EE4601
Communication Systems

Week 1

Introduction to Digital Communications
Channel Capacity

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Introduction

Digital communications is the exchange of information using a *finite* set of signal waveforms. This is in contrast to analog communication (e.g., AM/FM radio) which do not use a finite set of signals.
Why use digital communications?

- Natural choice for digital sources, e.g., computer communications.
- Source encoding or data compression techniques can reduce the required transmission bandwidth with a controlled amount of message distortion.
- Digital signals are more robust to channel impairments than analog signals.
  - noise, co-channel and adjacent channel interference, multipath-fading.
  - surface defects in recording media such as optical and magnetic disks.
- Higher bandwidth efficiency than analog signals.
- Data encryption and multiplexing is easier.
- Benefit from well known digital signal processing techniques.

Protocol Stack (3G cdma2000 EV-DO)

Overview 3GPP2 C.S0024 Ver 4.0

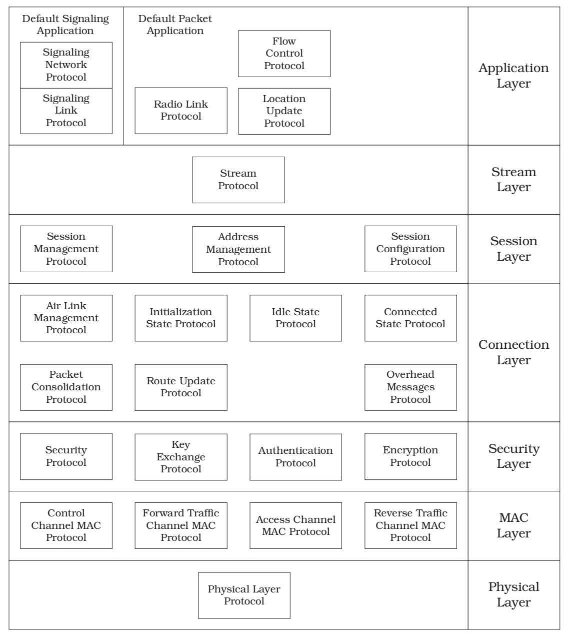
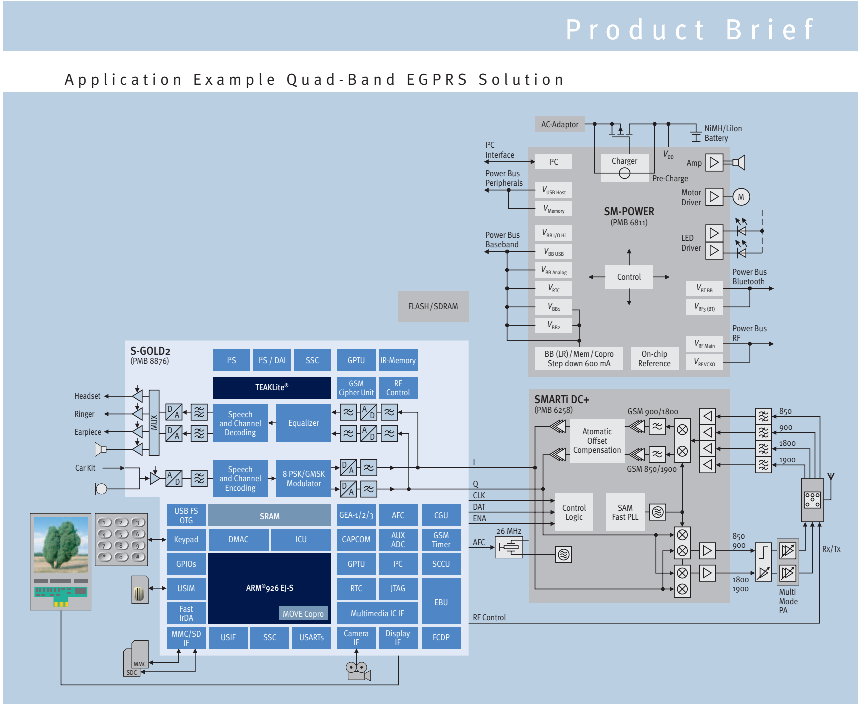


Figure 1.6.6-1. Default Protocols

- This course concentrates on the Physical Layer or PHY Layer.

