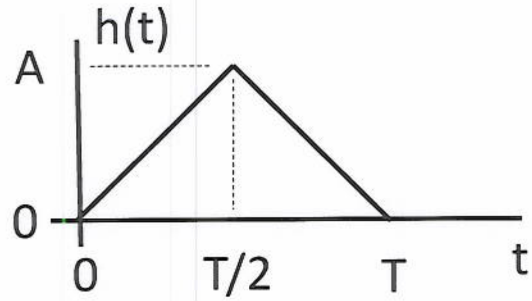
GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Quiz - Fall 2014
ECE 4601: Communication Systems

Aids Allowed: Course text, calculator
Attempt all questions
Questions are of equal value

DATE: Thursday October 2, 2014.
TIME: 12:05pm - 1:25pm
INSTRUCTOR: Prof. G.L. Stüber

1) Matched Filters: Consider the matched filter $h(t)$ shown below



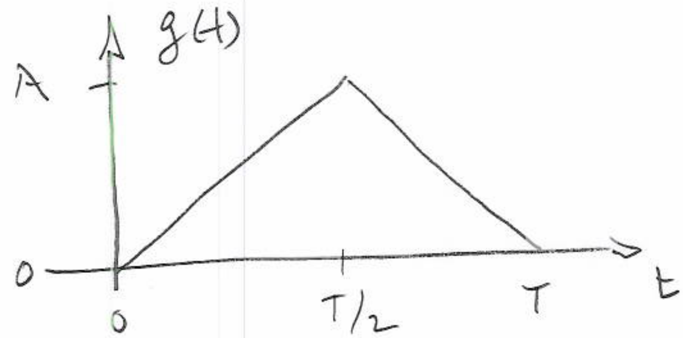
a) 3 marks: Sketch the pulse $g(t)$ that the filter $h(t)$ is matched to.

b) 2 marks: What is the energy of the pulse $g(t)$ in terms of A and T?

c) 3 marks: Construct a pulse $p(t)$ that is *orthogonal* to $g(t)$ and having the same energy as $g(t)$. *The solution is not unique.*

d) 2 marks: The pulse $g(t)$ is applied to input of the filter $q(t)$ that is matched to $p(t)$, and the output is sampled at time T. What is its value?

a)



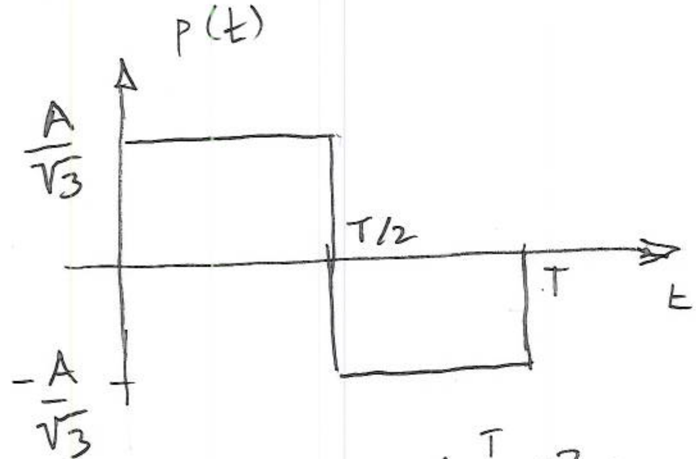
b)

$$E = \int_0^T g^2(t)\,dt = 2\int_0^{T/2} \left(\frac{2At}{T}\right)^2 dt$$

$$= 2\cdot\frac{4A^2}{T^2}\,\frac{t^3}{3}\Big|_0^{T/2} = \frac{8A^2}{T^2}\cdot\frac{T^3}{3\cdot 8}$$

$$= \frac{A^2T}{3}$$

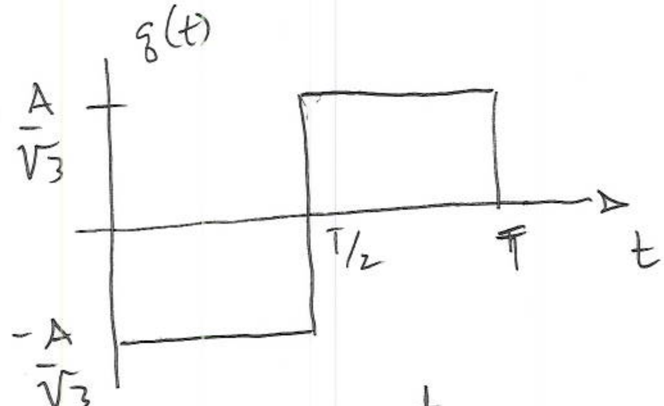
c)



