GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Quiz II - Fall, 2014
EE 4601: Communication Systems

Aids Allowed: Text, $8\frac{1}{2} \times 11$ crib sheet (one side), math tables, calculator

Attempt all 3 questions

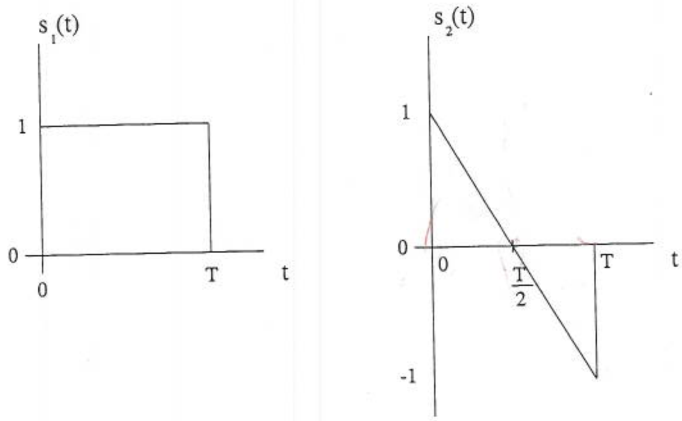
Questions are of equal value

DATE: Tuesday November 18, 2014.

TIME: 12:05 - 1:25

INSTRUCTOR: G.L. Stüber

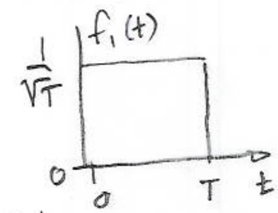
1) **Signal Space:** Suppose that the two waveforms $s_1(t)$ and $s_2(t)$ shown below are used for binary signaling on an additive white Gaussian noise channel.

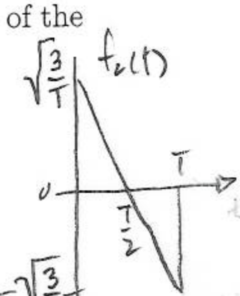


a) **2 marks:** Obtain a complete set of orthonormal basis functions, $\{f_i(t)\}_{i=1}^{N}$, for the signal set $\{s_1(t), s_2(t)\}$.

b) **3 marks:** Sketch the signal-space diagram and specify the signal vectors $\mathbf{s}_1$ and $\mathbf{s}_2$.

c) **5 marks:** Obtain an expression for the probability of *bit* error, P_b, in terms of the received *bit* energy-to-noise ratio, E_b/N_o.



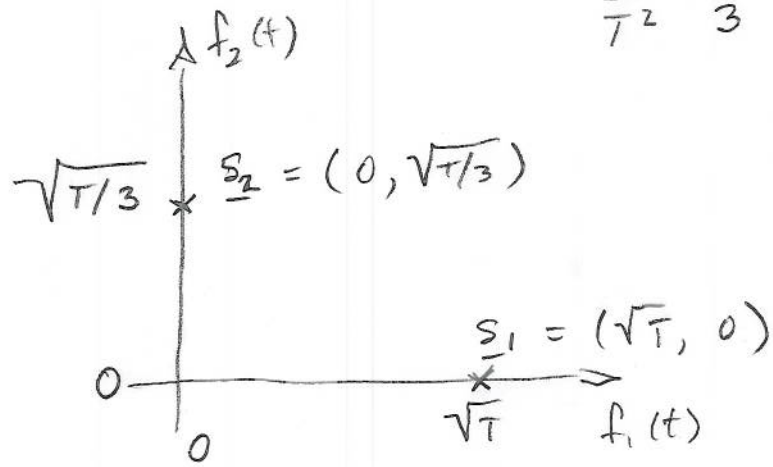


a) $f_1(t) = \dfrac{s_1(t)}{\sqrt{E_1}}$; $E_1 = T$

$f_2(t) = \dfrac{s_2(t)}{\sqrt{E_2}}$; $E_2 = 2\int_0^{T/2} \left(\frac{2t}{T}\right)^2 dt$