Cellular Radio Chipset



- This course concentrates on the digital baseband; baseband modulation/demodulation.

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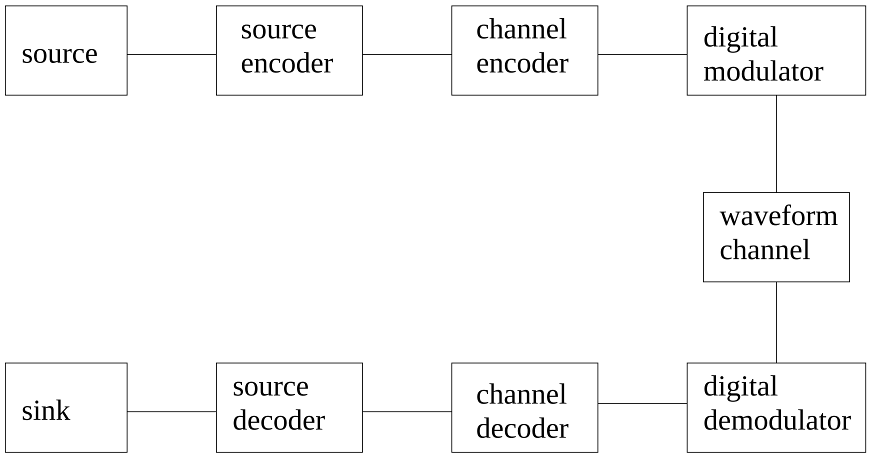
Course Objectives

1. Brief review of probability and introduction to random processes.
   - message waveforms, physical channels, noise and interference are all random processes.
2. Mathematical modelling and characterization of physical communication channels, signals and noise.
3. Design of digital waveforms and associated receiver structures for recovering channel-corrupted digital signals.
   - emphasis will be on waveform design, receiver processing, and performance analysis for "additive white Gaussian noise (AWGN) channels."
   - mathematical foundations are essential for effective physical layer modelling, waveform design, receiver design, etc.
   - communication signal processing is a key element of this course. Our focus will be on the "digital baseband" and not the "analog RF."

Basic Digital Communication System

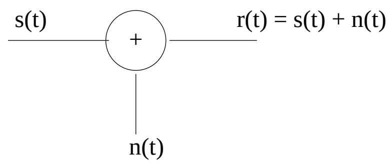


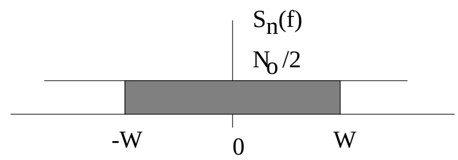
Some Types of Waveform Channels

- wireline channels, e.g., twisted copper pair, coaxial cable, power line
- fiber optic channels (optical communication is not considered in this course)
- wireless (radio) channels
  - line-of-sight (satellite, land microwave radio)
  - non-line-of-sight (cellular, wireless LAN, BAN, PAN)
- underwater acoustic channels (submarine communication)
- storage channels, e.g., optical and magnetic disks.
  - communication from the present to the future.

Mathematical Channel Models

Additive White Gaussian Noise Channel (AWGN):





Receiver thermal noise can be modeled as spectrally flat or "white."

