Quantum Computing

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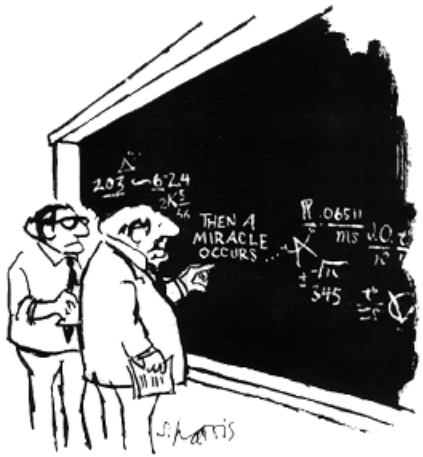
Qubit: The Quantum Unit of Information

- Conventional computers use **bit** (= 0 or 1)
- Quantum computers use a quantum bit, or **qubit**.
- Quantum mechanics allows the qubit to be 0, 1, **or somewhere in between**. How is this possible?



"I think you should be more explicit here in step two."

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Qubit: A Two-Level System

- Given Schrodinger's Equation,

$$\hat{H}\psi = E\psi$$

The solution may have the form

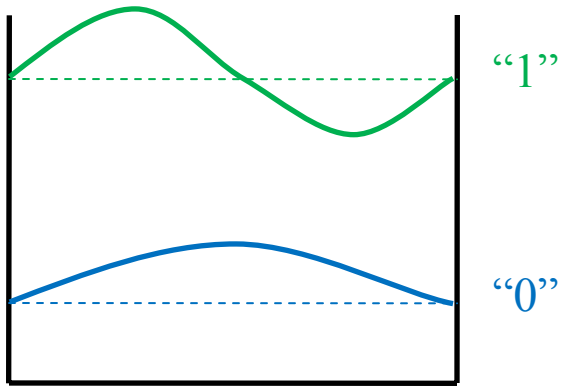
$$\psi = A\varphi_0 + B\varphi_1$$

- This is a **two-level system**, since there are two possible states. In quantum mechanics, the system can exist in both states at once with varying probability.

- Examples include energy levels of trapped ions, or quantum dots - basically 3D quantum wells.

Qubit: A Two-Level System

- Now decide that for our logical qubit,
  $\varphi_0 \rightarrow$ "0"
  $\varphi_1 \rightarrow$ "1"
- In this manner, the different states of our quantum system represent logical values, just like a bit.

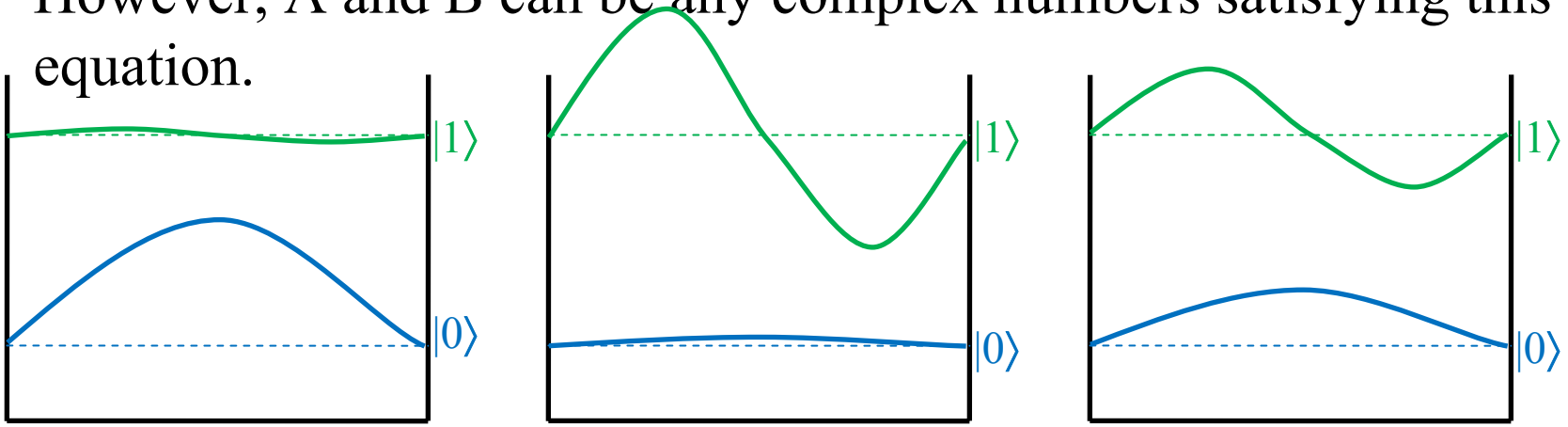


Qubit: Simple Representation

- For simplicity, we hide the quantum mechanics and simply represent the logical qubit states as $|0\rangle$ and $|1\rangle$, so the qubit state is $\psi = A|0\rangle + B|1\rangle$