$= \dfrac{8}{T^2} \dfrac{t^3}{3} \Big|_0^{T/2} = \dfrac{T}{3}$

b)



$$P_b = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)$$

$$d_{12}^2 = T + \frac{T}{3} = \frac{4T}{3}$$

However,

$$E_b = \frac{1}{2}\cdot T + \frac{1}{2}\cdot\frac{T}{3} = \frac{2T}{3}$$

so $d_{12}^2 = 2E_b$

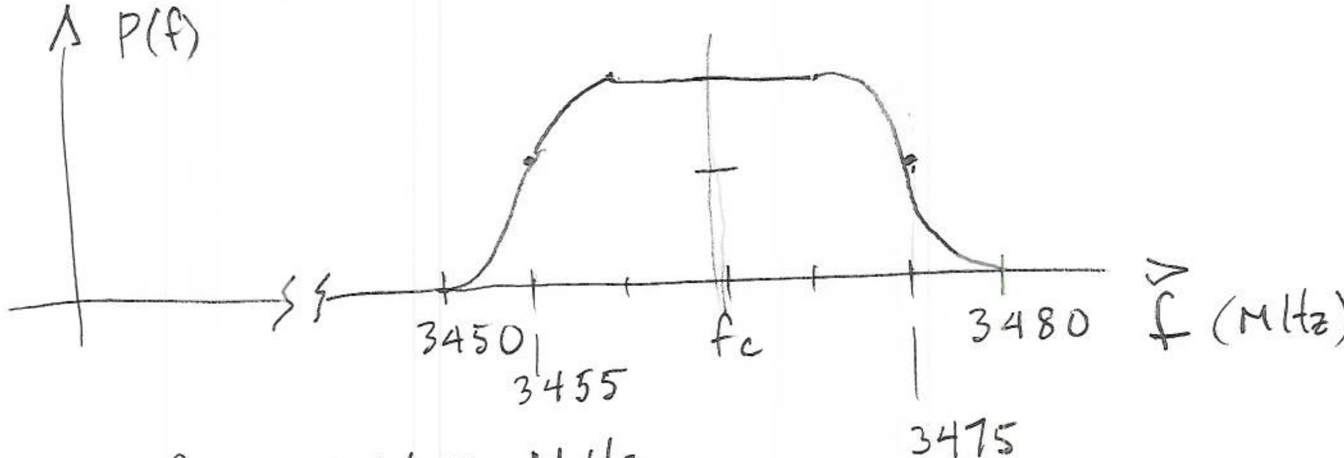
and $P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

2) Pulse Shaping: An available wireless channel passes the frequencies in the band from 3450 MHz to 3480 MHz. It is desired to design a modem that transmits information at a symbol rate of 20 Mbaud (20×10^6 symbols/second) and a bit rate of 80 Mbps (80×10^6 bits/second).

a) 6 marks: Select an appropriate QAM signal constellation size M, carrier frequency f_c, and roll-off factor β of a spectral raised cosine pulse that uses the entire frequency band.

b) 4 marks: Based on your results in part a) what are the *complex-valued* frequency responses of the transmit filter and receiver matched filters that you would implement?

a)



$$f_c = 3465 \text{ MHz}$$

$$f_N = \frac{R}{2} = 10 \text{ MHz}$$

$$\beta = \frac{f_x}{f_N} = \frac{5 \text{ MHz}}{10 \text{ MHz}} = 0.5$$

$$M = 16 \quad (4 \text{ bits/symbol})$$

b) The spectrum of the root raised cosine amplitude shaping pulse centered at $t = 0$ is $G_0(f)$, where $W = 1/2T$ and

$$\sqrt{2W}\,|G_0(f)| = \begin{cases} 1 & , 0 \le |f| \le W - f_x \\ \sqrt{\frac{1}{2}\left(1 - \sin \frac{\pi(|f| - W)}{2f_x}\right)} & , W - f_x \le |f| \le W + f_x \\ 0 & , |f| > W + f_x \end{cases}$$

To "implement" the pulse, we require a pulse that is causal. One possibility applies a rectangular window to the corresponding time domain pulse and time-shifting yields

$$g(t) = g_0(t - LT/2)\,\mathrm{rect}\left(\frac{t - LT/2}{LT}\right)$$

The corresponding $G(f)$ is given by

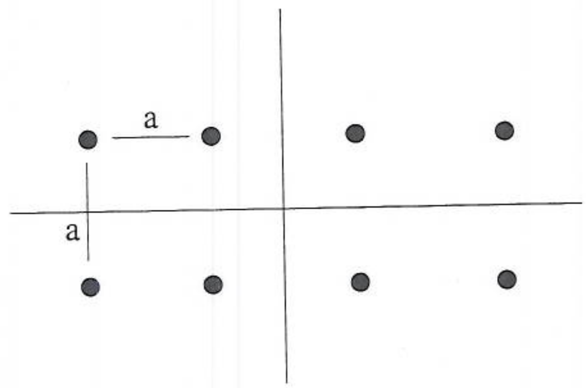
$$G(f) = \left[G_0(f) * \mathrm{sinc}(fLT)\right] e^{-j\pi f LT}$$

