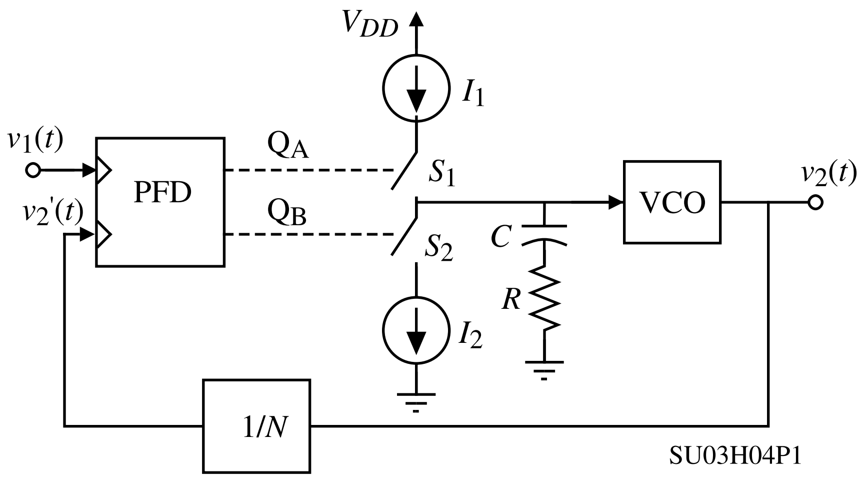
Homework Assignment No. 4 - Solutions

Problem 1 - (10 points)

For the DPLL shown assume that N = 1000 and the –3dB bandwidth is 1000 Hz. (a.) Assume that $\zeta = 0.2$ and solve for the natural pole frequency, ω_n, the filter time constant, $\tau = RC$, and the phase margin. (b.) Repeat part (a.) if $\zeta = 0.7$. (c.) Repeat part (a.) if $\zeta = 1$. Verify your answers with PSPICE.



Solution

The filter output can be written as,

$$V_f(s) = \frac{K_d}{s}(s\tau+1)(\theta_1-\theta_2') = \frac{K_d}{s}(s\tau+1)\left(\theta_1 + \frac{\theta_2}{N}\right) \qquad \text{where } \tau = RC$$

$$\theta_2(s) = \frac{K_o}{s}V_f(s) = \frac{K_oK_d}{s^2}(s\tau+1)\left(\theta_1 + \frac{\theta_2}{N}\right) = \frac{K_v\,(s\tau+1)}{s^2}\theta_1(s) + \frac{K_v\,(s\tau+1)}{Ns^2}\theta_2(s)$$

The closed-loop response is given as,

$$\frac{\theta_2(s)}{\theta_1(s)} = \frac{K_v\,(s\tau+1)}{s^2 + \frac{K_v\tau}{N}s + \frac{K_v}{N}} = \frac{K_v\,(s\tau+1)}{s^2 + 2\omega_n\zeta s + \omega_n^2}$$

$$\therefore \qquad \omega_n = \sqrt{\frac{K_v}{N}} \quad \text{and} \quad \tau = \frac{2\zeta}{\omega_n}$$

We know that the loop bandwidth, ω_{-3dB}, can be expressed as

$$\omega_{-3dB} = \omega_n\sqrt{2\zeta^2+1+\sqrt{(2\zeta^2+1)^2+1}} \quad \rightarrow \quad \omega_n = \frac{\omega_{-3dB}}{\sqrt{2\zeta^2+1+\sqrt{(2\zeta^2+1)^2+1}}}$$

$\zeta = 0.2$:

ω_n = 3933 rads/sec., τ = 102μs and PM = 0°+tan^{-1}(2000π·102μs) = 32.6°

$\zeta = 0.7$:

ω_n = 3066 rads/sec., τ = 457μs and PM = 0°+tan^{-1}(2000π·457μs) = 70.8°

$\zeta = 1$:

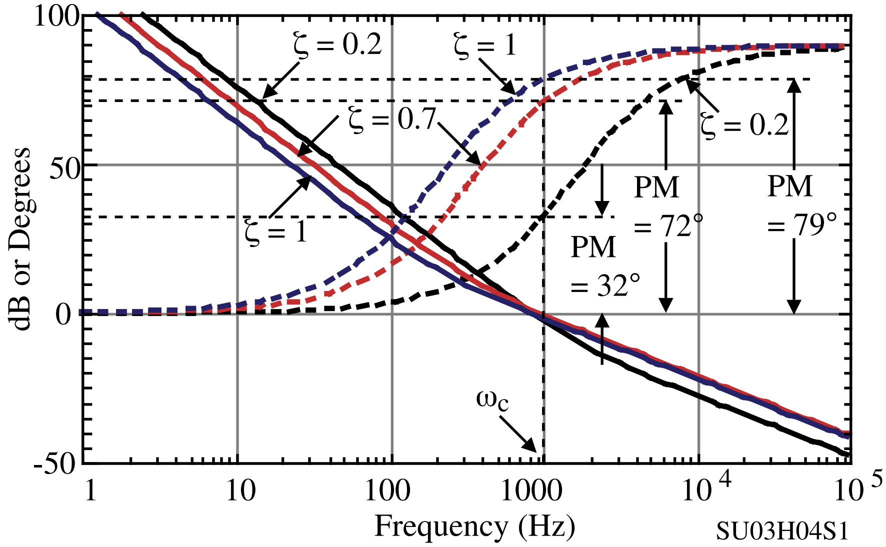
ω_n = 2531 rads/sec., τ = 790μs and PM = 0°+tan^{-1}(2000π·790μs) = 78.6°

Problem 1 – Continued

PSPICE Input File:

```
Homework4, Problem 1
VS 1 0 AC 1.0
R1 1 0 10K
ELPLL1 2 0 LAPLACE {V(1)}= {5044*5044*(1+102E-6*S)/(S+0.01)/(S+0.01)}
R2 2 0 10K
ELPLL2 3 0 LAPLACE {V(1)}= {3513*3513*(1+457E-6*S)/(S+0.01)/(S+0.01)}
R3 3 0 10K
ELPLL3 4 0 LAPLACE {V(1)}= {2531*2531*(1+790E-6*S)/(S+0.01)/(S+0.01)}
R4 4 0 10K
*Steady state AC analysis
.AC DEC 20 1 100K
.PRINT AC VDB(2) VP(2) VDB(3) VP(3) VDB(4) VP(4)
.PROBE
.END
```

Plot of Results:



Problem 2 – (10 points)

A type-I, second-order DPLL synthesizer is to be made with components having the following values:

$K_o = 4\text{x}10^8$ rads/sec./V $f_{ref} = 12.5$ kHz $K_d = 0.15$ V/rad $\beta = 2\pi$

Design a type-I, second-order synthesizer having the following specifications:

1.) Output frequency range = 50MHz

2.) Lock range = 10MHz at the output

3.) Damping factor = 0.75.

Determine the components for the loop filter. Let $C = 0.5\mu\text{F}$. Make a sketch of your filter with all components carefully labeled. Once your design is complete, determine the pull-in range in Hz (at the output) and the lock time of your loop.

Solution

$$N = \frac{f_{out}}{f_r} = \frac{50\text{MHz}}{12.5\text{kHz}} = 4000 \quad \text{and} \quad K_v = \frac{K_oK_d}{N} = \frac{4\text{x}10^8 \cdot 0.15}{4000} = 15{,}000 \text{ sec.}^{-1}$$

$$\Delta\omega_H = \beta K_v N = 2\pi \cdot 15\text{x}10^3 \cdot 4000 = 377\text{x}10^6 \text{ rads/sec.}$$

$$\Delta\omega_L = \frac{\tau_2}{\tau_1}\Delta\omega_H = \frac{\tau_2}{\tau_1} 377 \text{ Mrads/sec.} \rightarrow \quad \frac{\tau_2}{\tau_1} = \frac{\Delta\omega_L}{\Delta\omega_H} = \frac{62.8\text{Mrads/sec}}{377\text{Mrads/sec}} = \frac{1}{6}$$

$\therefore \quad \tau_1 = 6\tau_2$

$$\zeta = 0.5\sqrt{\frac{1}{K_v\tau_1}}(1+\tau_2K_v) \rightarrow 1.5 = \sqrt{\frac{1}{K_v\tau_1}}(1+\tau_2K_v) \rightarrow 2.25 = \frac{1}{K_v\tau_1}(1+\tau_2K_v)^2$$

