EE4601
Communication Systems

Week 9

Binary Modulated Signal Sets

Binary PSK (BPSK)

With BPSK information is transmitted in the carrier phase. Two sinusoids are used having a relative phase different of π radians. That is

$$\begin{aligned} s_1(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi f_c t) \\ s_2(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \pi) \end{aligned}$$

for $0 \le t \le T$. Note that

$$\begin{aligned} s_2(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \pi) \\ &= \sqrt{\frac{2E}{T}}\{\cos(2\pi f_c t)\cos\pi - \sin(2\pi f_c t)\sin\pi\} \\ &= -\sqrt{\frac{2E}{T}}\cos(2\pi f_c t) = -s_1(t) \end{aligned}$$

Binary PSK (BPSK)

Assuming that $f_c T \gg 1$, the energy in $s_1(t)$ and $s_2(t)$ is

$$\int_0^T s_i^2(t)dt = E$$

The vector representation of binary PSK signals requires only a single basis function because $s_1(t) = -s_2(t)$, i.e., the two signals are linearly dependent.

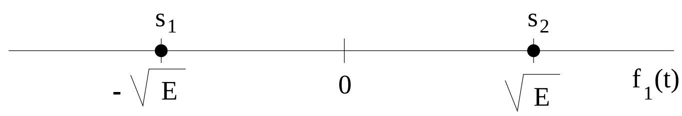
We have that

$$\begin{aligned} f_1(t) &= \frac{s_1(t)}{\sqrt{E}} \\ &= \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \end{aligned}$$

Then

$$\begin{aligned} s_1(t) &= \sqrt{E} f_1(t) \\ s_2(t) &= -\sqrt{E} f_1(t) \end{aligned}$$

Binary PSK (BPSK)



$$d_{12} = 2\sqrt{E}$$

The minimum distance decision rule is

$$\text{choose} \qquad \begin{cases} s_1 & \text{if } r < 0 \\ s_2 & \text{if } r > 0 \end{cases}$$

Assume s_1 is sent such that $r \sim N(s_1, N_0/2)$. The error probability is

$$\begin{aligned} P_e &= P_{e|s_1 \text{ sent}} P(s_1 \text{ sent}) + P_{e|s_2 \text{ sent}} P(s_2 \text{ sent}) \\ &= P_{e|s_1 \text{ sent}} \\ &= P(r < 0) \\ &= Q\left(\sqrt{\frac{2E}{N_o}}\right) \end{aligned}$$

Binary FSK (BFSK)

With binary FSK signals, information is transmitted in the carrier frequency. Two sinusoids are used that have different carrier frequencies.

$$
\begin{aligned}
s_1(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi f_c t) \\
s_2(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi (f_c + \Delta_f) t)
\end{aligned}
$$

for $0 \leq t \leq T$. The frequency difference is Δ_f.

Note that $s_1(t)$ and $s_2(t)$ both have energy E.

Depending on the choice of Δ_f, $s_1(t)$ and $s_2(t)$ may or may not be orthogonal.

Binary FSK (BFSK)

The vector representation of binary FSK signals requires two basis functions because the two signals $s_1(t)$ and $s_2(t)$ are linearly independent.

We have that

$$\begin{aligned} f_1(t) &= \frac{s_1(t)}{\sqrt{E}} \\ &= \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \end{aligned}$$

Then

$$\begin{aligned} s_{21} &= \int_0^T s_2(t) f_1(t) dt \\ &= \sqrt{E}\frac{2}{T}\int_0^T \cos(2\pi(f_c + \Delta_f)t)\cos(2\pi f_c t) dt \end{aligned}$$

Binary FSK (BFSK)

$$
\begin{aligned}
s_{21} &= \frac{\sqrt{E}}{T}\int_0^T \left\{\cos(2\pi\Delta_f)t + \cos(2\pi(2f_c + \Delta_f)t)\right\} dt \\
&= \frac{\sqrt{E}}{T}\int_0^T \cos(2\pi\Delta_f)t \; dt \\
&= \frac{\sqrt{E}}{T}\left.\frac{\sin(2\pi\Delta_f t)}{2\pi\Delta_f}\right|_0^T \\
&= \sqrt{E}\frac{\sin(2\pi\Delta_f T)}{2\pi\Delta_f T} \\
&= \sqrt{E}\mathrm{sinc}(2\Delta_f T)
\end{aligned}
$$

Hence,

$$
f_2(t) = \frac{\sqrt{\frac{2E}{T}}\cos(2\pi(f_c + \Delta_f)t) - \mathrm{sinc}(2\Delta_f T)\sqrt{\frac{2E}{T}}\cos(2\pi f_c t)}{\sqrt{E(1 - \mathrm{sinc}^2 2\Delta_f T)}}
$$

Binary Orthogonal FSK

Suppose that $\Delta_f = 1/(2T)$. Then it follows that

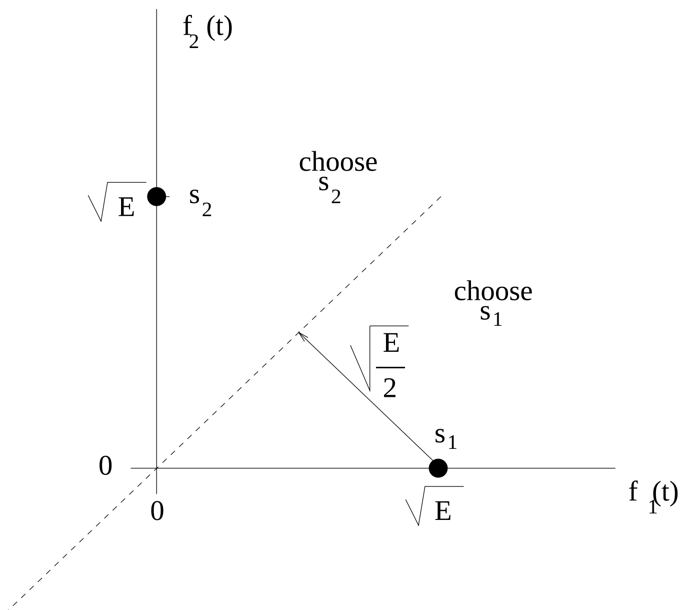
$$\begin{aligned} s_{21} &= \sqrt{E}\mathrm{sinc}(2\Delta_f T) \\ &= 0 \end{aligned}$$

In this case, $s_1(t)$ and $s_2(t)$ are *orthogonal*.

Hence,

$$f_2(t) = \sqrt{\frac{2}{T}}\cos(2\pi(f_c + \Delta_f)t)$$

Binary Orthogonal FSK



$$d_{12}^2 = 2E$$

Binary Orthogonal FSK

The minimum distance decision rule is

$$\text{choose} \quad \begin{cases} \mathbf{s}_1 & \text{if } \mathbf{r} \text{ is below dashed line} \\ \mathbf{s}_2 & \text{if } \mathbf{r} \text{ is above dashed line} \end{cases}$$

Using the *circular symmetric* property of the noise vector $\mathbf{n}$, the error probability is

$$\begin{aligned} P_e &= P_{e|s_1 \text{ sent}} \\ &= Q\left(\frac{\sqrt{E/2}}{\sqrt{N_o/2}}\right) \\ &= Q\left(\sqrt{\frac{E}{N_o}}\right) \end{aligned}$$

Note that coherent BFSK requires a factor of 2 (3 dB) increase in E/N_o to achieve the same error probability as BPSK.