ECE 3080 Microelectronic Circuits

Exam 1

February 21, 2008 10:05-10:55

Dr. W. Alan Doolittle

Print your name clearly and largely: Solutions

Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 new sheet of notes (1 page front and back) as well as a calculator. Turn in your note sheet with the exam. There are 100 total points **INCLUDING 1 TAKE HOME PROBLEM THAT SHOULD BE REMOVED FROM THE EXAM AND IS DUE FRIDAY FEBRUARY 25 AT THE START OF CLASS**. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Sign your name on **ONE** of the two following cases:

I DID NOT observe any ethical violations during this exam:

I observed an ethical violation during this exam:

First 25% Multiple Choice and True/False

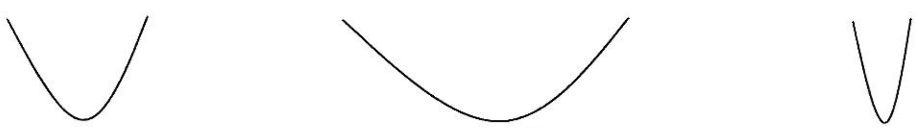
(Circle the letter of the most correct *answer or answers*)

1.) (3-points) True or False: If a semiconductor has a large inter-atomic spacing it will likely have a large bandgap.
2.) (3-points) True or False: Strain can modify the effective mass of an electron and this effect can be used to increase the speed of modern CMOS transistors.
3.) (3-points) True or False: The larger the slope of the Energy-Crystalline momentum (E-k) curve, the slower the velocity of the particle.
4.) (3-points) True or False: The effective mass of an electron or hole is proportional to the curvature of the E-k diagram.
5.) (3-points) True or False: Crystalline materials form due to cosmic forces combined with sheer luck as to where the atoms fall.

Select the **best** answer or answers for 6-8:

6.) (4-points) The Bloch-Wave theory...
- a.) ... treats the electron wavefunction as a simple plane wave modulated by the periodic potential.
- b.) ... predicts that the likelihood of finding the electron in each unit cell is identical.
- c.) ... predicts that the magnitude of the electron wavefunction is the same in every unit cell.
- d.) ... predicts that the phase of the electron wavefunction is the same in every unit cell.
- e.) Who knows!!!!

7.) (3-points) Which of the following E-k diagrams results in the largest effective mass ~~and smallest electron group velocity~~ at the minimum Energy value shown?



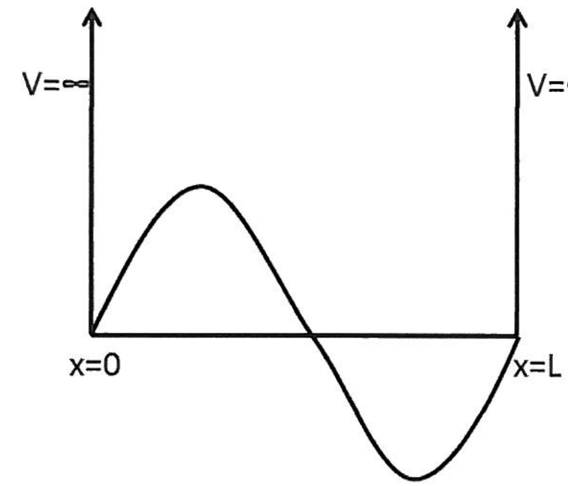
a) b) c)

8.) (3 points) Which of the following tools is used to grow crystalline semiconductors?
- a.) Molecular Beam Epitaxy (MBE)
- b.) Metal Organic Chemical Vapor Deposition (MOCVD)
- c.) Chemical Vapor Deposition (CVD)
- d.) Plasma Enhanced Chemical Vapor Deposition (PECVD)
- e.) Diffusion Furnaces

30 ~~20~~

10) (~~20~~ points total in 3 parts)

12 (~~8~~ 12 – Points) a) Plot the probability density function for and (~~8~~ – points) b) find the average (expected) value of the position, *x*, for the *n=2* state of the infinite potential well, where the wave function is $\psi_2 = A\sin(2\pi x/L)e^{(-jEt/h)}$ for $(0 \le x \le L)$. (~~4~~ 6 – Points) c) Explain the difference in the expected value versus the results expected from a single measurement.