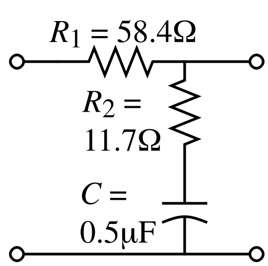
$$2.25 \cdot K_v(6\tau_2) = 13.5\ K_v\tau_2 = 1 + 2\ K_v\tau_2 + (K_v\tau_2)^2 \rightarrow 0 = 1 - 11.5x + x^2$$

where $x = K_v\tau_2$. Solving for x gives $x = K_v\tau_2 = \frac{11.5}{2} \pm \frac{1}{2}\sqrt{11.5^2 - 4} = 0.0876$

$$\therefore \quad \tau_2 = \frac{0.0876}{15{,}000} = 5.84\mu\text{s} = R_2C = R_2(0.5\mu\text{F}) \quad \rightarrow R_2 = \frac{5.84}{0.5\mu\text{F}} = \underline{\underline{11.7\Omega}}$$

$$\tau_1 = 6\tau_2 = 35\mu\text{s} = (R_1+R_2)C \quad \rightarrow R_1+R_2 = \frac{35}{0.5\mu\text{F}} = 70.08\Omega \quad \rightarrow \quad R_1 = \underline{\underline{58.4\Omega}}$$

Filter schematic:



SU03H04S2

$$\Delta\omega_P = N\beta\sqrt{2}\sqrt{2\zeta\omega_nK_vF(0) - \omega_n^2} \quad \text{and} \quad \omega_n = \sqrt{\frac{K_v}{\tau_1}} = 20{,}702 \text{ rads/sec}$$

$$\therefore \quad \Delta\omega_P = 4000 \cdot 2\pi\sqrt{2}\sqrt{2 \cdot 0.75 \cdot 20{,}702 \cdot 15{,}000 - (20{,}702)^2} = 216.85 \text{ Mrads/sec.}$$

$$\Delta f_P = \underline{\underline{34.51 \text{ MHz}}}$$

$$T_L = \frac{2\pi}{\omega_n} = \frac{6.283}{20{,}702} = \underline{\underline{303.5\mu\text{s}}}$$

Problem 3 – (10 points)

Given the DPLL described by

$K_d = 2.2$ V/rad $\qquad F(s) = \dfrac{1+\tau_2 s}{1+\tau_1 s} = \dfrac{1+5\text{x}10^{-6}s}{1+2\text{x}10^{-5}s}$

$f_{ref} = 12$ kHz $\quad K_o = 25$ MHz/V $\qquad \beta = 2\pi \qquad N = 15{,}000$

Determine the type number and order of the system and then find:

(a.) The output frequency in Hz.

(b.) The crossover frequency in Hz.

(c.) The noise bandwidth (Hz).

(d.) The closed-loop phase –3dB bandwidth in Hz

(e.) The steady-state phase error in response to a phase step of 0.1 radian.

(f.) The hold range (±Hz at the output).

(g.) The lock range (±Hz at the output).

(h.) The lock time.

(i.) The pull-in range (±Hz at the output)

(j.) The steady-state phase error in radians in response to a frequency step equal to the lock range.

Solution

This is a type-I, second-order system. The closed loop transfer function is,

$$\theta_2 = \frac{K_v F(s)}{s}\left(\theta_1 - \frac{\theta_2}{N}\right) \rightarrow \frac{\theta_2}{\theta_1} = \frac{K_v F(s)}{s + \frac{K_v F(s)}{N}} = \frac{K_v\left(\frac{1+\tau_2 s}{1+\tau_1 s}\right)}{s + \frac{K_v\left(\frac{1+\tau_2 s}{1+\tau_1 s}\right)}{N}} = \frac{K_v(1+\tau_2 s)}{s(1+\tau_1 s) + \frac{K_v(1+\tau_2 s)}{N}}$$

$$\frac{\theta_2}{\theta_1} = \frac{\frac{K_v}{\tau_1}(1+\tau_2 s)}{s^2 + \frac{s}{\tau_1}\left(1 + \frac{K_v}{N}\tau_2\right) + \frac{K_v}{\tau_1 N}} = \frac{K_v(1+\tau_2 s)}{s^2+2\zeta\omega_n s+\omega_n{}^2}$$

$$\therefore \qquad \omega_n = \sqrt{\frac{K_v}{N\tau_1}} = \sqrt{\frac{2\pi\cdot 25\text{x}10^6\cdot 2.2}{2\text{x}10^{-5}\cdot 15{,}000}} = 33.94 \text{ Krads/sec}$$