Thermal noise power in bandwith W is

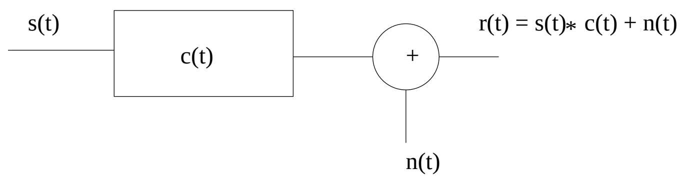
$$\frac{N_o}{2} \cdot 2 \cdot W = N_o W \quad \text{Watts}$$

At any time instant t_0, the noise waveform $n(t_0)$ is a Gaussian random variable with zero mean and variance N_oW, $n(t_0) \sim N(0, N_oW)$.

For a given channel input $s(t_0)$, the channel output $r(t_0)$ is also a Gaussian random variable with mean $s(t_0)$ and variance N_oW, $n(t_0) \sim N(s(t_0), N_oW)$.

Mathematical Channel Models

Linear Filter Channel:



An *ideal channel* has *impulse response* $c(t) = \alpha\delta(t - t_0)$ and, therefore,

$$r(t) = \alpha s(t - t_o) + n(t)$$

An ideal channel only attenuates and delays a signal, but otherwise leaves it undistorted. The channel *transfer function* is

$$C(f) = \mathcal{F}[c(t)] = \alpha e^{-j2\pi f t_o}, \quad |f| < B$$

where B is the system bandwidth.

- The *magnitude response* $|C(f)| = \alpha$ is flat in frequency f.
- The *phase response* $\angle C(f) = -2\pi f t_o$ is linear in frequency f.

Mathematical Channel Models

Two-Ray Multi-path Channel:

Suppose $r(t) = \alpha s(t) + \beta s(t - \tau)$.

Since $r(t) = s(t) * c(t)$, we have $c(t) = \alpha\delta(t) + \beta\delta(t - \tau)$.

Hence $C(f) = \alpha + \beta e^{-j2\pi f\tau}$.

Using $|C(f)|^2 = C(f)C^*(f)$, we can obtain

$$|C(f)| = \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta\cos(2\pi f\tau)}$$

Using the Euler identity, $e^{j\theta} = \cos(\theta) + j\sin(\theta)$ in $C(f)$ above, we can obtain

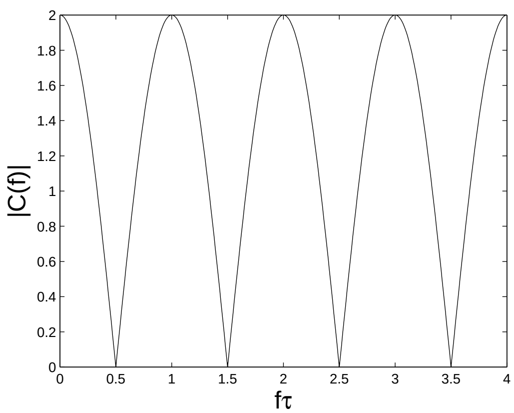
$$\angle(C(f) = -\mathrm{Tan}^{-1}\frac{\beta\sin(2\pi f\tau)}{\alpha + \beta\cos(2\pi f\tau)}$$

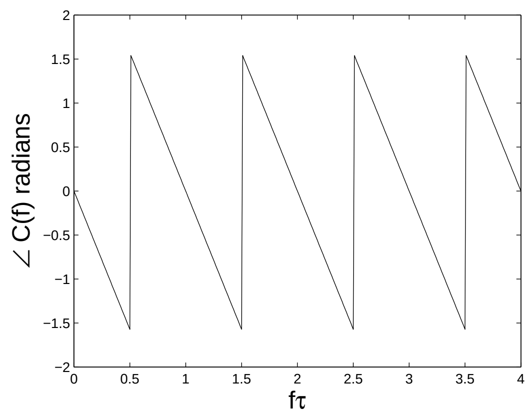
Mathematical Channel Models

Two-Ray Multi-path Channel:

Suppose $\alpha = \beta = 1$. Then

$$\begin{aligned} |C(f)| &= \sqrt{2 + 2\cos(2\pi f \tau)} \\ \angle C(f) &= -\mathrm{Tan}^{-1} \frac{\sin(2\pi f \tau)}{1 + \cos(2\pi f \tau)} \end{aligned}$$





Observe that the multi-path channel is *frequency selective*.

