LECTURE 090 – PLL DESIGN EQUATIONS AND PLL MEASUREMENTS

(Reference [2, Previous ECE6440 Notes])

Objective

The objective of this presentation is

1.) To provide a summary of relationships and equations that can be used to design PLLs.

2.) Illustrate the design of a DPLL frequency synthesizer

3.) Show how to make measurements on PLLs

Outline

- PLL design equations
- PLL design example
- PLL measurements
- Summary

PLL DESIGN EQUATIONS†

Introduction

The following design equations are to be used in designing PLLs and apply both to LPPLs and DPLLs with the following definitions:

LPLLs: $N = 1$ and $\beta = 1$

where N is the divider in the feedback loop and β is the loop *expansion factor* determined by the type of PFD.

$$\text{Loop gain} = K = \frac{K_d K_o F(0)}{N} = \frac{K_v F(0)}{N}$$

Goal of these equations:

Permit the basic design of an LPLL or DPLL.

† These notes are taken from PLL Design Equations Notes by R.K Feeney, July 1998

Type – I, First-Order Loop ($F(0) = 1$)

Crossover frequency (frequency at which the loop gain is 1 or 0dB):

$$\boxed{\omega_c = K \text{ (radians/sec.)}}$$

-3dB Bandwidth (frequency at which the closed-loop gain is equal to –3dB):

Closed loop transfer function $= \dfrac{K}{s+K} \quad \rightarrow \quad \boxed{\omega_{-3dB} = K \text{ (radians/sec.)}}$

Noise Bandwidth:

$$B_n = \int_0^\infty |H(j2\pi f)|^2 df = \int_0^\infty \frac{K^2}{K^2+(2\pi f)^2} df = \frac{K}{2\pi}\int_0^\infty \frac{K}{K^2+(2\pi f)^2} d(2\pi f) = \frac{K}{2\pi}\frac{\pi}{2} = \frac{K}{4} \text{ (Hz)}$$

Hold Range:

$$\boxed{\Delta\omega_H = \beta NK}$$

Lock (Capture) Range:

$$\boxed{\Delta\omega_L = \Delta\omega_H = \beta NK}$$

Type-I, First-Order Loop ($F(0) = 1$) - Continued

Steady-State Phase Error:

For a sinusoidal phase detector, $\quad \varepsilon_{ss} = \lim\limits_{t\to\infty} \theta(t) = \sin^{-1}\left(\dfrac{\Delta\omega_{osc}}{NK}\right)$

For a nonsinusoidal (digital) phase detector, $\varepsilon_{ss} = \lim\limits_{t\to\infty} \theta(t) = \dfrac{\Delta\omega_{osc}}{NK} \le \beta$

The steady-state error is never larger than β. A larger error indicates a failure to lock.

Frequency Acquisition Time:

$$\boxed{T_a = \frac{1}{K} \text{ (sec.)}}$$

For a Type-I loop,

Lock Range and Acquisition Time = Hold Range and Acquisition Time.

Type-I, Second-Order Loop

This type of loop is generally implemented with a lag-lead filter as shown below.

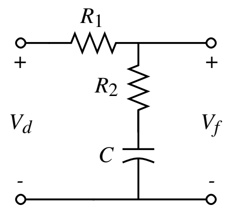
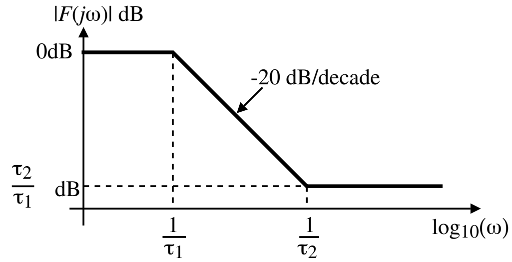


Fig. 090-01



Filter Transfer Function:

$$F(s) = \frac{1+\tau_2 s}{1+\tau_1 s} \quad \text{where} \quad \tau_1 = (R_1+R_2)C \quad \text{and} \quad \tau_2 = R_2C$$

(Note: The definition for $\tau_1 = (R_1+R_2)C$ which is different from that in Lecture 050)

