ECE 3080 Microelectronic Circuits

Exam 1

September 23, 2004

Dr. W. Alan Doolittle

Print your name clearly and largely: Solutions

Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 new sheet of notes (1 page front and back) as well as a calculator. There are 100 total points. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Sign your name on **ONE** of the two following cases:

I DID NOT observe any ethical violations during this exam:

____

I observed an ethical violation during this exam:

____

First 30% **Multiple Choice and True/False (Circle the letter of the most correct answer / answers)**

1.) (2-points) True or False: If two of your professors collide in the hallway, there is a finite but very small probability that they will pass through each other due to the wavelike nature of particles.
2.) (2-points) True or False: A semiconductor material has a wider bandgap than an insulator.
3.) (2-points) True or False: A wave function by itself does not have any physically observable features (energy, position, momentum etc...) – Physical observables must have "operators" acting on the wave function.
4.) (2-points) True or False: Direct bandgap semiconductors have conduction band electron minima's at the same momentum as the valence band maxima.
5.) (2-points) True or False: A zincblende crystal structure has more atoms in the unit cell than does the diamond crystal structure.

Select the **best** answer or answer<u>s</u> for 6-10:

6.) (3-points) A "new" semiconductor consists of equal compositions of fictitious group II elements Dr, Do, Is and group VI elements Go and Od also present in equal composition. To within 1%, what is the correct reduced semiconductor notation for this remarkable compound?
   a.) DrDoIsGoOd
   b.) $Dr_{0.333}Do_{0.333}Is_{0.333}Go_{0.5}Od_{0.5}$
   c.) $Dr_{0.20}Do_{0.20}Is_{0.20}Go_{0.20}Od_{0.20}$
   d.) Cannot be determined from the information given.
   e.) You cannot have more than 4 elements making up a semiconductor.

7.) (3-points) Given that the energy bandgap of AlN is 6.2 eV, GaN is 3.4 eV and InN is ~0.65 eV, which material is likely to have the shortest chemical bonds?
   a.) AlN
   b.) GaN
   c.) InN
   d.) None of the above
   e.) McDonalds is accepting applications for engineers who have no clue

8.) (3-points) If an electron is confined in a small 3D box by infinite potential outside the box, which of the following are true?
   a.) The energy of the electron can take on any value
   b.) The energy of the electron can only take on discrete values
   c.) the electron wavelengths must be resonant with the box dimensions
   d.) The wavefunction derivative must be continuous at the box boundaries
   e.) The electron wavelengths are quantized

9.) (3-points) What is the miller index of the plane intersecting the major axis at x=10, y=20 and z=40.
   a.) 111
   b.) 124
   c.) 421
   d.) None of the above

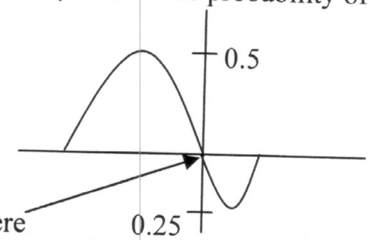
$\frac{1}{10} \quad \frac{1}{20} \quad \frac{1}{40} \quad \times 40 = 4 \; 2 \; 1$

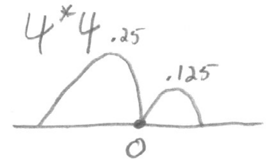
10.) (2-points) Which of the following E-k diagrams results in a ~~higher~~ larger effective mass?

a) b) c)

11.)(2 points) Based on the wavefunction shown below, what is the probability of finding an electron at the point located below?

a.) 0
b.) 0.75
c.) 1
d.) 0.25
e.) 0.5





12.)(2 points) If all the following atoms have the same lattice constant and are all covalently bonded, which is likely to have the smallest bandgap? (Hint: think in terms of the geometry factors you found in your homework).

a.) Simple Cubic
b.) Body Centered Cubic
c.) ~~Face Centered~~ Cubic
d.) ~~Diamond~~ → higher density / shortest distance between atoms

13.) (2 points) If a particle has fictitious wavefunction of $\Psi = Ax^2$ from $-1 \leq x < 1$ and 0 for all other x, what is the value of A?