The following math identities may (or may not) be helpful:

$$\int u(\sin(u))^2 du = \frac{u^2}{4} - \frac{u(\sin(2u))}{4} - \frac{\cos(2u)}{8} \qquad \int (\sin(u))^2 du = \frac{u}{2} - \frac{(\sin(2u))}{4}$$

a) 

$\sin^2$ functions

b)
$$\langle x\rangle = \frac{\int_0^L A^* \sin\left(\frac{2\pi x}{L}\right) e^{j\frac{E}{\hbar}t} (x) A \sin\left(\frac{2\pi x}{L}\right) e^{-j\frac{E}{\hbar}t} dx}{\int_0^L A^* \sin\left(\frac{2\pi x}{L}\right) e^{j E/\hbar t} \sin\left(\frac{2\pi x}{L}\right) e^{-j E/\hbar t} dx}$$

normalization condition

Note: You could have found A by the Normalization

$$\langle x\rangle = \frac{\int_0^L x \sin^2\left(\frac{2\pi x}{L}\right) dx}{\int_0^L \sin^2\left(\frac{2\pi x}{L}\right) dx}$$

let $u = \frac{2\pi x}{L} \Rightarrow du = \frac{2\pi}{L} dx \quad x = \frac{Lu}{2\pi} \quad dx = \frac{L}{2\pi} du$

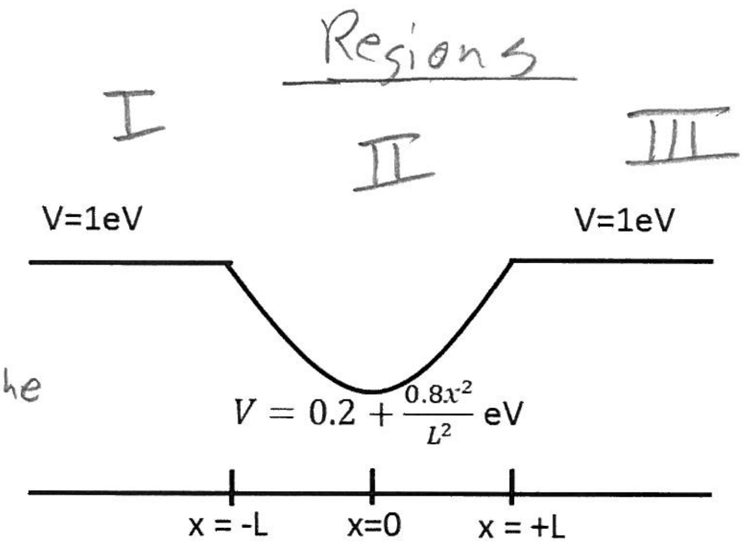
$$\langle x\rangle = \frac{\left(\frac{L}{2\pi}\right)^2 \int_0^{2\pi} u \sin^2 u \, du}{\frac{L}{2\pi}\int_0^{2\pi} \sin^2 u \, du} = \frac{\frac{L}{2\pi}\left[\frac{u^2}{4} - \frac{u \sin 2u}{4} - \frac{\cos 2u}{8}\right]_0^{2\pi}}{\left[\frac{1}{2}u - \frac{1}{4}\sin 2u\right]_0^{2\pi}}$$

$$= \frac{L}{2\pi}\left(\frac{\left[\frac{(2\pi)^2}{4} - 0 - \frac{1}{8}\right] - \left[0 - 0 - \frac{1}{8}\right]}{\left[\frac{2\pi}{2} - 0\right] - [0 - 0]}\right) = \boxed{\frac{L}{2}}$$

Use this page for additional work.

c) If you measure the electron position most of the time, you will find it centered near $x = \frac{L}{4}$ or $\frac{3L}{4}$. You will never find it at $x = \frac{L}{2}$. However, since $\psi^*\psi$ is centered about $x = \frac{L}{2}$, the average of <u>all</u> (many) measurements will be $\frac{L}{2}$ $\left(\langle x \rangle = \frac{L}{2}\right)$.

10) (25 – Points Total) Given a parabolic potential well with finite potential barriers as shown below, ...



a) (6 points) ... write down the Schrodinger equation for ~~all~~ the region~~s~~ and identify what the Hamiltonian for this problem would be. (DO NOT SOLVE IT – JUST WRITE IT DOWN).