$$\zeta = 0.5\sqrt{\frac{N}{K_v\tau_1}}\left(1+\frac{\tau_2 K_v}{N}\right) = 0.5\sqrt{\frac{15{,}000}{2\pi\cdot 25\text{x}10^6\cdot 2.2\cdot 2\text{x}10^{-5}}}\left(1 + \frac{2\pi\cdot 25\text{x}10^6\cdot 2.2\cdot 5\text{x}10^{-6}}{15{,}000}\right)$$

$$= (0.5)(1.4732)(1.115) = 0.821$$

(a.) $f_o = Nf_r = 15{,}000\cdot 12\text{kHz} = \underline{\underline{180\text{MHz}}}$

(b.) $f_c = \dfrac{\omega_n}{2\pi}\sqrt{2\zeta^2+ + \sqrt{4\zeta^4+1}} = \dfrac{33{,}940}{2\pi}\sqrt{2(0.821)^2+\sqrt{4(0.821)^4+1}} = \underline{\underline{9{,}397 \text{ Hz}}}$

Problem 3 - Continued

(c.) $B_n = \frac{\omega_n}{2}\left(\zeta + \frac{1}{4\zeta}\right) = \frac{33{,}940}{2}\left(0.821 + \frac{1}{4\cdot 0.821}\right) = \underline{\underline{19.1\text{kHz}}}$

(d.) $\omega_{-3\text{dB}} = \omega_n\sqrt{b + \sqrt{b^2+1}}$ where $b = 2\zeta^2 + 1 - \frac{N\omega_n}{K_v}\left(4\zeta - \frac{N\omega_n}{K_v}\right)$

$$b = 2(0.821)^2 + 1 - \frac{33{,}940\cdot 15{,}000}{2\pi\cdot 25\text{x}10^6\cdot 2.2}\left(4\cdot 0.821 - \frac{33{,}940\cdot 15{,}000}{2\pi\cdot 25\text{x}10^6\cdot 2.2}\right) = -0.320$$

$$\omega_{-3\text{dB}} = 33{,}940\sqrt{-0.320 + \sqrt{(-0.320)^2+1}} = 29{,}003 \text{ rads/sec.} \rightarrow f_{-3\text{dB}} = \underline{\underline{4615\text{Hz}}}$$

(e.) Because this is a Type-I, second-order system, the phase error in response to a phase step is zero provided that $\Delta\theta < \beta$.

(f.) $\Delta\omega_H = \beta NK = \beta K_v \quad \rightarrow \quad \Delta f_H = \frac{\beta K_v}{2\pi} = (2\pi\cdot 25\text{x}10^6\cdot 2.2) = \underline{\underline{\pm 345.6\text{MHz}}}$

(g.) $\Delta\omega_L = \frac{\tau_2}{\tau_1}\Delta\omega_H \quad \rightarrow \quad \Delta f_L = \pm 345.6\text{MHz}\left(\frac{5\text{x}10^{-6}}{2\text{x}10^{-5}}\right) = \underline{\underline{\pm 86.39\text{MHz}}}$

(h.) Lock time $= T_L = \frac{2\pi}{\omega_n} = \frac{2\pi}{33{,}940} = \underline{\underline{185\mu\text{s}}}$

(i.) Pull-in range.

$$\Delta\omega_P = N\beta\sqrt{2}\sqrt{\frac{2\zeta\omega_n K_v F(0)}{N} - \omega_n^2} \text{ where } F(0) = 1 \text{ for the filter selected.}$$

$$\therefore \quad \Delta f_P = \frac{15{,}000\cdot 2\pi\sqrt{2}}{2\pi}\sqrt{\frac{2\cdot 0.821\cdot 33{,}940\cdot 2\pi\cdot 25\text{x}10^6\cdot 2.2}{15{,}000} - (33{,}940^2)} = \underline{\underline{243.7\text{MHz}}}$$

Note that,

$$\Delta f_L < \Delta f_P < \Delta f_H$$

(j.) $\varepsilon_{ss} = \frac{\Delta\omega_{osc}}{K_v} = \frac{2\pi\cdot 86.39\text{x}10^6}{2\pi\cdot 25\text{x}10^6\cdot 2.2} = \underline{\underline{1.57 \text{ rads}}} < \beta$ as required