a.) ¼
b.) ½
c.) ~~5/2~~
d.) $\sqrt{5/2}$
e.) None of the above

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

$$\int_{-\infty}^{-1} 0 \, dx + \int_{-1}^{1} A^2 x^4 \, dx + \int_{1}^{\infty} 0 \, dx = 1$$

$$A^2 \frac{1}{5} x^5 \Big|_{-1}^{1} = 1$$

$$A^2 = \frac{5}{2}$$

$$A = \sqrt{5/2}$$

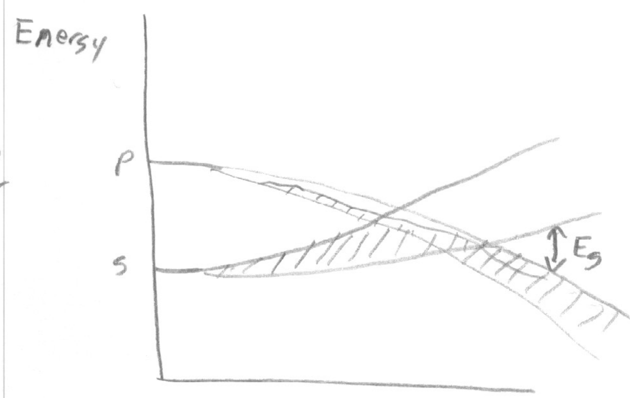
Second 20% Short Answer:
All of the following problems relate to different ways of describing the energy bandgap and quantization effects. All descriptions should be less than 3 sentences for full credit:

14.) (5-points) Describe what the energy bandgap is in terms of the atomic electron bond model (NOT the Pauli Exclusion / s-p hybridization model).

The energy bandgap is the energy required to have the valence electron (outermost electron) "stripped" (removed) from the atom and conduct freely through the crystal.

15.) (5-points) When many atoms are brought together to form a crystal, how does the energy bandgap result (this is the Pauli Exclusion / s-p hybridization) ?

As atoms come together to bond, forming a crystal, the Energy States must split to satisfy the Pauli-Exclusion principle (no 2 electrons can have the same energy @ the same position.



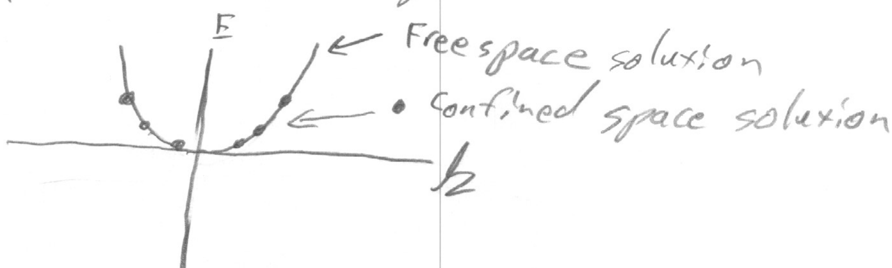
When the interatomic spacing is small enough to have a range of disallowed energies (see figure) an energy band gap forms.

16.) (5-points) Describe the formation of the energy bandgap based on the Kronig-Penney model.

As the charged electron moves through the crystal, it experiences a series of periodic potentials, each of which allow for transmission and reflection of the electron's wave function. Since the potentials are periodic, a standing wave pattern forms where only electrons with certain wavelengths (i.e. certain energies) can freely conduct. The disallowed energies (wavelengths) constitute the energy band gap.

17.) (5-points) Describe the relationship Energy and momentum in free space and in space confined (quantum wells) systems.

In free space the particle can have any Energy, E, where $E \propto k^2$ (where k is momentum). In a confined space, E can only have discrete values, so k can only have discreet values. The locus of discrete values is a subset of the free space E vs k.

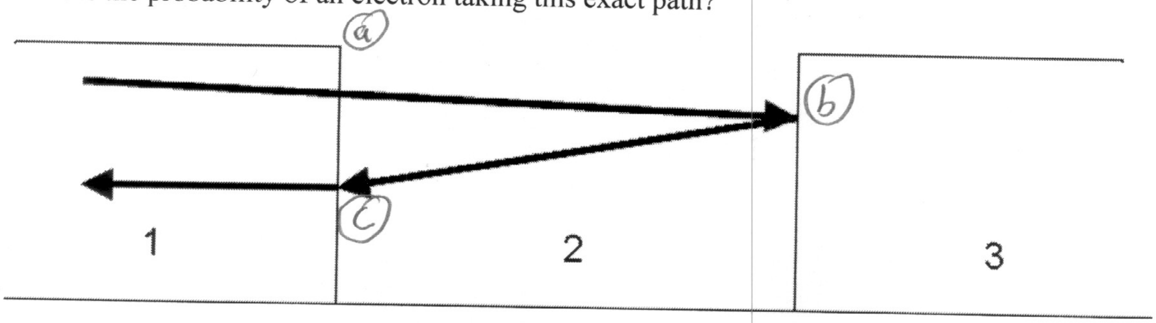


Third 25% Problems (3rd 25%)