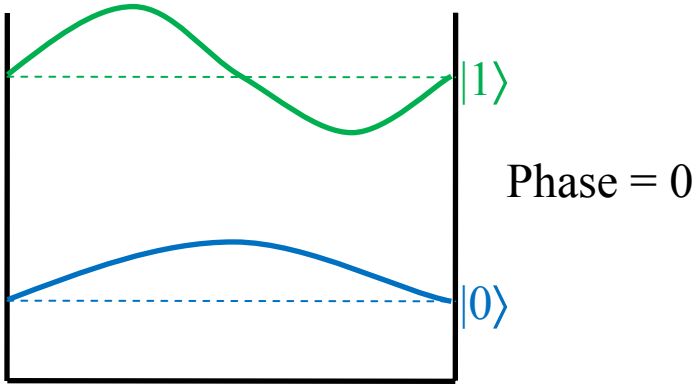
Qubit: Probability Amplitude

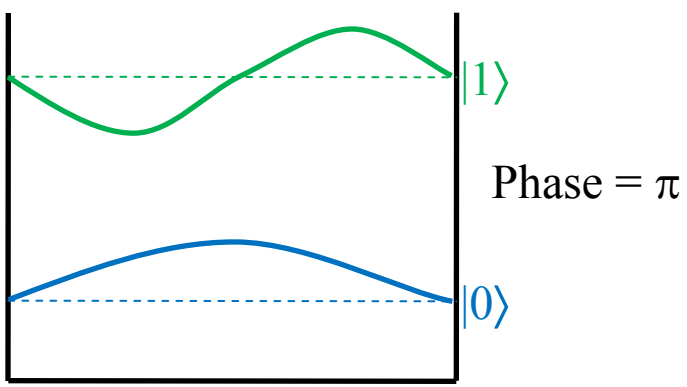
- The coefficients A and B are **probability amplitudes**, meaning they indicate the probability the qubit is in state $|0\rangle$ or $|1\rangle$.
- $|A|^2$ = probability the system is in state $|0\rangle$, etc.
- Because they represent probability, $|A|^2 + |B|^2 = 1$.
- However, A and B can be any complex numbers satisfying this equation.



Qubit: Quantum Phase

- A qubit's **phase** is the relative phase between complex coefficients A and B. Consider the following quantum states:
  $\psi_a = 2^{-½}(|0\rangle + |1\rangle)$ and $\psi_b = 2^{-½}(|0\rangle - |1\rangle)$
- Both states have a 50% chance of being logical $|0\rangle$ or $|1\rangle$, **but they are different states**.
- The distinction lies in their **phases**, which are 0 and π, respectively. The use of phase is what makes quantum computers very powerful.





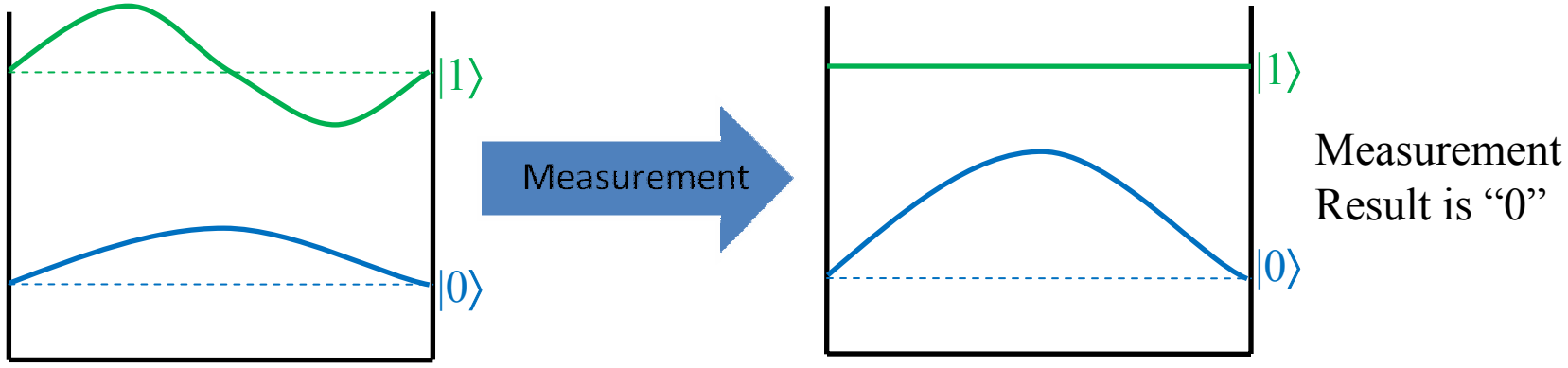
Qubit: Exponential Scaling

- We've seen already that a single qubit can represent two values simultaneously; this is known as **superposition**.

- Amazingly, superposition scales exponentially. Two qubits can represent 4 states, three qubits can represent 8, etc. Just 40 qubits could represent $2^{40} \cong 1.1$ Trillion states **at once**, which is far more than a modern PC can handle.