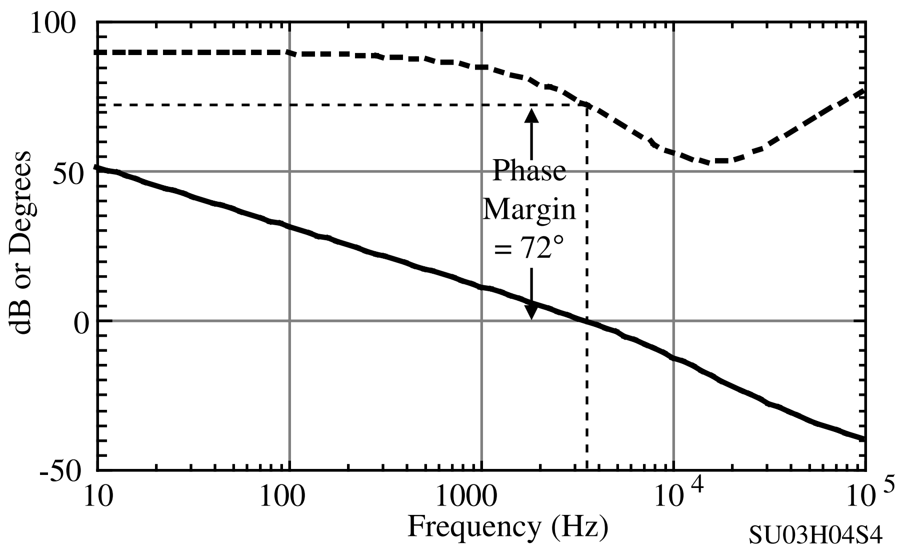
Problem 4 – (10 points)

Construct an accurate Bode plot of the synthesizer in Problem 3. Use this Bode plot to determine the phase margin.

Solution

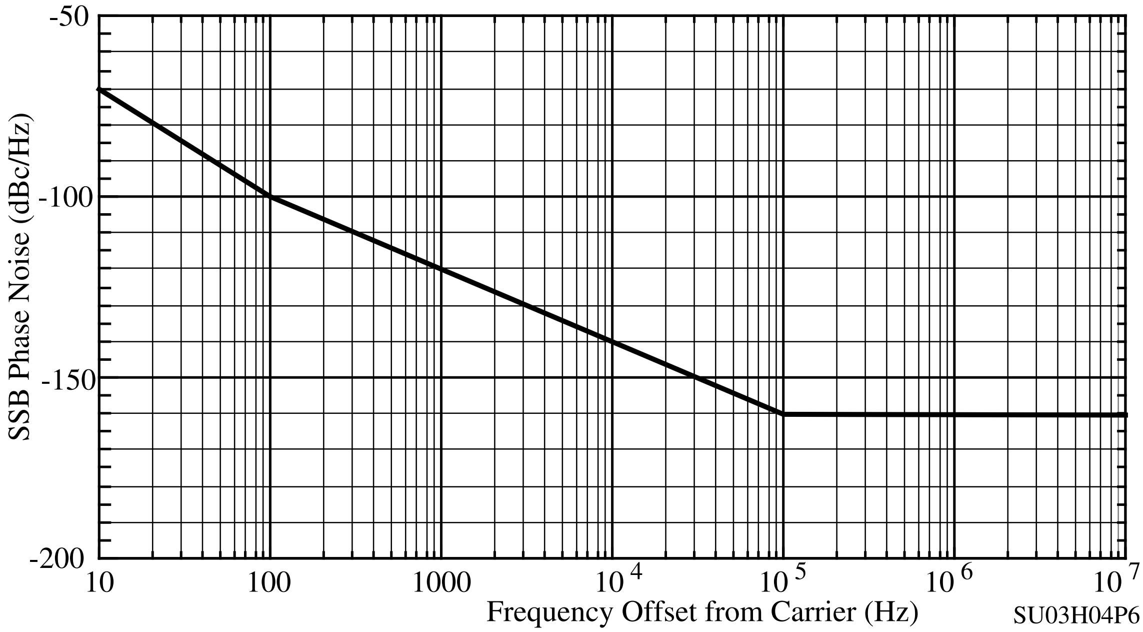
PSPICE was used to solve this problem. The input file and the results are shown below.

```
Problem H4P4-Open Loop Response of an DPLL with Lead-Lag
Filter
VS 1 0 AC 1.0
R1 1 0 10K
ELPLL 2 0 LAPLACE {V(1)}= {23.04E+3*(1+5E-6*S)/(1+2E-
5*S)/(S+0.001)}
R2 2 0 10K
*Steady state AC analysis
.AC DEC 20 10 100K
.PRINT AC VDB(2) VP(2)
.PROBE
.END
```



