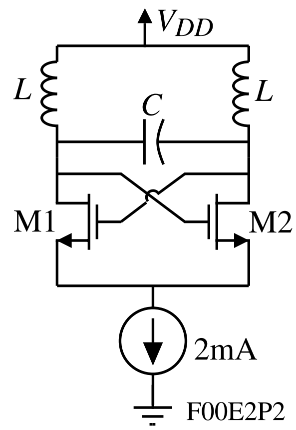
Homework Assignment No. 6 - Solutions

Problem 1 - (10 points)

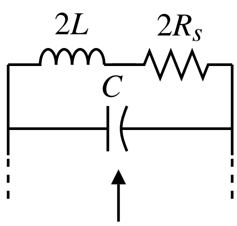
An LC oscillator is shown. The value of the inductors, L, are 5nH and the capacitor, C, is 5pF. If the Q of each inductor is 5, find (a.) the frequency of oscillation, (b.) the value of negative resistance that should be available from the cross-coupled, source-coupled pair (M1 and M2) for oscillation and (c.) design the W/L ratios of M1 and M2 to realize this negative resistance.



F00E2P2

Solution

(a.) The equivalent circuit seen by the negative resistance circuit is:



F00E2S2A

The frequency of oscillation is given as $1/\sqrt{2LC}$ or $\omega_o = 2\pi \text{x} 10^9$ radians/sec.

Therefore the series resistance, R_s, is found as

$$R_s = \frac{\omega L}{Q} = \frac{2\pi \text{x} 10^9 \cdot 5\text{x}10^{-9}}{5} = 2\pi\ \Omega$$

Converting the series impedance of $2L$ and $2R_s$ into a parallel impedance gives,

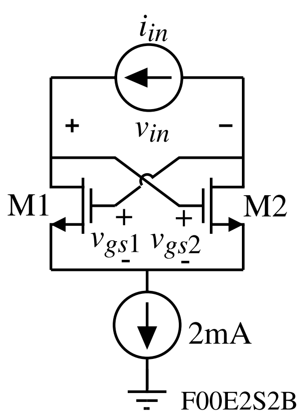
$$Y = \frac{1}{2R_s + j\omega 2L} = \frac{0.5}{R_s + j\omega L} \cdot \frac{R_s - j\omega L}{R_s - j\omega L} = \frac{0.5R_s}{R_s{}^2 + \omega^2 L^2} - j\frac{0.5\omega L_s}{R_s{}^2 - \omega^2 L^2}$$

The reciprocal of the conductance is the parallel resistance, R_p, given as

$$R_p = \frac{R_s{}^2 + \omega^2 L^2}{0.5R_s} = \frac{4\pi^2 + 4\pi^2 \cdot 25}{\pi} = 4\pi(26) = 326.7\Omega$$

$\therefore\ \underline{\underline{R_{neg} = -104\pi\ \Omega = -326.7\Omega}}$

(b.) The negative resistance seen by the RLC circuit is found as follows.



F00E2S2B

$$i_{in} = g_{m1}v_{gs1} - g_{m2}v_{gs2} = g_m(v_{gs1} - v_{gs2}) = -\ g_m v_{in}$$

$$\therefore\ R_{in} = \frac{-1}{g_m}$$

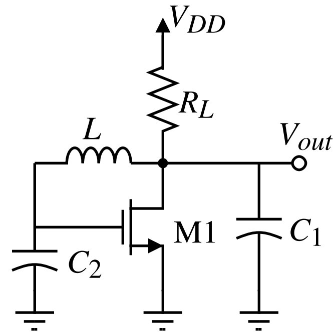
Assuming the 2mA splits evenly between M1 and M2 for the negative resistance calculation gives,

Thus, $g_m = g_{m1} = g_{m2} = \dfrac{1}{104\pi} = \sqrt{2\text{mA} \cdot 110\text{x}10^{-6}\ (W/L)} = \dfrac{\sqrt{W/L}}{2132}$

$\therefore\ W/L = \left(\dfrac{2132}{104\pi}\right)^2 = 42.6 \quad \Rightarrow \quad \underline{\underline{W/L = 42.6}}$

Problem 2 - (10 points)

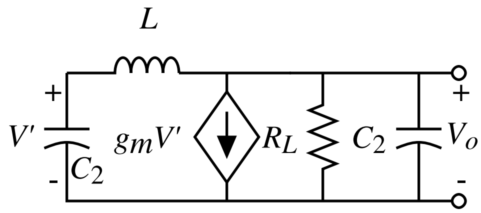
An LC oscillator is shown. Find an expression for the frequency of oscillation and the value of g_mR_L necessary for oscillation. Assume that the output resistance of the FET, r_{ds}, can be neglected.