-L<x<L

* b) (6 points) ... solve the ~~time dependent~~ Schrodinger equation in regions x<-L and x>L only (DO NOT solve in the region –L<x<L and DO NOT solve for the coefficients).

c) (9 points) ... write down what the boundary conditions for this entire system would be (do not solve them).

d) (4 points) ... At what location or locations will the electron wavelength be the largest (support your answer with an equation).

a) $$\underbrace{\left[-\frac{\hbar^2}{2m}\nabla^2 + \left(0.2 + \frac{0.8x^2}{L^2}\right)(1.6e{-19})\right]}_{H}\psi = E\psi$$

b) $$\psi_{1,3}(x,t) = Ae^{-j(\omega t - k_1 x)} + Be^{-j(\omega t + k_1 x)}$$

where $k_1 = \sqrt{\frac{2m(E-q)}{\hbar^2}}$ → converts from 1eV to 1.6e-19 J

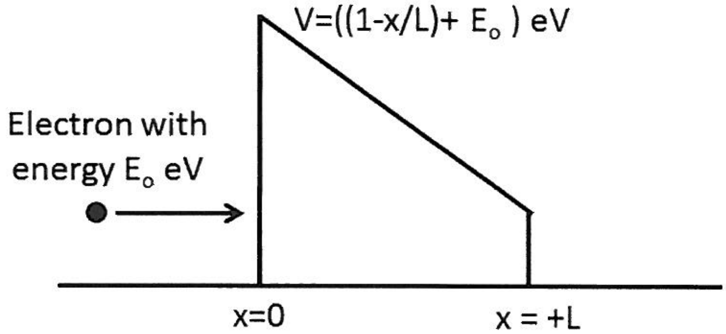
c) $$\boxed{\begin{array}{ll} \psi_I(x=-L) = \psi_{II}(x=-L) & \psi_{II}(x=L) = \psi_{III}(x=L) \\ \left.\frac{d\psi_I}{dx}\right|_{x=-L} = \left.\frac{d\psi_{II}}{dx}\right|_{x=-L} & \left.\frac{d\psi_{II}}{dx}\right|_{x=L} = \left.\frac{d\psi_{III}}{dx}\right|_{x=L} \end{array}}$$

d) $$k = \frac{2\pi}{\lambda} = \sqrt{\frac{2m}{\hbar^2}(E-V)}$$

$k_{min} \Rightarrow \lambda_{max} \Rightarrow V_{max} \Rightarrow$ Regions I + II

Take home problem (20 Points):

Carefully remove from exam and work ALONE at home. This is an open book – open notes question but you must reference any materials you use. Due Friday February 25th at the start of class. Note: In order to encourage ethical behavior (i.e. no discussions among students and no group projects), I am offering a 10 point bonus for credible evidence of cheating on this assignment – can you really trust “that person” to keep quite or will they turn you in to get the points?????



In semiconductor-insulator barriers such as in a MOSFET or superconductor-insulator barriers such as in some supercomputers or even “Electric field emission devices” such as found in plasma displays, electron tunneling through a barrier is important. On a previous exam, a barrier without an electric field was examined. Here I am asking you to exam the effect of tunneling using a triangular barrier that results from the presence of an electric field. Consider an electron with energy E_o, incident on a barrier of thickness L and potential as shown in the figure above. Determine the tunneling probability in terms of the electric field needed to create the potential difference described in the figure. The tunneling probability for this problem is defined as:

$$T = \frac{\Psi^*(x = L)\Psi(x = L)}{\Psi^*(x - 0)\Psi(x - 0)}$$

NOTE: Until now, we have not developed the tools to handle “complex” potentials beyond, i.e. anything other than simple constant potentials. Thus, here we introduce a very useful approximation based on the idea that the wavelength of an electron is so small, that for most potential forms, the wavelength of the electron is much smaller than the dimensions for which the potential varies. This leads to an approximation known as the Wigner, Kramers, Brillouin (WKB) approximation which relates the wavefunction at a given point to the wavefunction at another point. To solve this problem, you will need to research the WKB approximation and correctly apply it here.