System Parameters:

$$\boxed{\omega_n = \sqrt{\frac{K}{\tau_1}}} \quad \text{and} \quad \boxed{\zeta = \frac{\omega_n}{2}\left(\tau_2 + \frac{1}{K}\right) = \frac{1}{2}\sqrt{\frac{1}{K\tau_1}}\,(1+\tau_2 K)}$$

Note that because $\tau_2 < \tau_1$, we see that

$$\frac{\omega_n}{2K} < \zeta < \frac{K^2+\omega_n^2}{2\omega_n K}$$

Type-I, Second-Order Loop – Continued

Crossover Frequency:

The general close-loop frequency response for high-gain loops is,

$$H(s) = \frac{2s\zeta\omega_n + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{1+\dfrac{s^2}{2\zeta\omega_n s + \omega_n^2}} = \frac{1}{1+\text{Loop Gain}}$$

The crossover frequency, ωc, is the frequency when the loop gain is unity.

$$\therefore \qquad \frac{\omega_c^4}{\omega_n^4+4\zeta^2\omega_n^2\omega_c^2} = 1 \quad \rightarrow \quad \omega_c^4 - (4\zeta^2\omega_n^2)\omega_c^2 - \omega_n^4 = 0$$

Solving for ω_c gives,

$$\boxed{\omega_c = \omega_n\sqrt{2\zeta^2 + \sqrt{4\zeta^4+1}}}$$

3dB Bandwidth:

$$\boxed{\omega_{-3\text{dB}} = \omega_n\sqrt{b + \sqrt{b^2+1}} \quad \text{where } b = 2\zeta^2 + 1 - \frac{\omega_n}{K}\left(4\zeta - \frac{\omega_n}{K}\right)}$$

Noise Bandwidth:

$$\boxed{B_n = \frac{\omega_n}{2}\left(\zeta + \frac{1}{4\zeta}\right) \quad \text{(Hz)}}$$

Type-I, Second-Order Loop – Continued

Hold Range:

$$\Delta\omega_H = \beta NK \text{ at the output} \qquad \Delta\omega_H = \beta K \text{ at the input}$$

Lock Range:

$$\Delta\omega_L = \frac{\tau_2}{\tau_1}\Delta\omega_H = \frac{\tau_2}{\tau_1}\beta NK$$

Lock Time:

The lock time is set by the loop natural frequency, ω_n and is

$$T_L = \frac{2\pi}{\omega_n}$$

Pull-in Range:

$$\Delta\omega_P = N\beta\sqrt{2}\sqrt{2\zeta\omega_n KF(0)-\omega_n^2} \text{ at the output} \qquad \Delta\omega_P = \beta\sqrt{2}\sqrt{2\zeta\omega_n KF(0)-\omega_n^2} \text{ at the input}$$

This formula is only valid for moderate or high loop gains, i.e. $KF(0) \leq 0.4\omega_n$.

Pull-in Time:

$$T_P \approx \frac{4\left(\frac{\Delta f_{osc}}{N}\right)^2}{B_n^3} \approx \frac{\pi^2}{16}\frac{\Delta\omega_{osc}^2}{\zeta\omega_n^3}$$

Note that $\Delta\omega_H \leq \Delta\omega_{osc} \leq \Delta\omega_P$

Type-I, Second-Order Loop – Continued

Frequency Acquisition Time:

$$T_a = T_P + T_L$$

If the frequency step is within the lock limit (one beat), then the pull-in time is zero.

Steady-State Phase Error to a frequency step of $\Delta\omega_{osc}$:

For a sinusoidal phase detector, $\varepsilon_{ss} = \lim_{t\to\infty}\theta(t) = \sin^{-1}\left(\frac{\Delta\omega_{osc}}{NK}\right)$

For a nonsinusoidal (digital) phase detector, $\varepsilon_{ss} = \lim_{t\to\infty}\theta(t) = \frac{\Delta\omega_{osc}}{NK} \leq \beta$

The steady-state error is $\leq \beta$. A larger error indicates a failure to lock.