Qubit: Measurement

- Retrieving data encoded in a quantum system requires **measurement**. The measurement process destroys a quantum state, giving as output one of the possible states.
- For example, the state $2^{-½}(|0\rangle + |1\rangle)$ will give "0" or "1", each with 50% probability, **but not both**.
- The measurement output is always a series of bits, just as many as there are qubits.



Quantum Computers: Algorithms

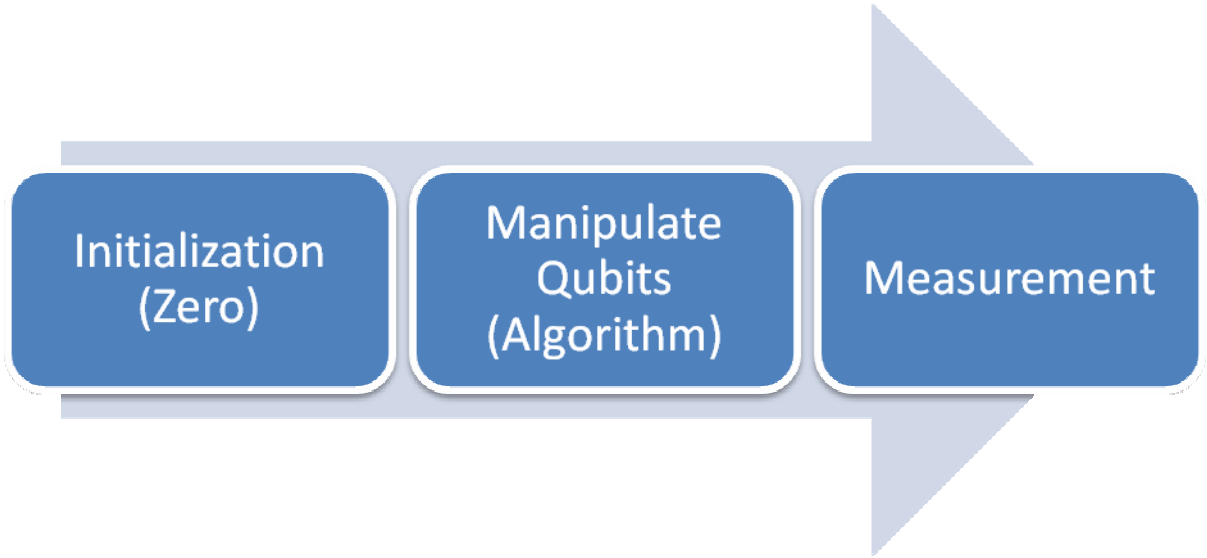
- The advantage of quantum computing is that it can solve some problems faster than conventional computers

- Shor's algorithm can factor very large numbers in polynomial time, which if realized would break most cryptography today. Conventional computers require exponential time.

- Grover's algorithm can search a list of N values in $O(\sqrt{N})$ time, as opposed to O(N) for a conventional computer. The algorithm also uses O(log(N)) qubits.

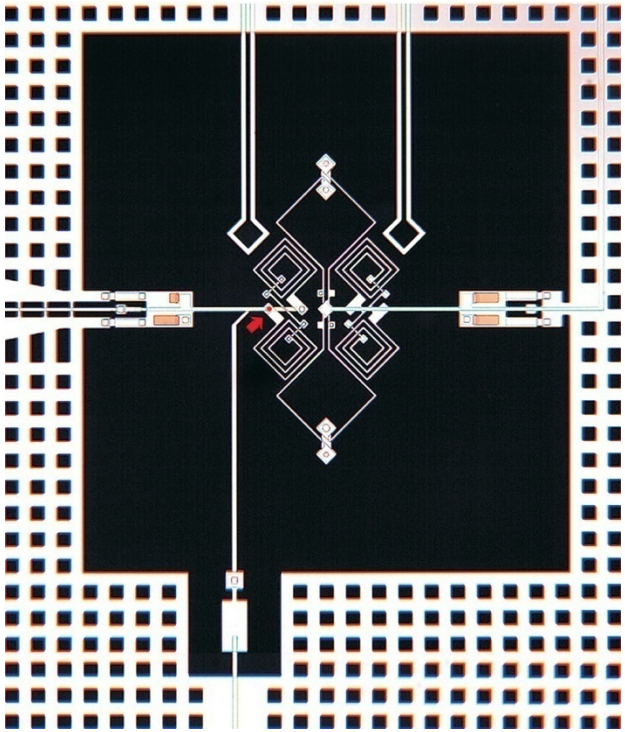
Quantum Computers: Computation

- Executing a quantum algorithm follows a process similar to conventional computers:



Quantum Computers: DiVincenzo Criteria

- DiVincenzo established five criteria that a viable quantum computer must satisfy:
  1. Qubits must be clearly defined.
  2. The computer must have the ability to initialize qubits, e.g., set to zero.
  3. Qubits must be stable throughout the computation.
  4. Operations must be controllable and universal.
  5. Accurate measurement must be possible.



Superconducting Qubit

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