Mathematical Channel Models

Two-Ray Fading Channel:

Suppose we transmit $s(t) = \cos(2\pi f_o t)$ and the received waveform is $r(t) = \alpha \cos(2\pi f_o t) + \beta \cos(2\pi (f_o + f_d)t)$, where f_d is a “Doppler” shift.

$f_d = (v/\lambda_o)\cos(\theta)$, where v is velocity, λ_o is the carrier wavelength, θ is the angle of arrival at the receiver. Note that $c = f_o \lambda_o$, where c is the speed of light.

Using the complex phaser representation of sinusoids, we can write

$$r(t) = A(t)\cos(2\pi f_o t + \phi(t))$$

where

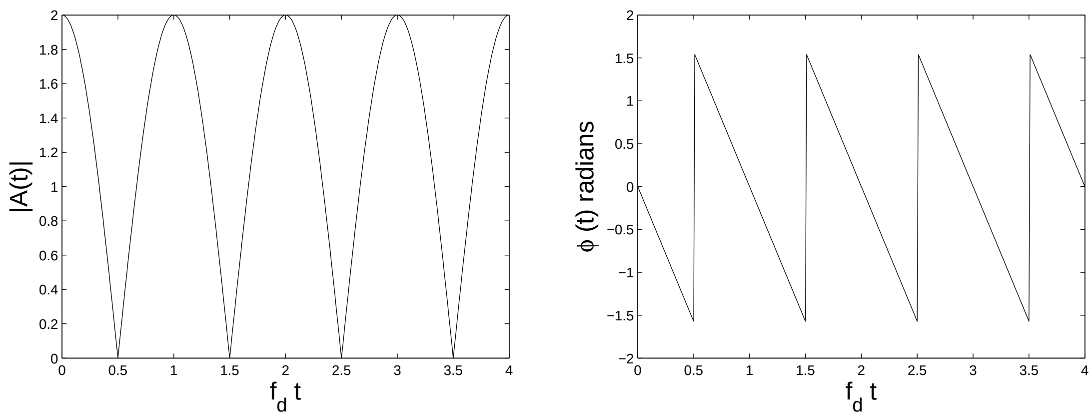
$$\begin{aligned} A(t) &= \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta\cos(2\pi f_d t)} \\ \phi(t) &= -\mathrm{Tan}^{-1}\frac{\beta \sin(2\pi f_d t)}{\alpha + \beta\cos(2\pi f_d t)} \end{aligned}$$

Mathematical Channel Models

Two-Ray Fading Channel:

Suppose $\alpha = \beta = 1$. Then

$$\begin{aligned} A(t) &= \sqrt{2 + 2\cos(2\pi f_d t)} \\ \phi(t) &= -\mathrm{Tan}^{-1} \frac{\sin(2\pi f_d t)}{1 + \cos(2\pi f_d t)} \end{aligned}$$



Observe that the fading channel is *time varying*.

Shannon Capacity of a Channel

Claude Shannon in his paper "A Mathematical Theory of Communication" BSTJ, 1948, proved that every physical channel has a **capacity**, C, defined as the maximum possible rate that information can be transmitted over the channel with an **arbitrary reliability**.

Arbitrary reliability means that the probability of information bit error or bit error rate (BER) can be made as small as desired.

Conversely, information cannot be transmitted reliably over a channel at any rate greater than the channel capacity, C. The BER will be bounded from zero.

The channel capacity depends on the channel impulse response or channel transfer function, and the received bit energy-to-noise ratio (E_b/N_o).

Arbitrary reliability can be realized in practice by using error control coding techniques.