Maximum Sweep Rate of the Input Frequency:

Largest sweep rate of the input frequency for which the loop remains in lock.

$$\frac{d(\Delta\omega)}{dt} = \omega_n^2 \left(\frac{\text{radians/sec}}{\text{sec}}\right)$$

Maximum Sweep Rate for Aided Acquisition:

This is the case when the VCO is externally swept to speed up acquisition.

$$\frac{d(\Delta\omega)}{dt} = \frac{\omega_n^2}{2} \left(\frac{\text{radians/sec}}{\text{sec}}\right)$$

Type-2, Second-Order Loop

This type of PLL system generally uses the active PI filter as shown below.

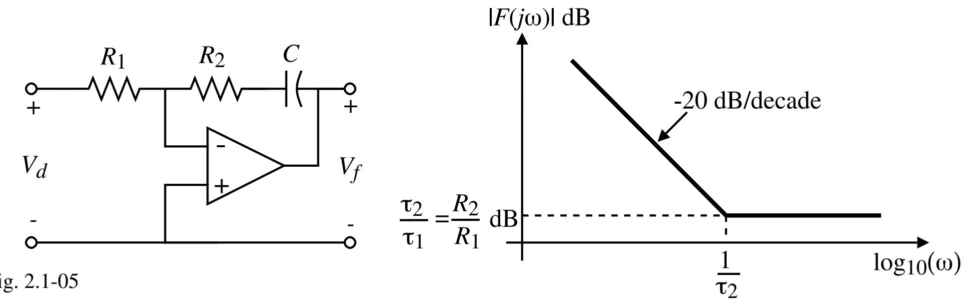


Fig. 2.1-05

Filter Transfer Function:

$$F(s) = -\frac{1+s\tau_2}{s\tau_1} = -\left(\frac{\tau_2}{\tau_1}\right)\left(\frac{s+1/\tau_2}{s}\right) = -\left(\frac{R_2}{R_1}\right)\left(\frac{s+1/\tau_2}{s}\right) \quad \text{where} \quad \tau_1 = R_1C \text{ and } \tau_2 = R_2C$$

System Parameters:

$$\omega_n = \sqrt{\frac{K}{\tau_1}} \quad \text{and} \quad \zeta = \frac{1}{2}\sqrt{K\tau_2\frac{R_2}{R_1}} = \frac{1}{2}\tau_2\sqrt{\frac{K}{\tau_1}} = \frac{\tau_2\omega_n}{2}$$

3dB Bandwidth:

$$\omega_{-3dB} = \omega_n\sqrt{2\zeta^2+1+\sqrt{(2\zeta^2+1)^2+1}}$$

Type-2, Second-Order Loop – Continued

Noise Bandwidth:

$$B_n = \frac{\omega_n}{2}\left(\zeta + \frac{1}{4\zeta}\right) \quad \text{or} \quad B_n = \frac{1}{4}\left(K\frac{R_2}{R_1} + \frac{1}{\tau_2}\right)$$

Hold Range:

Limited by the dynamic range of the loop components.

Lock (Capture) Range:

$$\Delta\omega_H = \beta N 2\zeta\omega_n$$

Lock (Capture) Time:

$$T_L = \frac{2\pi}{\omega_n}$$

Pull-in Range:

The pull-in range is the frequency range beyond the lock (capture) range over which the loop will lock after losing lock (skipping cycles).

- The pull-in range for a 2nd or higher order, type-2 loop is theoretically infinite and limited by the amplifier and phase detector offsets and by the dynamic range of the loop.
- A system with large offsets and a large frequency error may never lock.

Type-2, Second-Order Loop – Continued

Pull-in Time:

$$T_P = \tau_2\left(\frac{\frac{\Delta\omega_{osc}}{N}}{K\frac{R_2}{R_1}} - \sin\theta_o\right)$$

where θ_o is the initial phase difference between the reference and VCO signals. Assume $\sin\theta_o = -1$ for the worst case.

Pull-out Range:

$$\Delta\omega_{PO} \approx 1.8N\beta\omega_n(1+\zeta) \text{ at the output}$$

$$\Delta\omega_{PO} \approx 1.8\beta\omega_n(1+\zeta) \text{ at the input}$$

Frequency Acquisition Time:

$$T_a = T_P + T_L$$

If the frequency step is within the lock limit (one beat), then the pull-in time is zero.

