ECE 3080 Microelectronic Circuits

Exam 1

85 points

February 20, 2008

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Print your name clearly and largely: Solutions

Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 new sheet of notes (1 page front and back) as well as a calculator. Turn in your note sheet with the exam. There are 100 total points **PLUS 1 TAKE HOME PROBLEM THAT SHOULD BE REMOVED FROM THE EXAM AND IS DUE MONDAY FEBRUARY 25 AT THE START OF CLASS**. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Sign your name on **ONE** of the two following cases:

I DID NOT observe any ethical violations during this exam:

I observed an ethical violation during this exam:

First 25% **Multiple Choice and True/False**
(<u>Circle</u> the letter of the most correct *<u>answer or answers</u>*)

1.) (3-points) True or False: If a semiconductor has a small inter-atomic spacing it will likely have a large bandgap.

2.) (3-points) True or False: Since the lattice constant of a given material is fixed, the energy bandgap of that material is also constant since the energy bandgap depends on the inter-atomic spacing. strain changes lattice constant and thus E_g

3.) (3-points) True or False: For both direct and indirect bandgap materials both the energy and momentum between electrons and holes must be conserved during the recombination or generation process.

4.) (3-points) True or False: The probability of occupying a state located at the fermi-energy is always ½.

5.) (3-points) True or False: The most probable configuration of a given system of electrons and energy states is the one that has the most ways it can be reconfigured while remaining indistinguishable (i.e. you cannot tell the difference between the reconfigured arrangements).

Select the **<u>best</u>** answer or answer<u>s</u> for 6-8:

6.) (4-points) The Bloch-Wave theory...
- a.) ... treats the electron wavefunction as a simple plane wave modulated by the periodic potential.
- b.) ... predicts that the likelihood of finding the electron in each unit cell is identical.
- c.) ... predicts that the magnitude of the electron wavefunction is the same in every unit cell.
- d.) ... predicts that the phase of the electron wavefunction is the same in every unit cell.
- e.) Who knows!!!!

7.) (3-points) Which of the following E-k diagrams results in the smallest effective mass and highest electron group velocity at the maximum k value shown (rightmost point on each curve)?

a) b) c)

8.) [redacted]

2nd Section: Short answer worth 25%

9) (10 - Points) Explain in 4 sentences or less how the energy bandgap forms in terms of the electron represented as a wave interacting with the periodic potential.

Minimum expected points for full credit:

The electron traveling through the crystal ~~undergoe~~s experinences repeatative potentials from the atoms it encounters.

Since the electron has wavelike character, at each ~~a~~ change in potential (atoms) the probability wave ~~a~~ can be reflected or transmitted. ~~resulting in a~~

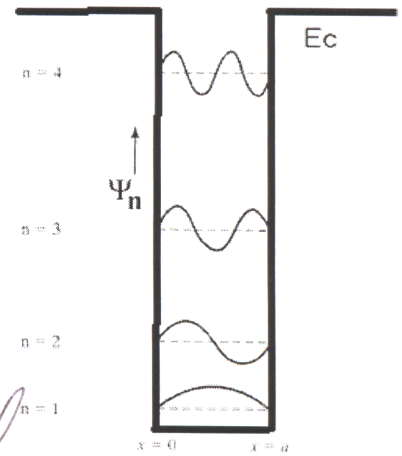
The resultant probability wave pattern from multiple reflections/transmissions sets up wavelengths that are likely to pass through the crystal and others that are not (the energy bandgap).

optional

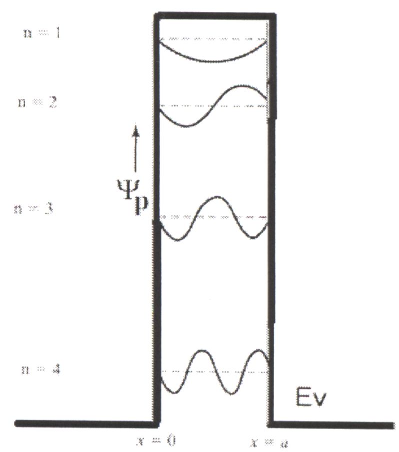
This is very similar to optical anti-reflection coatings.

Note: $E = \frac{hc}{\lambda}$

10) (15 – Points) The quantum well to the right is to be used as a light emitting diode (LED) and has 4 quantized states in the conduction band well and 4 quantized states in the valence band well as pictured. Each state has the shown wavefunction, Ψ_n for electrons and Ψ_p for holes for each state, n=1, 2, 3 and 4. Explain why transitions from odd to even quantum number (n) wells are forbidden (a drawing may help you explain your answer).