15.) (25-points total)

An electron travels through a series of potential steps along the path shown below from region 1 into region 2, reflecting off of the barrier at region 3 then penetrating the barrier at region 2 where it travels on it's way toward negative infinity. The barriers both have a potential energy of, V while the electron has an energy E=3V.

What is the probability of an electron taking this exact path?



$$k_1 = k_3 = \sqrt{\frac{2m}{\hbar^2}(E-V)} = \sqrt{\frac{2m}{\hbar^2}(3V-V)} = \sqrt{2}\sqrt{\frac{2m}{\hbar^2}V}$$

$$k_3 = \sqrt{\frac{2m}{\hbar^2}E} = \sqrt{3}\sqrt{\frac{2m}{\hbar^2}V}$$

A+ a) $T_{a} = \dfrac{2k_1}{k_1 + k_2} = \dfrac{2\sqrt{2}\sqrt{\frac{2mV}{\hbar^2}}}{\sqrt{2}\sqrt{\frac{2mV}{\hbar^2}} + \sqrt{3}\sqrt{\frac{2mV}{\hbar^2}}}$

$$= \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{3}}$$

$$= 0.898$$

Probability of trans. $T_{a}^{*} T_{a} = (0.898)^2 = 0.808$

A+ b) $R_{b} = \dfrac{k_2 - k_3}{k_2 + k_3} = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 0.101$

Probability of reflection: $R_{b}^{*} R_{b} = (0.101)^2 = 0.0102$

A+ c) Same as at a) $T_{c}^{*} T_{c} = 0.808$

$\boxed{\text{Total Probability of Path} = 0.808\,(0.0102)\,0.808}$

Third 25% Problems (3^{rd} 25%)

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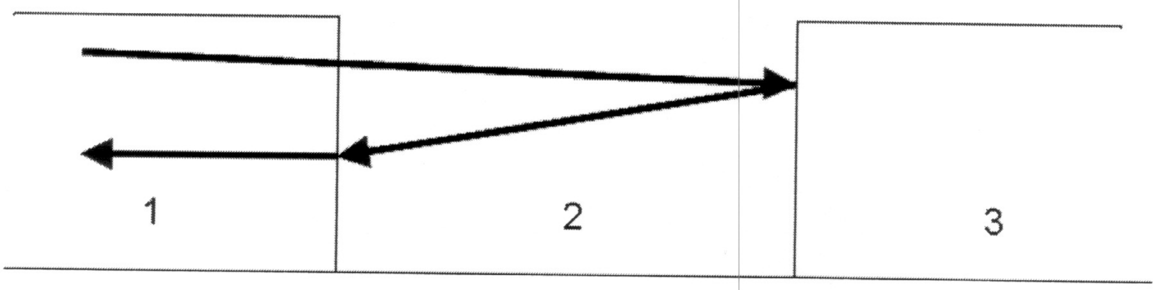


Note: As discussed in class, this solution results in the possibility of Transmission probabilities greater than 1 due to improper normalization. While acceptable for our purposes the correct solution would be:

Since $T^*T + R^*R = 1$

$$T^*T = 1 - \left(\frac{k_a - k_b}{k_a + k_b}\right)^2$$

$$= \frac{4 k_a k_b}{(k_a + k_b)^2}$$

Using this T^*T, the correct answer would be,

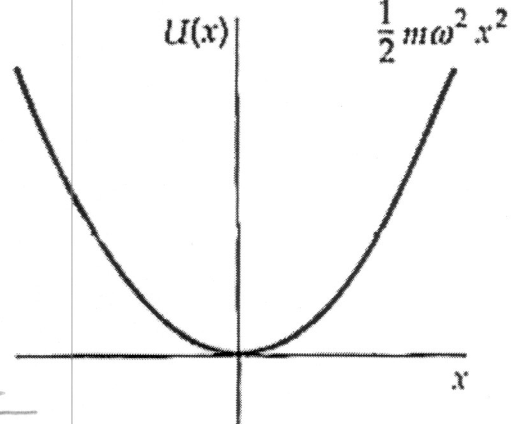
$$\boxed{\text{Total probability} = (0.989)(0.0102)(0.989) = 0.00999 \quad \text{or} \approx 1\%}$$