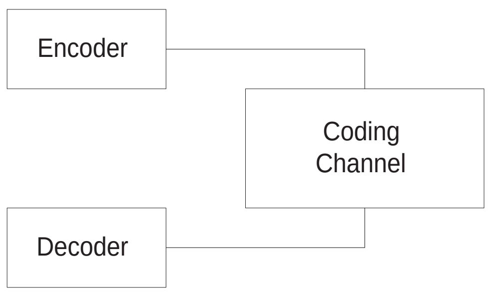
Coding Channel and Capacity

The channel capacity depends only on the *coding channel*, defined as the portion of the communication system that is "seen" by the coding system.

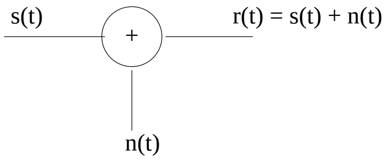
The input to the coding channel is the output of the channel encoder.

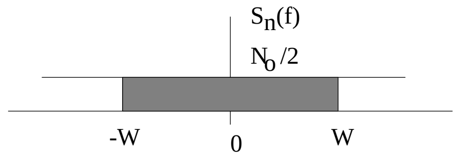
The output of the coding channel is the input to the channel decoder.

In practice, the coding channel inputs are often chosen from a *digital modulation alphabet*, while the coding channel outputs are continuous valued decision variables generated by sampling the corresponding *matched filter outputs* in the receiver.



AWGN Channel Capacity





For the AWGN channel, the capacity is

$$C = W \log_2 \left(1 + \frac{P}{N_o W}\right)$$

$$\begin{aligned} W &= \text{channel bandwidth (Hz)} \\ P &= \text{constrained input signal power (watts)} \\ N_o &= \text{one-sided noise power spectral density (watts/Hz)} \\ N_o/2 &= \text{two-sided noise power spectral density (watts/Hz)} \end{aligned}$$

Capacity of the AWGN Channel

Dividing both sides by W

$$\frac{C}{W} = \log_2\left(1 + \frac{P}{N_o W}\right) = \log_2\left(1 + \frac{E_b}{N_o} \cdot \frac{R}{W}\right)$$

$$\begin{aligned}
R &= 1/T = \text{data rate (bits/second)} \\
E_b &= \text{energy per data bit (Joules)} = PT \\
E_b/N_o &= \text{received bit energy-to-noise spectral density ratio (dimensionless)} \\
R/W &= \text{bandwidth efficiency (bits/s/Hz)}
\end{aligned}$$

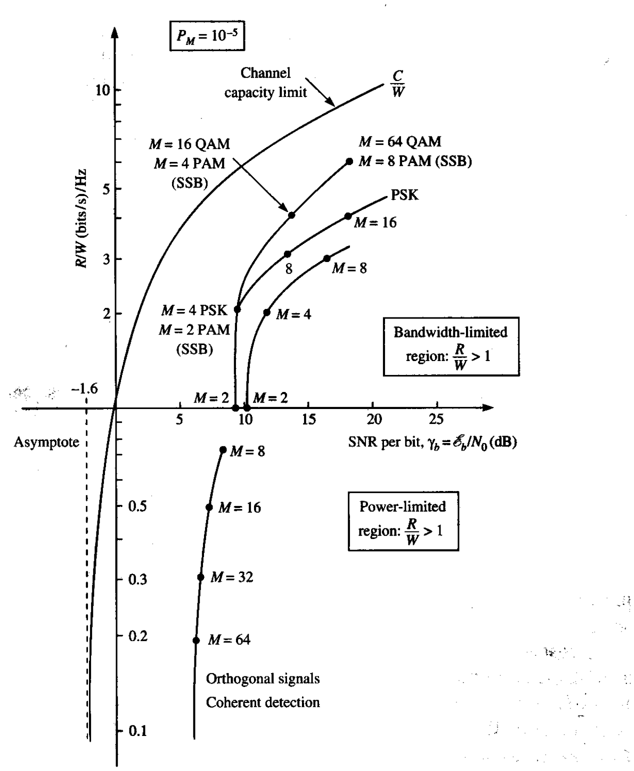
If $R = C$, i.e., we transmit at a rate equal to the channel capacity, then

$$\frac{C}{W} = \log_2\left(1 + \frac{E_b}{N_o} \cdot \frac{C}{W}\right)$$

or inverting this equation we get E_b/N_o in terms of C, viz.