When 4×4 is considered the electrons + holes do not occupy the same positions. For example:



n = 2 (even)

n = 1 (odd)

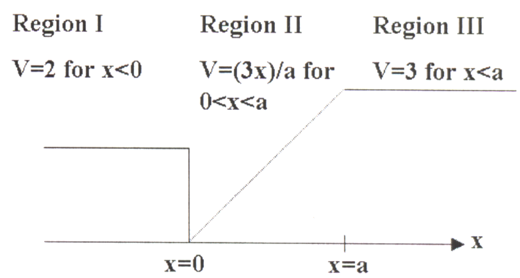
has little* spatial overlap. whereas n = 2 conduction to n = 2 Valence band is perfectly alighed

n = 2 (E_c)

n = 2 (E_v)

* Some overlap is possible. To understand parity selection rules completely, one must consider the detailed nature of Ψ_n. See Libboff et.al. if interested.

11) (35 – Points Total) Given a triangular potential well with finite potential barriers separated by a linear potential as shown below, ...



a) (9 points) ... write down the Schrodinger equation for region II ONLY and identify what the Hamiltonian for this problem would be. (DO NOT SOLVE IT – JUST WRITE IT DOWN).

b) (12 points) ... solve the Schrodinger equation in regions I and III only (DO NOT in region II). Do not solve for the coeficients

c) (9 points) ... write down what the boundary conditions for this system would be.

d) (5 points) ... At what location or locations will the electron wavelength be the largest (support your answer with an equation).

a) $$\underbrace{\left(-\frac{\hbar^2}{2m}\nabla^2 + \overbrace{\frac{3x}{a}}^{V(x)}\right)}_{\text{Hamiltonian operator}}\Psi = E\Psi$$

b) $$\Psi_{I,III}(x,t) = Ae^{-j(\omega t - k_{I,III}x)} + Be^{-j(\omega t + k_{I,III}x)}$$

where $k_I = \sqrt{\frac{2m(E-2)}{\hbar^2}}$ & $k_{III} = \sqrt{\frac{2m(E-3)}{\hbar^2}}$

and $E_I = \frac{\hbar^2 k^2}{2m} + 2$ and $E_{III} = \frac{\hbar^2 k^2}{2m} + 3$

c)
$$\left.\begin{aligned} \Psi_I(x=0) &= \Psi_{II}(x=0) \\ \frac{\partial \Psi_I(x=0)}{\partial x} &= \frac{\partial \Psi_{II}(x=0)}{\partial x} \end{aligned}\right\} \text{1 set}$$

$$\left.\begin{aligned} \Psi_{II}(x=a) &= \Psi_{III}(x=a) \\ \frac{\partial \Psi_{II}(x=a)}{\partial x} &= \frac{\partial \Psi_{III}(x=a)}{\partial x} \end{aligned}\right\} 2^{nd} \text{ set}$$

Note: Since I asked you Not to solve in Region II, This is the best you can do.

Use this page for additional work.

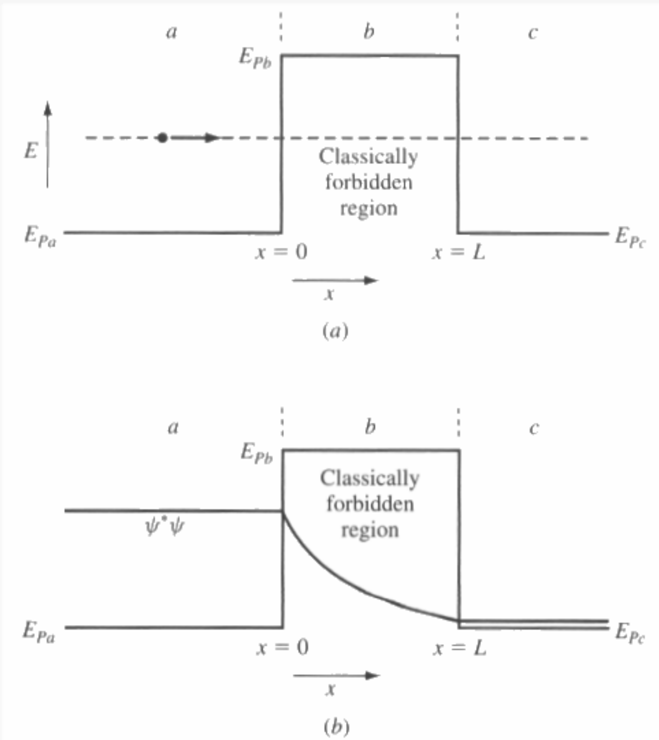
d) $k_i = \frac{2\pi}{\lambda} \Longrightarrow \lambda$ largest $\Rightarrow k$ smallest

In region III k is smallest.