Steady-state Phase Error:

The steady-state phase error of a type-2 system is zero for both a phase step and a frequency step.

Type-2, Second-Order Loop – Continued

Steady-state Phase Error – Continued:

The steady-state phase error due to a frequency ramp of $\Delta\omega_{osc}$ radians/sec./sec. is,

For a sinusoidal phase detector, $\varepsilon_{ss} = \lim_{t\to\infty}\theta(t) = \sin^{-1}\left[\left(\frac{R_1}{R_2}\right)\frac{\tau_2\frac{d\Delta\omega_{osc}}{dt}}{NK}\right]$

For a nonsinusoidal (digital) phase detector, $\varepsilon_{ss} = \lim_{t\to\infty}\theta(t) = \left[\left(\frac{R_1}{R_2}\right)\frac{\tau_2\frac{d\Delta\omega_{osc}}{dt}}{NK}\right] \leq \beta$

The steady-state error is $\leq \beta$. A larger error indicates a failure to lock.

Maximum Sweep Rate of Input Frequency:

Largest sweep rate of the input frequency for which the loop remains in lock.

$$\frac{d(\Delta\omega_{in})}{dt} = \beta\omega_n^2 \left(\frac{\text{radians/sec}}{\text{sec}}\right)$$

Maximum Sweep Rate for Aided Acquisition:

This is the case when the VCO is externally swept to speed up acquisition.

$$\frac{d(\Delta\omega_{osc})}{dt} = \frac{N\beta}{2\tau_2}\left(4B_n - \frac{1}{\tau_2}\right)\left(\frac{\text{radians/sec}}{\text{sec}}\right)$$

DESIGN OF A 450-475 MHz DPLL FREQUENCY SYNTHESIZER

Specifications

Design a DPLL frequency synthesizer that meets the following specifications:

Frequency Range:	450 – 475 MHz
Channel Spacing:	25 kHz
Modulation:	FM from 300 to 3000 Hz
Modulation Deviation:	±5kHz
Loop Type:	Type 2
Loop Order:	Second order
VCO Gain:	K_o = 1.25MHz/V = 7.854 Mradians/sec./V
Phase Detector Type:	PFD ($\beta = 2\pi$)
Phase Detector Gain:	K_d = 0.796 V/radian

(This example will be continued later in more detail concerning phase noise and spurs)

Note on channel spacing:

Carson's rule → BW of an FM signal is ≈ $2[\Delta f_c + f_m(\max)]$ = 2(±5kHz+3kHz) = 16kHz

If we assume a 9 kHz guard band, then

Channel Spacing = 9 kHz + 16 kHz = 25 kHz

PLL System

Block Diagram:

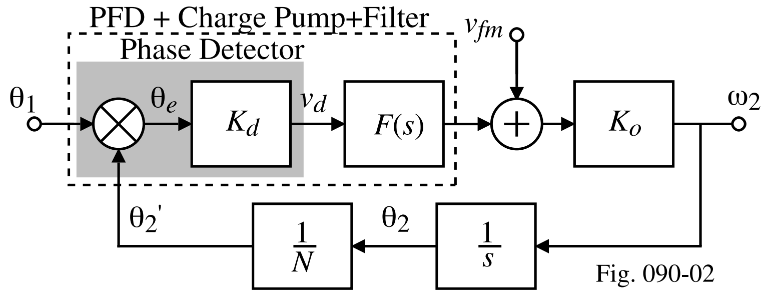


Fig. 090-02

The pertinent transfer function for this problem is given as $\frac{\omega_2(s)}{V_{fm}(s)}$ which is found as

$$\omega_2(s) = K_o\left[V_{fm}(s) + F(s)K_d\left(\theta_1 - \frac{\omega_2(s)}{sN}\right)\right] = K_o\left[V_{fm}(s) + F(s)K_d\theta_1 - \frac{F(s)K_d}{sN}\,\omega_2(s)\right]$$