Pulling all the concepts together for a useful purpose: (4th 25%)

15.) (30-points total in 2 parts)
In nature, the "harmonic oscillator potential" is often observed. Specifically, a potential of the form,

$V(x)=\frac{1}{2}m\varpi^2x^2$ as shown here

is often found for oscillation of atoms (phonons) and electrons around a central point, x=0.



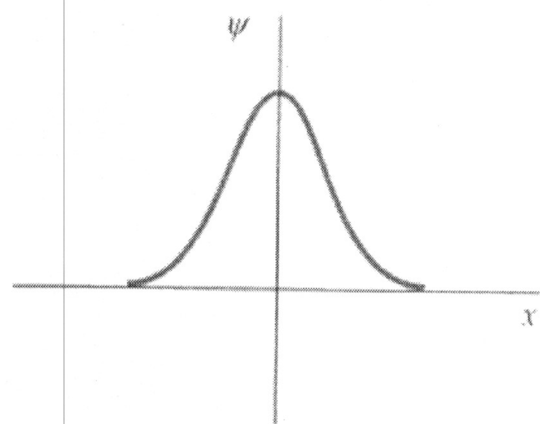
"Hamiltonian" for

a.) (9 points) Write out the Schrodinger equation using this potential.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left(A_o e^{-\alpha x^2}\right)+\left(\frac{1}{2}m\omega^2x^2\right)A_o e^{-\alpha x^2}=E A_o e^{-\alpha x^2}$$

b.) (21 points) If one solution (actually one of many solution) for the wavefunction is a Gaussian function given below,

$$\Psi(x)=A_o e^{-\alpha x^2}$$



what is the value of Energy obtained from the Schrodinger equation?
Hints: The Schrodinger equation must hold for all values of x. Thus, certain choices of x (i.e. x=0) make for simpler solutions. For full credit, you should eliminate α from the solution.

See next page

$\psi(x) = Ae^{-\alpha x^2}$

Extra work can be done here, but clearly indicate which problem you are solving.

Noting that: $\frac{d\psi}{dx} = -2\alpha x A_0 e^{-\alpha x^2}$

and $\frac{d^2\psi}{dx^2} = -2\alpha A_0 e^{-\alpha x^2} + 4\alpha^2 x^2 A_0 e^{-\alpha x^2}$

the Hamiltonian becomes,

$$-\frac{\hbar^2}{2m}\left(-2\alpha A_0 e^{-\alpha x^2} + 4\alpha^2 x^2 A_0 e^{-\alpha x^2}\right) + \frac{1}{2}m\omega^2 x^2 A_0 e^{-\alpha x^2}$$

$$= E A_0 e^{-\alpha x^2}$$

or dividing by $Ae^{-\alpha x^2}$ ...

$$\circledast \quad -\frac{\hbar^2}{2m}\left(-2\alpha + 4\alpha^2 x^2\right) + \frac{1}{2} m\omega^2 x^2 = E$$

Using the hint, making $x=0$ we get

$$\circledast\circledast \quad E = \frac{\hbar^2 \alpha}{m}$$

But what is α?

Plug $\circledast\circledast$ into $\circledast$,

$$-\frac{\hbar^2}{2m}\left(4\alpha^2 x^2\right) + \frac{1}{2} m\omega^2 x^2 = 0$$

Divide by x^2 and solve for α

$$\alpha^2 = \frac{m^2\omega^2}{4\hbar^2} \quad \text{or} \quad \alpha = \frac{m\omega}{2\hbar}$$

$$\therefore \quad \boxed{E = \frac{\hbar^2 \alpha}{m} = \frac{1}{2}(\hbar\omega)}$$

Note: correct energy unit

ECE 3080 Microelectronic Circuits

→ *Take Home Exam 1* ← Given to raise averages.