Take home problem (15 Points):

Carefully remove from exam and work ALONE at home. This is an open book – open notes question but you must reference any materials you use. Due Monday February 25th at the start of class. Note: In order to encourage ethical behavior (i.e. no discussions among students and no group projects), I am offering a 10 point bonus for credible evidence of cheating on this assignment – can you really trust "that person" to keep quite or will they turn you in to get the points??????

In semiconductor-insulator barriers such as in a MOSFET or superconductor-insulator barriers such as in some supercomputers or even "Electric field emission devices" such as found in plasma displays, electron tunneling through a barrier is important. Consider an electron moving in a potential E_{pa} incident on a barrier of thickness L and potential E_{pb}.
The electron energy is such that $E_{pa}=E_{pc}<E<E_{pb}$ and **ONLY electron motion in the +x direction is considered, (i.e. there are no reflections at either the x=0 or x=L boundary)**.



Tunneling. (a) An electron approaches a finite potential barrier; (b) the probability density function shows there is a small chance the electron will appear on the far side of the barrier.

Part a) determine the probability of the electron tunneling from x=0 to x=L (your answer will be written in terms of algebraic variables). Hint: use the exponential form of the wave function and your boundary conditions between regions.

Part b) If the electron has energy E= ½kT eV (equal to the peak in the energy distribution above the conduction band) and the barrier E_{pb}-E_{pa}=0.34 eV as could be found for an GaAs-AlGaAs barrier, what is the probability of tunneling (numeric answer) for the electron having an effective mass of $0.067m_o$ at room temperature for a 2 nm barrier.

Take home Solution

Since no reflections:

$\psi_1(x=0) = \psi_2(x=0)$ + e^{+kx} components are zero.

Thus,

$$\psi_1 = Ae^{-jk_1x} + \cancel{Be^{+jk_1x}} \to 0$$
$$\psi_2 = Ce^{-jk_2x} + \cancel{De^{+jk_2x}} \to 0$$
$$\psi_3 = Ee^{-jk_3x} + Fe^{+jk_3x}$$

where k is of the form:

$$k = \sqrt{\left(\frac{2m}{\hbar^2}\right)(E - E_{pi})}$$

where $E_{pi} = 0$ for 1 + 3 + $E_{pi} \neq 0$ for 2.

Note for region 2, $\psi_2(x) \Rightarrow$ damped exponential

Thus @ $x = 0$

$$\psi_1(x=0) = \psi_2(x=0)$$
$$Ae^{-j0} = Ce^{-j0} \implies A = C$$

Thus @ $x = L$

$$\psi_2(x=L) = \psi_3(x=L)$$
$$Ce^{-k_2'L} = Ee^{-jk_3L}$$

but letting $k_2' = jk_2$

$$\Rightarrow E = \left(Ae^{-k_2'L}\right)e^{+jk_3L}$$

$k_2' = \sqrt{\left(\frac{2m}{\hbar^2}\right)(E_{ph_2} - E)}$

~~damped amplitude~~

$$T = \frac{\psi_3^*(L)\,\psi_3(L)}{\psi_1^*(x=0)\,\psi_1(0)} = \frac{Ce^{-k_2'L}\,Ce^{-k_2'L}}{Ce^{-jk_1x}\,Ce^{+jk_1x}} = e^{-2k_2'L}$$

$$\boxed{T = e^{-2 k_2' L} \quad \text{where } L = 2\,nm \quad k_2' = \sqrt{\left(\frac{2m}{\hbar^2}\right)(E_{ph_b} - E)}}$$

b) for $E = \frac{1}{2} kT$ @ RT

$$E = \frac{0.0259}{2} = 0.01295\,eV$$

$$E_{ph_b} = 0.34\,eV$$

$$k_2' = \sqrt{\frac{2(0.067 m_0)}{\hbar^2}\left(0.34 - \frac{0.0259}{2}\right) eV * 1.6\times10^{-19}\,J/eV}$$

$$= 7.54\times10^{8}\left(\frac{1}{m}\right) = \frac{2\pi}{\lambda}$$

$$T = e^{-2 k_2' (2\times10^{-9})}$$

$$\simeq e^{-3}$$

$$\boxed{T = 0.0489 \text{ or } \approx 5\% \text{ Probability}}$$