Setting $\theta_1 = 0$ gives

$$\frac{\omega_2(s)}{V_{fm}(s)} = \frac{K_o}{1+\frac{F(s)K_dK_o}{sN}}$$

PLL System - Continued

The charge-pump + filter combination has a transfer function given as

$$F(s) = \frac{1+\tau_2 s}{\tau_1 s}$$

The final form of the closed-loop transfer is given as

$$\frac{\omega_2(s)}{V_{fm}(s)} = \frac{K_o}{1+\frac{(1+\tau_2 s)K_d K_o}{s^2 N\tau_1}} = \frac{s^2 K_o}{s^2 + \frac{K_d K_o \tau_2}{N\tau_1}s + \frac{K_d K_o}{N\tau_1}} = \frac{s^2 K_o}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$$

where,

$$\omega_n = \sqrt{\frac{K_d K_o}{N\tau_1}} \quad \text{and} \quad \zeta = \frac{\tau_2}{2}\sqrt{\frac{K_d K_o}{N\tau_1}}$$

Finding the Loop Parameters

1.) Division Ratio

$$N_{min} = \frac{450 \text{ MHz}}{25 \text{ kHz}} = 18{,}000 \quad \text{and} \quad N_{max} = \frac{475 \text{ MHz}}{25 \text{ kHz}} = 19{,}000$$

2.) Loop Bandwidth

To pass the 300Hz lower frequency limit, we require that the maximum –3dB frequency is 300Hz. Therefore, B_L = 300Hz.

3.) Damping Constant

For reasons discussed previously, we select ζ = 0.707. Let us check to see if this is consistent with the design.

We know that,

$$\zeta = \frac{\tau_2}{2}\sqrt{\frac{K_d K_o}{N\tau_1}} \quad \rightarrow \quad \zeta = \frac{k}{\sqrt{N}}$$

$$\therefore \quad \zeta_{max} = \frac{k}{\sqrt{N_{min}}} \quad \text{and} \quad \zeta_{min} = \frac{k}{\sqrt{N_{max}}} \quad \rightarrow \quad \zeta_{max} = \zeta_{min}\sqrt{\frac{N_{max}}{N_{min}}} = 1.0274\zeta_{min}$$

Also, $\zeta = \sqrt{\zeta_{max}\cdot\zeta_{min}} = 0.707$, which gives

$$\zeta_{min}{}^2(1.0274) = 0.5 \quad \rightarrow \quad \zeta_{min} = 0.6976 \quad \text{and} \quad \zeta_{max} = 1.0274\cdot 0.6976 = 0.7167$$

Finding the Loop Parameters – Continued

4.) Natural frequency, ω_n

$$\omega_{-3dB} = \omega_n\sqrt{2\zeta^2 + 1 + \sqrt{(2\zeta^2 + 1)^2 + 1}} \rightarrow \qquad \omega_n = \frac{\omega_{-3dB}}{\sqrt{2\zeta^2 + 1 + \sqrt{(2\zeta^2 + 1)^2 + 1}}}$$

The maximum ω_n will occur at the minimum value of N and the minimum damping factor. Therefore,

$$\omega_n(\max) = \frac{\omega_{-3dB}}{\sqrt{2\zeta_{min}^2 + 1 + \sqrt{(2\zeta_{min}^2 + 1)^2 + 1}}}$$

$$= \frac{2\pi\cdot 300}{\sqrt{2(0.6976)^2 + 1 + \sqrt{(2(0.6976)^2 + 1)}}} = 980 \text{ radians/sec.}$$

$$\omega_n(\min) = \frac{\omega_{-3dB}}{\sqrt{2\zeta_{max}^2 + 1 + \sqrt{(2\zeta_{max}^2 + 1)^2 + 1}}} = 910 \text{ radians/sec.}$$

$$\therefore \quad \omega_n = \sqrt{\omega_n(\max)\cdot\omega_n(\min)} = 944$$

Loop Parameter Summary:

Frequency (MHz)	N	ω_n (rads./sec.)	ζ	Bandwidth (Hz)
450.00	18,000	910	0.7167	300
475.00	19,000	980	0.6976	300

Design of the Loop Filter

The loop filter selected is the active PI using the single-ended realization below.