F99E2P4

Solution

An open-loop, small-signal model of this oscillator is shown below.



F99P3S2

Writing a nodal equation at the output and input gives,

$$g_m V' + G_L V_o + sC_2 V_o + \frac{V_o - V'}{sL} = 0 \quad \text{and} \quad \frac{V_o - V'}{sL} = sC_2 V' \quad \rightarrow \quad V_o = V'(1+s^2LC_2)$$

$$\therefore \quad g_m V' + \left(G_L + sC_1 + \frac{1}{sL}\right)(1+s^2LC_2)V' - \frac{V'}{sL} = 0$$

Assuming a non-zero value of V' gives,

$$g_m + \left(G_L + sC_1 + \frac{1}{sL}\right)(1+s^2LC_2) - \frac{1}{L} = g_m + G_L + sC_1 + \frac{1}{sL} + s^2LC_2G_L + s^3LC_1C_2 + sC_2 - \frac{1}{sL} = 0$$

or $\quad (g_m + G_L - \omega^2 LG_LC_2) + j\omega[C_1 - \omega^2 LC_1 + C_2] = 0$

Therefore, the frequency of oscillation is,

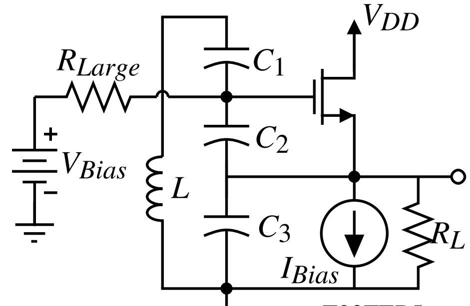
$$\boxed{\omega_{osc} = \sqrt{\frac{C_1+C_2}{LC_1C_2}} = \frac{1}{\sqrt{\frac{LC_1C_2}{C_1+C_2}}}}$$

The value of g_mR_L necessary for oscillation is

$$g_m + G_L = LG_LC_2\omega_{osc}^2 = G_L\left(1 + \frac{C_2}{C_1}\right) \quad \rightarrow \quad \boxed{g_mR_L = \frac{C_2}{C_1}}$$

Problem 3 – (10 points)

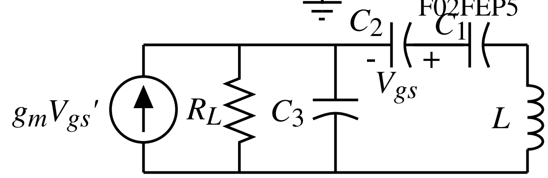
A Clapp oscillator which is a version of the Colpitt's oscillator is shown. Find an expression for the frequency of oscillation and the value of g_mR_L necessary for oscillation. Assume that the output resistance of the FET, r_{ds}, and R_{Large} can be neglected (approach infinity).



F02FEP5

Solution

The small-signal model for this problem is shown below.



F02FES5

The loop gain will be defined as V_{gs}/V_{gs}'.
Therefore,

$$V_{gs} = \frac{-g_mV_{gs}'\,R_L\|(1/sC_3)}{R_L\|(1/sC_3)+\frac{1}{sC_1}+\frac{1}{sC_2}+sL}\left(\frac{1}{sC_2}\right)$$

$$= \frac{-g_mV_{gs}'\,\frac{R_L(1/sC_3)}{R_L+(1/sC_3)}\,\frac{1}{sC_2}}{\frac{R_L(1/sC_3)}{R_L+(1/sC_3)}+\frac{1}{sC_1}+\frac{1}{sC_2}+sL}$$

$$T(s) = \frac{V_{gs}}{V_{gs}'} = \frac{\frac{-g_m R_L}{sR_LC_3+1}\,\frac{1}{sC_2}}{\frac{R_L}{sR_LC_3+1}+\frac{1}{sC_1}+\frac{1}{sC_2}+sL} = \frac{\frac{-g_m R_L}{sC_2}}{R_L+(sR_LC_3+1)\left(\frac{1}{sC_1}+\frac{1}{sC_2}+sL\right)}$$

$$T(s) = \frac{-g_m R_L}{sC_2R_L+(sR_LC_3+1)(s^2LC_2+\frac{C_2}{C_1}+1)}$$

$$T(s) = \frac{-g_m R_L}{sC_2R_L+s^3R_LC_3LC_2+sR_L\frac{C_2C_3}{C_1}+sC_3R_L+s^2LC_2+\frac{C_2}{C_1}+1}$$

$$T(j\omega) = \frac{-g_m R_L}{[1+\frac{C_2}{C_1}-\omega^2LC_2]+j\omega[R_L(C_2+C_3)+R_L\frac{C_2C_3}{C_1}-\omega^2R_LC_3LC_2]} = 1 + j0$$

$$\therefore \quad C_2+C_3+\frac{C_2C_3}{C_1} = \omega_{osc}{}^2C_3LC_2 \quad \rightarrow \quad \boxed{\omega_{osc} = \sqrt{\frac{1}{L}\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)}}$$