September 29, 2004

Dr. W. Alan Doolittle

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Instructions:
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Sign your name on **ONE** of the two following cases:

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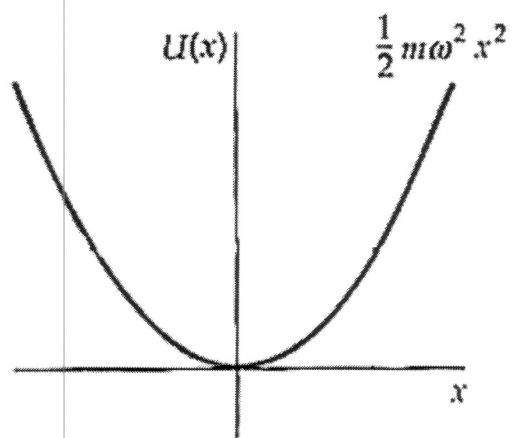
I observed an ethical violation during this exam:

(10-points total)

In nature, the "harmonic oscillator potential" is often observed. Specifically, a potential of the form,

$V(x)=\frac{1}{2}m\varpi^2x^2$ as shown here ⟶



is often found for oscillation of atoms (phonons) and electrons around a central point, x=0. For the in class exam, you solved the problem for the ground state energy using the given ground state wave function. Now I ask you to solve the problem for the first excited state energy. Thus, if one solution (actually one of many solutions) for the wave function is a function given below,

$$\Psi(x) = A_1 x e^{-\alpha x^2}$$

What is the value of Energy, E, obtained from the Hamiltonian form of the Schrodinger equation?

Hints: The Hamiltonian Schrodinger equation must hold for all values of x. Thus, certain choices of x (i.e. x=0) make for simpler solutions. For full credit, you should eliminate α from the solution.

Hamiltonian:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

$$\psi(x) = A_1\, x e^{-\alpha x^2}$$

$$\frac{d\psi}{dx} = A_1 x(-2\alpha x)e^{-\alpha x^2} + A_1 e^{-\alpha x^2}$$

$$= A_1\left[-2\alpha x^2 + 1\right]e^{-\alpha x^2}$$

$$\frac{d^2\psi}{dx} = A_1\left[-2\alpha x^2 + 1\right]\left[-2\alpha x\right]e^{-\alpha x^2} + A_1 e^{-\alpha x^2}\left[-4\alpha x\right]$$

$$= A_1 e^{-\alpha x^2}\left[4\alpha^2x^3 - 2\alpha x - 4\alpha x\right]$$

$$= A_1 e^{-\alpha x^2}\left[4\alpha^2x^3 - 6\alpha x\right]$$

$$= \psi(x)\left[4\alpha^2x^2 - 6\alpha\right]$$

$$* \quad -\frac{\hbar^2}{2m}\psi(x)\left[4\alpha^2x^2 - 6\alpha\right] + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

Divide by ψ,

$$** \quad -\frac{\hbar^2}{2m}\left[4\alpha^2x^2 - 6\alpha\right] + \frac{1}{2}m\omega^2x^2 = E$$

at $x=0$, $E = \frac{\hbar^2 6\alpha}{2m} = \frac{3\hbar^2\alpha}{m}$

cont'd on next page

sub stituting E into **,

$$-\frac{\hbar^2}{2m}\left[4\alpha^2x^2-6\alpha\right]+\frac{1}{2}m\omega^2x^2=\frac{3\hbar^2\alpha}{m}$$

$$-\frac{\hbar^2}{2m}4\alpha^2x^2+\frac{3\hbar^2\alpha}{m}+\frac{1}{2}m\omega^2x^2=\frac{3\hbar^2\alpha}{m}$$

$$\frac{1}{2}m\omega^2=\frac{\hbar^2}{2m}4\alpha^2$$

$$\alpha^2=\frac{m^2\omega^2}{\hbar^2 4}$$

$$\alpha=\frac{m\omega}{\hbar(2)}$$

$$\therefore E=\frac{3\hbar^2\alpha}{m}=\frac{3\hbar^{\cancel{2}}}{\cancel{m}}\left(\frac{\cancel{m}\omega}{\cancel{\hbar}(2)}\right)$$

$$\boxed{E=\frac{3}{2}\hbar\omega}$$