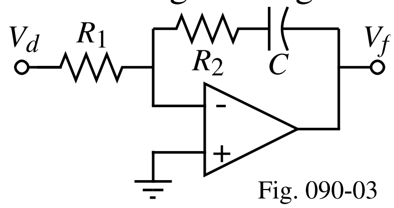


Fig. 090-03

The transfer function is,

$$F(s) = \frac{sR_2C + 1}{sR_1C} = \frac{s\tau_2 + 1}{s\tau_1} \rightarrow \quad \tau_1 = R_1C \quad \text{and} \quad \tau_2 = R_2C$$

1.) Time constants

We will use the date for $N = 18{,}000$ to design the filter.

$$\tau_1 = \frac{K_dK_o}{N\omega_n^2} = \frac{0.796\cdot 7.854\text{x}10^6}{18{,}000(910)^2} = 0.419 \text{ ms}$$

$$\tau_2 = \frac{2\zeta}{\omega_n} = \frac{2\cdot 0.7167}{910} = 1.575 \text{ ms}$$

2.) Loop filter design

Select $R_1 = 2.4\text{k}\Omega$ which gives

$$C = \frac{\tau_1}{R_1} = \frac{0.419\text{x}10^{-3}}{2.4\text{x}10^3} = 0.175\ \mu\text{F} \quad \text{and} \quad R_2 = \frac{\tau_2}{C} = \frac{1.575\text{x}10^{-3}}{0.175\text{x}10^{-6}} = 9.0\ \text{k}\Omega$$

Design of the Loop Filter – Continued

3.) Simulated response of the filter.

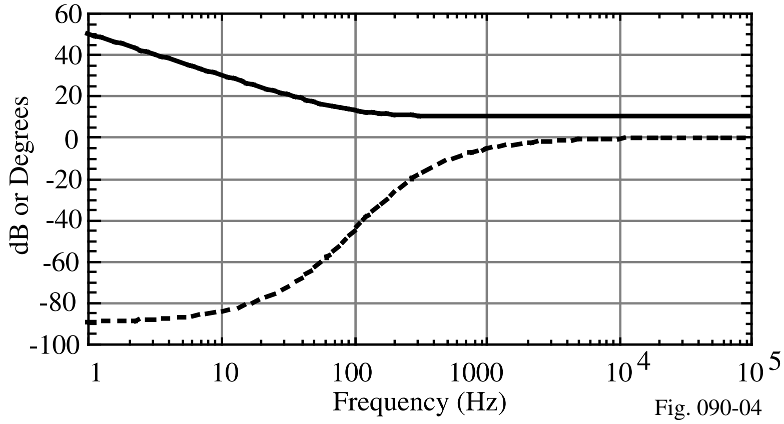


Fig. 090-04

4.) Differential version of the filter.

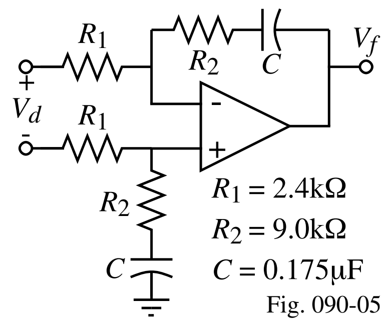


Fig. 090-05

Loop Stability

1.) Loop Gain.

The loop gain for $N = 18{,}000$ is given by

$$LG(s) = \frac{K_d K_o F(s)}{Ns} = \frac{K(s\tau_2 + 1)}{s^2 N \tau_1} = \frac{7.854\text{x}10^6 \cdot 0.796\ (1+1.575\text{x}10^{-3}s)}{0.419\text{x}10^{-3} \cdot 18{,}000\ s^2}$$