Also, $g_m R_L = \omega_{osc}{}^2LC_2 - 1 - \frac{C_2}{C_1} = C_2\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)-\frac{C_2}{C_1}-1 = \frac{C_2}{C_3} \rightarrow$

$$\boxed{g_m R_L = \frac{C_2}{C_3}}$$

Problem 4 – (50 points maximum)

The objective of this problem is to use passive LC tank and negative feedback circuit to design an LC oscillator that meets the GSM specification. At first, show the condition that the ideal circuit oscillates at $\omega_{osc} = \frac{1}{\sqrt{LC}}$ and find quality factor, Q. The transistors should be modeled with the standard small-signal model using g_m and r_{ds} or r_{out} in this part of the problem. Second, use SPICE to obtain a transient simulation. Third, simulate the oscillator that replaces the ideal inductor with the lumped inductor model shown, and use the program referenced below [1] to layout the inductor. Use the model parameters given in [2] for this problem.

Fig.1. Ideal LC VCO

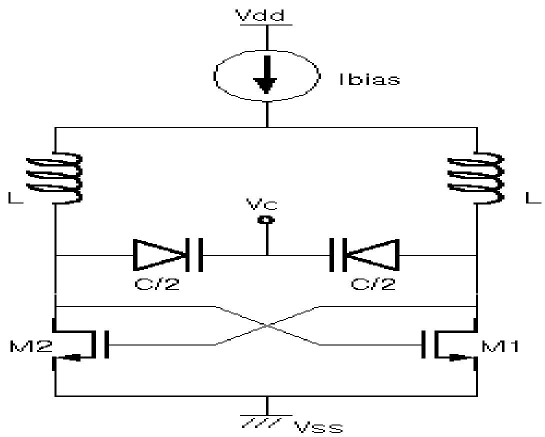
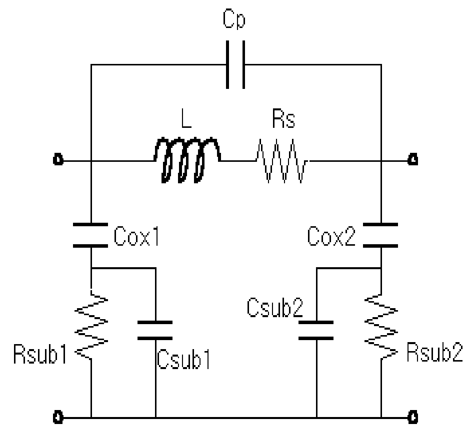


Fig.2. Lumped Inductor Model



GSM specifications:

Frequency range = 935 ~ 960MHz v_c = 0.75 ~ 1.75V

Switching time = 800μsec V_{DD} = 2.5V

Technology parameter:

Metal sheet resistance = 35 mΩ/sq.

Substrate layer resistivity = 0.015 Ω-cm

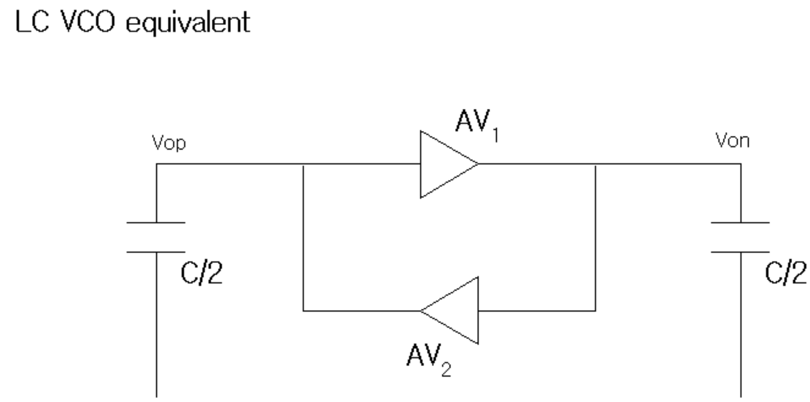
Metal to substrate capacitance = 5.91 aF/μm^2

Metal to metal capacitance = 98.0 aF/μm

C_{sub}, R_{sub}, C_p can be ignored

Problem 4 - Continued

Solution



$A_{v1} = A_{v2} = G_m * (r_{out} \,||\, L \,||\, C/2)$

$A_{v1}(s) = A_{v2}(s) = G_m \dfrac{1}{\dfrac{1}{r_{out}} + \dfrac{1}{Ls} + \dfrac{C}{2}s} = \text{Gm } s L \dfrac{1}{1 + s\dfrac{L}{r_{out}} + s^2 L \dfrac{C}{2}}$ where G_m and r_{out} are the transconductance and the output resistance of M1 and M2 transistors respectively, C/2 is the total output capacitance at the outputs V_{on} and $V_{op.}$ For this circuit to oscillate, the gain around the loop must be equal to negative one; therefore, each cross coupled gain stage can be presented as shown below. Hence, the total gain equation around the loop is equal to $H(s) = \dfrac{A_{v1}(s)}{1 - A_{v1}(s)}$.