$$\frac{E_b}{N_o} = \frac{2^{C/W} - 1}{C/W}$$

AWGN Channel Capacity



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Capacity of the AWGN Channel

Example: Suppose that $W = 6$ MHz (TV channel bandwidth) and the received SNR $\triangleq P/(N_o W) = 20$ dB. What is the channel capacity?
Answer: C $= 6 \times 10^6 \log_2(1 + 100) = 40$ Mbps. It is impossible to transmit information reliably on this channel with a rate greater than 40 Mbps.

Asymptotic behavior: as $C/W \to 0$.
Using L'Hôpital's rule

$$\begin{aligned}
\lim_{C/W \to 0} \frac{E_b}{N_o} &= \lim_{C/W \to 0} 2^{C/W} \ln 2 \\
&= \ln 2 \\
&= 0.693 \\
&= -1.6\text{dB}
\end{aligned}$$

Conclusion: It is impossible to communicate on an AWGN channel with arbitrary reliability if $E_b/N_o < -1.6$ dB, regardless of how much bandwidth we use.

AWGN Channel Capacity

Power Efficient Region: $R/W < 1$ bits/s/Hz. In this region we have bandwidth resources available, but transmit power is limited, e.g., deep space communications.

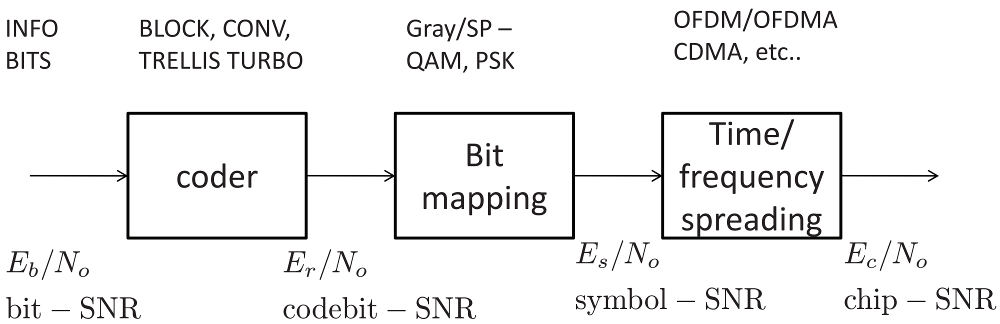
Bandwidth Efficient Region: $R/W > 1$ bits/s/Hz. In this region we have power resources available, but bandwidth is limited, e.g., commercial wireless communications. *Note: we still want to use power efficiently, i.e., bandwidth and power efficient communication*

Observe that most uncoded modulation schemes operate about 10 dB from the Shannon capacity limit for an error rate of 10^{-5}.

State-of-the-art "turbo" coding schemes can close this gap to less than 1 dB, with the cost of additional receiver *processing complexity* and *delay.*

Generally, we can tradeoff *power*, *bandwidth*, *processing complexity*, *delay.*

What is SNR?



$$
\begin{aligned}
E_b &= \text{energy per information bit} \\
E_r &= \text{energy per code bit} \\
E_s &= \text{energy per modulated symbol} \\
E_c &= \text{energy per spreading chip}
\end{aligned}
$$

The term signal-to-noise ratio (SNR) used by itself is vague:
It could mean Bit-SNR, Code-bit-SNR, Symbol-SNR, Chip-SNR.

We always need to compare different systems on the basis of received Bit-SNR, E_b/N_o.