so

$$|G(f)| = |G_0(f) * \mathrm{sinc}(fLT)|$$

$$\angle G(f) = -\pi f LT$$

3) **Signal Space:** A 8-PAM signal constellation is shown in the figure below. Assume the symbols are used with equal probability and the receiver makes minimum distance decisions.

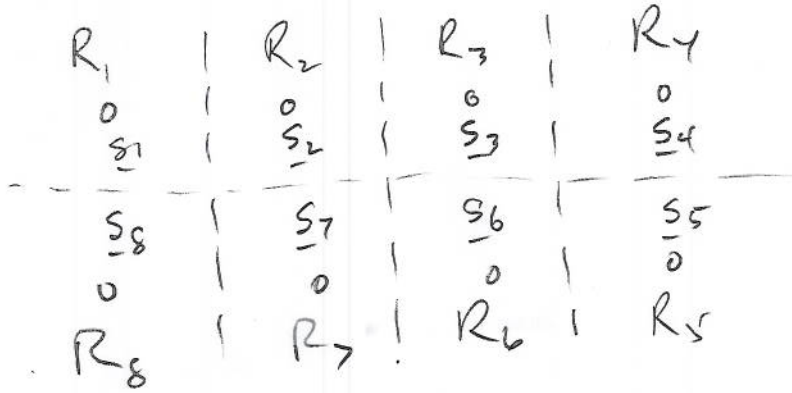


a) **2 marks:** Determine and sketch the decision boundaries and identify the Voronoi regions, R_i.

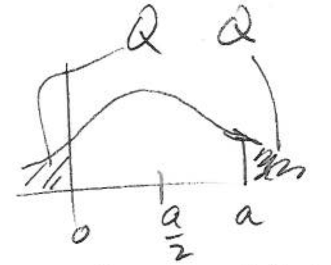
b) **6 marks:** Find the *exact* probability of 8-PAM *symbol* error, P_M, in terms of the received *bit* energy-to-noise ratio E_b/N_o.

c) **2 marks:** By using the minimum distance, $d_{\min}$, between signal vectors in the signal constellation, and relating $d_{\min}$ to the energy per bit, E_b, obtain a simple upper bound on the probability of *symbol* error, P_M, in terms of the received *bit* energy-to-noise ratio, E_b/N_o.

a)



b) There are two cases.



Let $S_1 = \{\underline{s}_1, \underline{s}_4, \underline{s}_5, \underline{s}_8\}$

$$P_{c|S_1} = (1-Q)^2 = 1-2Q+Q^2$$

$$S_2 = \{\underline{s}_2, \underline{s}_3, \underline{s}_6, \underline{s}_7\}$$

$$P_{c|S_2} = (1-Q)(1-2Q) = 1-3Q+2Q^2$$

$$P_c = \frac{1}{2} P_{c|S_1} + \frac{1}{2} P_{c|S_2} = 1 - \frac{5}{2}Q + \frac{3Q^2}{2}$$

$$Q = Q\left(\frac{a}{2\sigma}\right)$$

$$\sigma^2 = \frac{N_0}{2}$$

$$P_M = 1 - P_c = \frac{5}{2}Q - \frac{3Q^2}{2}$$

However,

$$E_s = \frac{1}{2}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right] + \frac{1}{2}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{3a}{2}\right)^2\right]$$

$$= \frac{a^2}{4} + \frac{5a^2}{4} = \frac{6a^2}{4} = \frac{3a^2}{2}$$

$$E_b = E_s/3 = a^2/2$$

$$Q = Q\left(\sqrt{\frac{a^2}{2N_0}}\right) = Q\left(\sqrt{E_b/N_0}\right)$$

$$P_M = \frac{5}{2}Q - \frac{3Q^2}{2}$$

c) $$P_M < (M-1)\,Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right) = 7Q\left(\sqrt{E_b/N_0}\right)$$

The above follows from the union bound

$N = 42$

$\bar{X} = 17.45/25 \quad 70\%$

$\sigma_X = 5.1$