$\therefore \qquad V_{IF}(j\omega) \approx \left(\frac{R_s}{2R_s+2R_{ON}}\right) \frac{4A_{RF}}{\pi} cos(\omega_{RF}t) \cdot cos(\omega_{LO}t)$$

$$= \left(\frac{R_s}{2R_s+2R_{ON}}\right) \frac{2A_{RF}}{\pi} cos[\omega_{RF} - \omega_{LO})t]$$

The conversion gain in general is written as

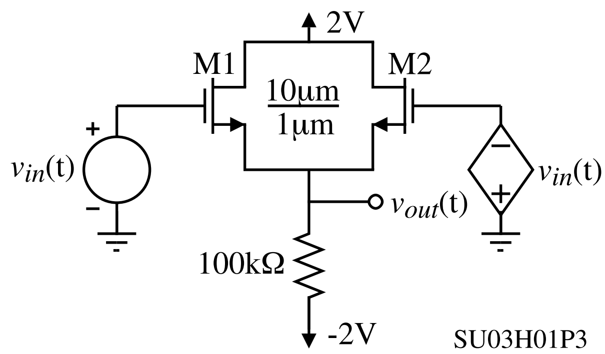
$$G_c = \frac{|V_{IF}|}{|V_{RF}|} = \left(\frac{R_s}{2R_s+2R_{ON}}\right) \frac{2}{\pi}$$

(a.) For $R_{ON} = 0$, $G_c = \frac{1}{\pi}$ $\rightarrow$ $\boxed{G_c = \frac{1}{\pi} = -9.943\text{dB}}$

(b.) For $R_{ON} = 0.5R_s$, $G_c = \frac{2}{3\pi}$ $\rightarrow$ $\boxed{G_c = \frac{2}{3\pi} = -13.465\text{dB}}$

Problem 3 – (10 points)

Use SPICE to demonstrate that the following circuit is a frequency doubler. If $v_{in}(t)$ is a sinusoid of 10kHz and 1.5V peak, show $v_{in}(t)$ and $v_{out}(t)$ as a function of time. The model parameters of the MOSFETS are $K_N' = 110\mu A/V^2$, $V_{TN} = 0.7V$, and $\lambda_N = 0.04V$-1.



Solution

The results of this problem are below.

SPICE Input File:

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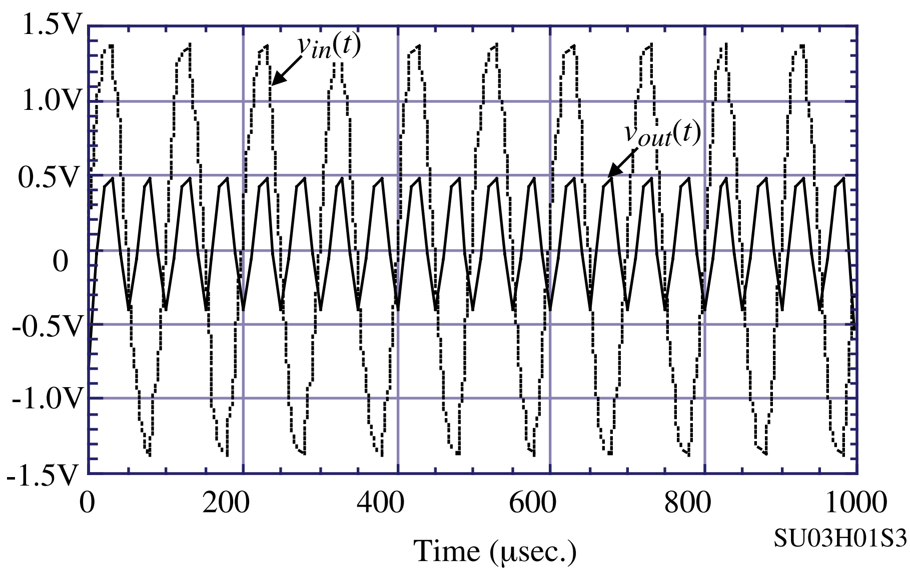
Homework H01P3 - Frequency Doubler
VIN 1 0 DC 0.0 SIN(0 1.5 10KHz)
EVIN 0 2 1 0 1.0
VDD 4 0 DC 2.0
VSS 5 0 DC -2.0
M1 4 1 3 3 NMOS1 W=10U L=1U
M2 4 2 3 3 NMOS1 W=10U L=1U
RTAIL 3 5 100K
.MODEL NMOS1 NMOS VTO=0.7 KP=110U LAMBDA=0.04
.OP
.TRAN (10U 1000U)
.PRINT TRAN V(1) V(2) V(3)
.PROBE
.END
```

Output Plots:



Problem 4 – (10 points)

An 10nH inductor has a Q of 5 and is used to create a tank circuit with a 10pF capacitor. Assume the capacitor is ideal. (a.) What is the resonant frequency of this circuit? (b.) What value of parallel negative resistance should be used to create an oscillator? (c.) If C is changed to 20 pF, what is the new value of the parallel negative resistance?

Solution

C = 10pF:

$$L_p = \left(1+\frac{1}{Q^2}\right) = \frac{26}{25}\cdot 10\text{nH} = 10.4\text{nH}$$

$$\omega_o = \frac{1}{\sqrt{L_pC}} = \frac{1}{\sqrt{10.4\text{nH}\cdot 10\text{pF}}} = 3.1623\text{x}10^9 \text{ radians/sec.}$$

$$Q = \frac{\omega_o L_s}{R_s} \quad \rightarrow \quad R_s = \frac{\omega_o L_s}{Q} = 6.201\Omega$$

$$\therefore \quad R_p = (1+Q^2)R_s = 26\cdot 6.201\Omega = \underline{\underline{161.245\Omega}}$$

C = 20pF:

$$\omega_o = \frac{1}{\sqrt{L_pC}} = \frac{1}{\sqrt{10.4\text{nH}\cdot 20\text{pF}}} = 2.1926\text{x}10^9 \text{ radians/sec.}$$

$$Q = \frac{\omega_o L_s}{R_s} \quad \rightarrow \quad R_s = \frac{\omega_o L_s}{Q} = 4.3853\Omega$$

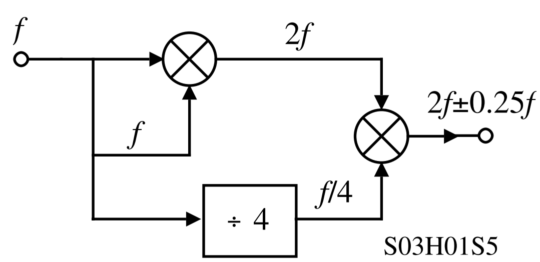
$$\therefore \quad R_p = (1+Q^2)R_s = 26\cdot 4.3853\Omega = \underline{\underline{114.017\Omega}}$$

Problem 5 – (10 points)

Give a block diagram of simple brute-force coherent direct synthesizer that will generate $1.75f$ from f. The input frequency f is to vary from 12 MHz to 15MHz. Since f is variable, you cannot use frequency multipliers (integer frequency dividers and mixers are allowed) in your design. A simple design will receive more credit. What other frequencies will be present at the output?

Solution

Approach: $f_{out} = f\text{x}f - f/4 = 1.75f$



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The frequency $2.25f$ will also be present at the output.