Substituting for $A_{v1}(s)$,

$$H(s) = \frac{\dfrac{\text{Gm } s L}{1 + s\dfrac{L}{r_{out}} + s^2 L \dfrac{C}{2}}}{1 - \dfrac{\text{Gm } s L}{1 + s\dfrac{L}{r_{out}} + s^2 L \dfrac{C}{2}}} = \frac{G_m s L}{1 - s L (G_m - G_{out}) + s^2 L \dfrac{C}{2}}$$

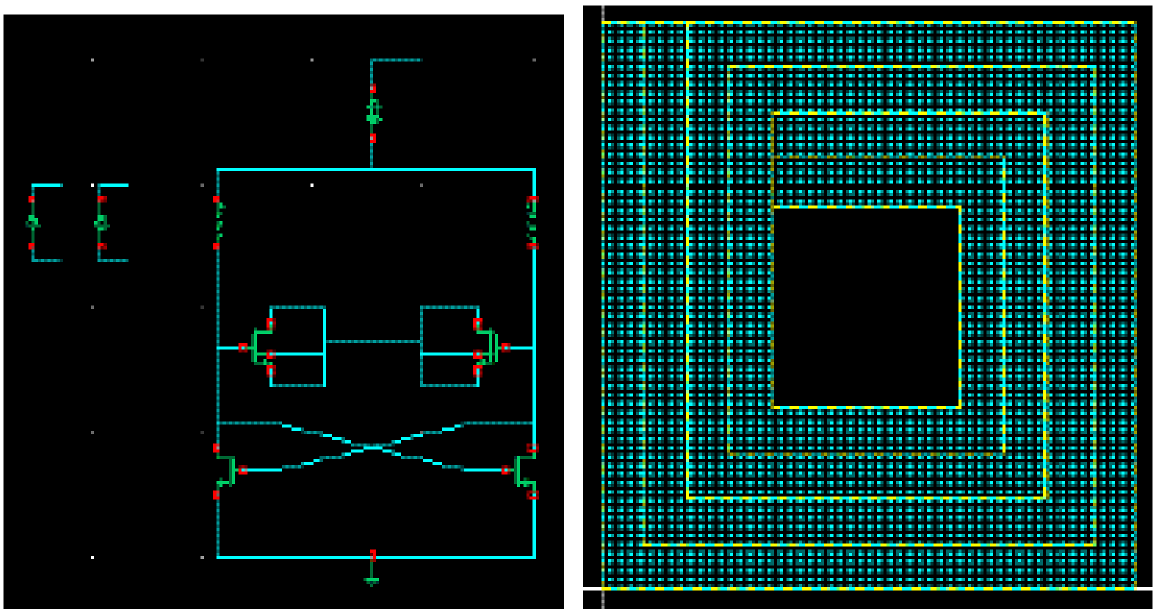
For this circuit to oscillate at $\omega_{osc} = \dfrac{1}{\sqrt{L\dfrac{C}{2}}}$, it is necessary for the s term in the denominator to be equal to zero; hence, $G_m = \dfrac{1}{r_{out}} = G_{out}$

By forcing G_m greater than Gout a pair of complex poles are forced in the right side plane. This is the condition to start oscillation. Once the oscillation starts, the G_{eff} parameter (G_{eff} = $G_m - G_{out}$) approaches zero and the oscillation becomes sustaining, giving a $Q = \dfrac{\sqrt{L\dfrac{C}{2}}}{L\, G_{eff}} = \sqrt{\dfrac{C}{2L}} R_{eff}$

<u>Problem 4 – Continued</u>

Inductor Layout:

LC VCO schematic and inductor layout



Each element size

Negative resistance transistor(M1 and M2): L=.25um, W=25um, M=20

Accumulation Capcitor(C1 and C2): L=.5um, W=345um, M=50

Ideal inductor size: .865nH

Inductor's layout figure: turns=2, spacing=.5um, width=40um, diameter=250um

Inductor's parasitic resistor, R_s=1.186 Ohms (from layout and its technology parameter)

Inductor's parasitic cap, C_{ox1} and C_{ox2}=160.22fF (from layout and its technology parameter)

Current source: 1mA, Tunning voltage: 0.75~1.75