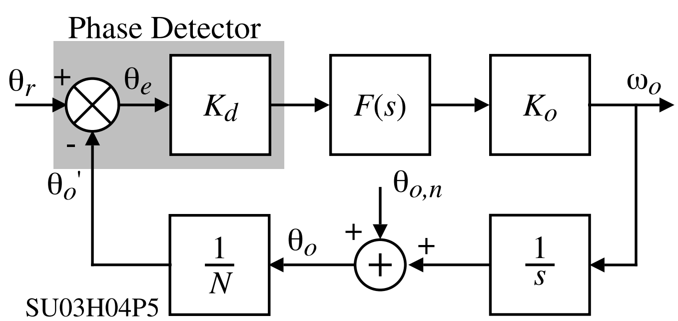
Problem 5 – (10 points)

Write the transfer functions giving: (1) The VCO phase noise in the output, (2) the reference oscillator phase noise in the output. Use the literal form of the equations. The phase noise of the VCO used in the synthesizer of Problem 3 is shown below. Make an accurate plot of the VCO phase noise in the output of the synthesizer.



Solution

The following block diagram will be used to find the phase noise in the output due to the VCO phase noise.



$$\theta_o = \left[\theta_{o,n} - \frac{K_v F(s)\theta_o}{sN}\right]$$

$$\frac{\theta_o}{\theta_{o,n}} = \frac{s}{s+\frac{K_v F(s)}{N}}$$

$$\boxed{\frac{\theta_o}{\theta_{o,n}} = \frac{s}{s+\frac{K_v}{N}\left(\frac{1+\tau_2 s}{1+\tau_1 s}\right)} = \frac{s(1+\tau_1 s)}{s^2\tau_1 + s\left(1+\frac{K_v\tau_2}{N}\right) + \frac{K_v}{N}}}$$

From Problem 3 of this assignment we get,

$$\boxed{\frac{\theta_o}{\theta_{r,n}} = \frac{1}{N}\,\frac{\frac{K_v}{\tau_1}(1+\tau_2 s)}{s^2 + \frac{s}{\tau_1}\left(1 + \frac{K_v}{N}\tau_2\right) + \frac{K_v}{\tau_1 N}}}$$

Problem 5 – Continued

The following PSPICE input file gives the results plotted below.

```
Homework 4, Problem 5 -In/Out VCO Phase Noise, Transfer Function
.PARAM N=15000, KVCO=157.1E6, T1=2E-5, T2=5E-6, KD=2.2, E=0.001
*Input Phase Noise
vphasenoise 1 0 ac 1.0
R1 1 0 10k
EPN 2 0 freq {v(1)} = (1,-40,0) (10,-70,0) (100,-100,0)
+(1E5,-160,0) (1E6,-160,0)
RPN 2 0 10k
*VCO Noise Transfer Function
EDPLL1 3 0 LAPLACE {V(1)}=
+{S*(T1*S+1)/(S*S*T1+KD*KVCO*T2/N*S+S+KD*KVCO/N)}
RDPLL1 3 0 10K
*VCO Noise at the Output
EDPLL2 4 0 LAPLACE {V(2)}=
+{S*(T1*S+1)/(S*S*T1+KD*KVCO*T2/N*S+S+KD*KVCO/N)}
RDPLL2 4 0 10K
*Reference Noise Transfer Function
EDPLL3 5 0 LAPLACE {V(1)}=
+{KD*KVCO*(1+T2*S)/(S*S*T1+S+KD*KVCO/N*S+KD*KVCO/N)/N}
RDPLL3 5 0 10K
*Reference Noise at the Output
EDPLL4 6 0 LAPLACE {V(2)}=
+{KD*KVCO*(1+T2*S)/(S*S*T1+S+KD*KVCO/N*S+KD*KVCO/N)/N}
RDPLL4 6 0 10K
*Steady state AC analysis
.AC DEC 20 1 1000K
.PRINT AC VDB(2) VDB(3) VDB(4) VDB(5) VDB(6)
.PROBE
.END
```

VCO Output Noise (and Reference Output Noise):