$$= \frac{828.83\text{x}10^3(1+1.575\text{x}10^{-3}s)}{s^2}$$

2.) Bode Plot

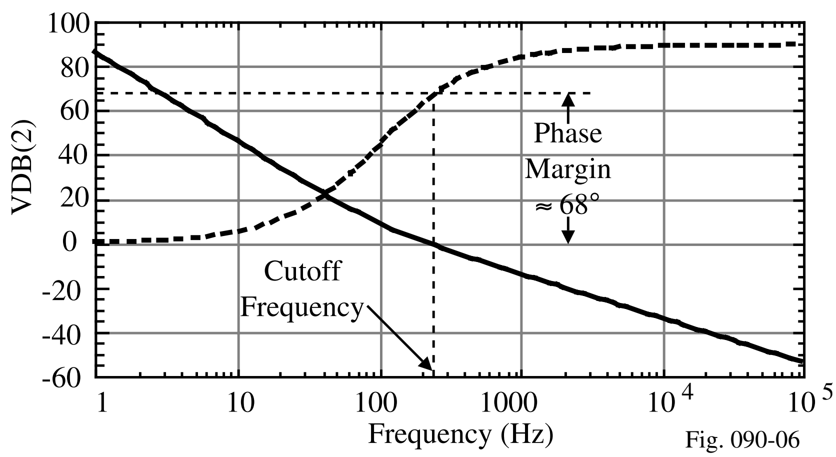


Fig. 090-06

Closed Loop Gain

A plot of the closed-loop transfer function of $\frac{\omega_2(j\omega)}{V_{fm}(j\omega)}$ is shown below.

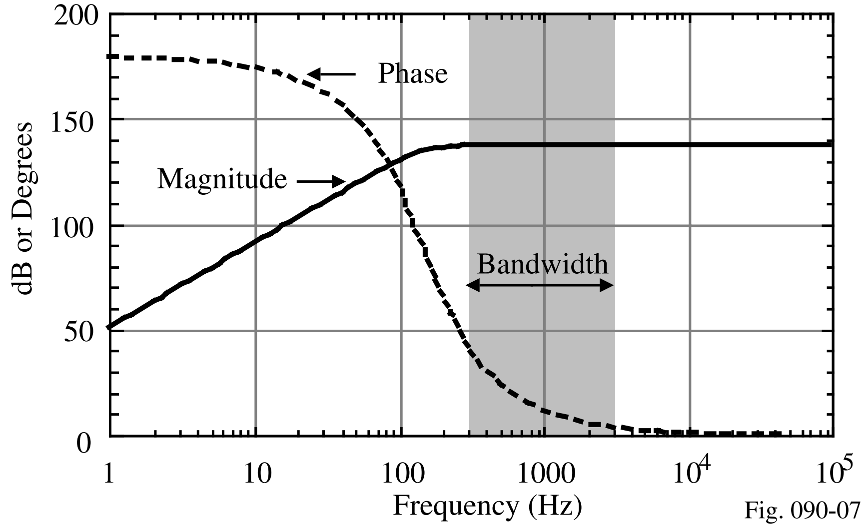


Fig. 090-07

(We will continue this example later.)

MEASUREMENT OF PLL PERFORMANCE

(The device under test in this section is the Exar XR-S200.)

Measurement of Center Frequency, f_o

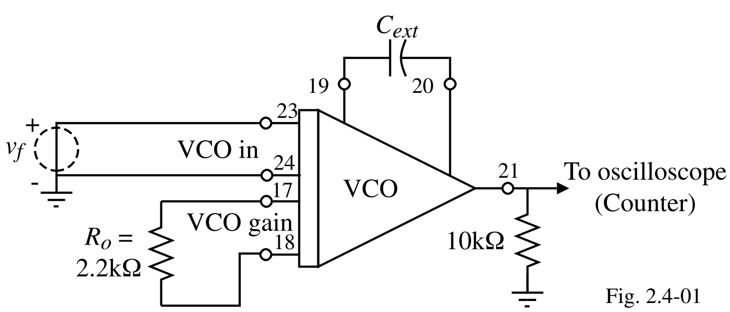
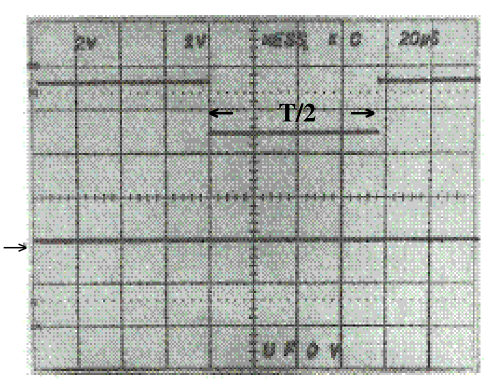