$$E = \int_0^T \frac{A^2}{3} dt = \frac{A^2 T}{3}$$

$$\int_0^T g(t)\, p(t) = 0$$

d)



$$y(t) = \int_0^t g(\tau)\, g(t-\tau)\, d\tau$$

$$y(T) = \int_0^T g(\tau)\, g(T-\tau)\, d\tau$$

$$= \int_0^T g(\tau)\, p(\tau)\, d\tau$$

$$= 0$$

2) Random Processes: Suppose that white Gaussian noise process $X(t)$ having a power spectral density of $N_o/2$ watts/Hz is input to a linear time invariant filter $h(t)$, having transfer function $H(f)$, and the output is $Y(t)$. The output power spectral density is observed to be

$$\Phi_{YY}(f) = \frac{N_o a}{a^2 + (2\pi f)^2}$$

a) 2 marks What is the output autocorrelation function $\phi_{YY}(\tau)$?

b) 3 marks Find the filter transfer function $H(f)$ and impulse response $h(t)$.

c) 2 marks What is the total power in the output process $Y(t)$?

d) 3 marks What is the joint probability density function of the random variables $Y_1 = Y(t_1)$ and $Y_2 = Y(t_1 + 1/a)$?

a)

$$\phi_{YY}(\tau) = \mathcal{F}^{-1}\{\Phi_{YY}(f)\}$$

$$= \frac{N_o}{2} e^{-a|\tau|}$$

b)

$$\Phi_{YY}(f) = \Phi_{XX}(f)\,|H(f)|^2$$

$$= \frac{N_o}{2} \cdot |H(f)|^2 = \frac{N_o a}{a^2 + (2\pi f)^2}$$

$$|H(f)|^2 = \frac{2a}{a^2 + (2\pi f)^2} = H(f) H^*(f)$$

$$= \frac{\sqrt{2a}}{a + j2\pi f} \cdot \frac{\sqrt{2a}}{a - j2\pi f}$$

$$H(f) = \frac{\sqrt{2a}}{a + j2\pi f} \qquad h(t) = \sqrt{2a}\, e^{-at} u(t)$$

c) $P_t = \phi_{yy}(0) = \frac{N_0}{2}$

d) Since the input is Gaussian y_1 and y_2 have a joint Gaussian density

$$\mu_{y_1} = \mu_{y_2} = 0$$

$$\sigma_{y_1}^2 = \sigma_{y_2}^2 = \phi_{yy}(0) = \frac{N_0}{2}$$

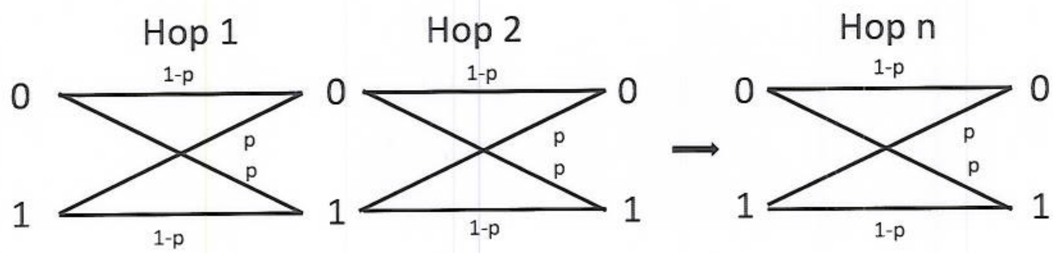
$$\mu_{y_1 y_2} = E[Y_1 Y_2]$$

$$= \phi_{yy}(1/a) = \frac{N_0}{2} e^{-1}$$

$$\Lambda = \frac{N_0}{2} \begin{bmatrix} 1 & e^{-1} \\ e^{-1} & 1 \end{bmatrix}$$

$$f_{Y_1 Y_2}(y_1, y_2) = \frac{1}{\pi N_0 (1 - e^{-2})} \exp\left\{ - \frac{y_1^2 - 2e^{-1} y_1 y_2 + y_2^2}{N_0 (1 - e^{-2})} \right\}$$

3) Error Probability: A multi-hop communication system consists of the serial concatenation of n binary symmetric channels each having crossover probability p as shown below, where the number of hops, n, is odd.



a) 5 marks: What is the end-to-end probability of bit error in terms of n and p?

b) 3 marks: Evaluate the expression obtained in part a) with $n = 5$ and $p = 0.01$.

c) 2 marks: Suppose that each hop uses on-off keying, where a "1" is transmitted as a pulse $g(t)$ and a "0" is transmitted as nothing. What is the required E_b/N_o in decibel (dB) units required to give $p = 0.01$?

a) P_b = Prob (odd number of hops in error)

$$= \sum_{\substack{i=1 \\ i \text{ odd}}}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

b)

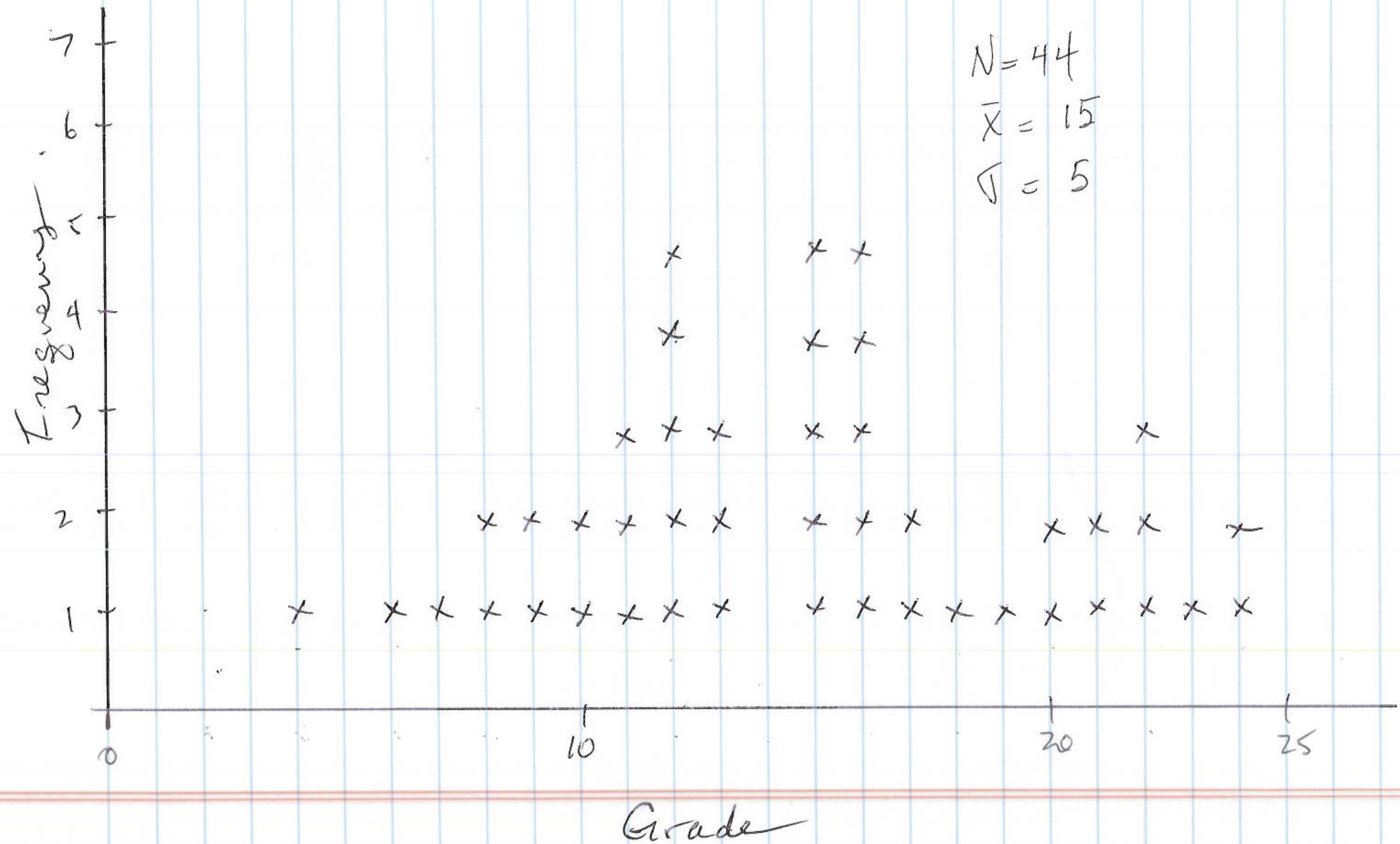
$$P_b = \binom{5}{1} p(1-p)^4 + \binom{5}{3} p^3(1-p)^2 + \binom{5}{5} p^5$$

$$= 5p(1-p)^4 + 10p^3(1-p)^2 + p^5$$

$$= 4.804 \times 10^{-2}$$

c) $0.01 = Q\left(\sqrt{\frac{E_b}{N_o}}\right) \Rightarrow \sqrt{\frac{E_b}{N_o}} = 2.33$ $\quad \frac{E_b}{N_o} = 5.429$

$\frac{E_b}{N_o} = 7.35\text{ dB}$



Frequency
1
2
3
4
5
6
7
0
10
20
25
Grade
N = 44
X̄ = 15
σ = 5