Fig. 2.4-01

Results:

$v_f = 0$ →



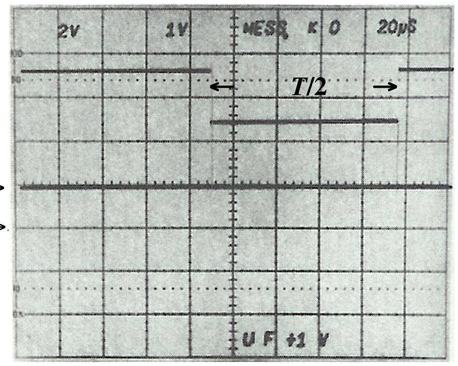
$T/2 \approx 76\mu s$

or $T = 152\mu s$

$\therefore f_o = 1/T = \underline{6.54 \text{ kHz}}$

Measurement of the VCO Gain, K_o

Use the same measurement configuration as for f_o. Vary v_f and measure the output frequency of the VCO.



$T/2 \approx 85\mu s$

or $T = 170\mu s$

$\therefore\ f_o = 1/T = 5.88$ kHz

Calculation of K_o.

$$K_o = \frac{\Delta\omega}{\Delta v_f} = \frac{2\pi(6.54\text{-}5.88)}{1\text{-}0}\text{x}10^3 = 4.13\text{x}10^3\ \frac{1}{\text{V}\cdot\text{sec}}$$

Measurement of the Phase Detector Gain, K_d

Test circuit:

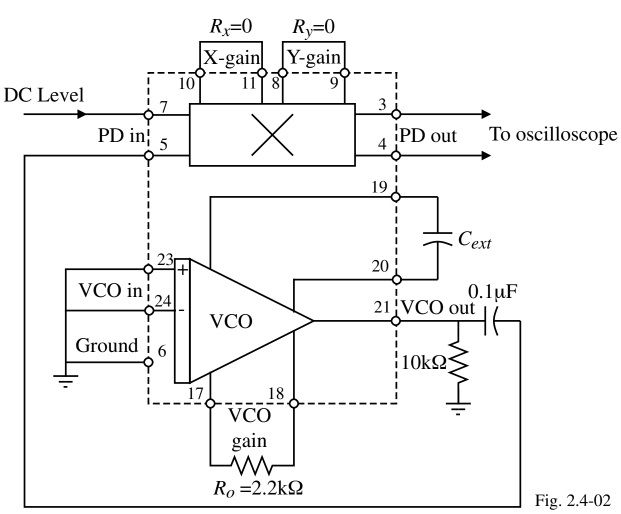


Fig. 2.4-02

Measurement Principle:

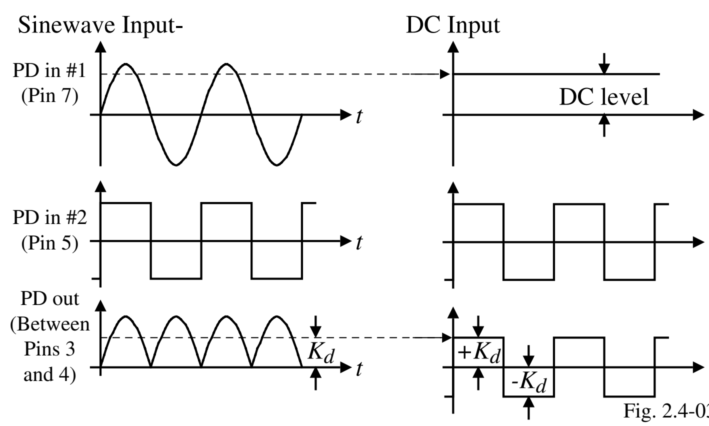


Fig. 2.4-03

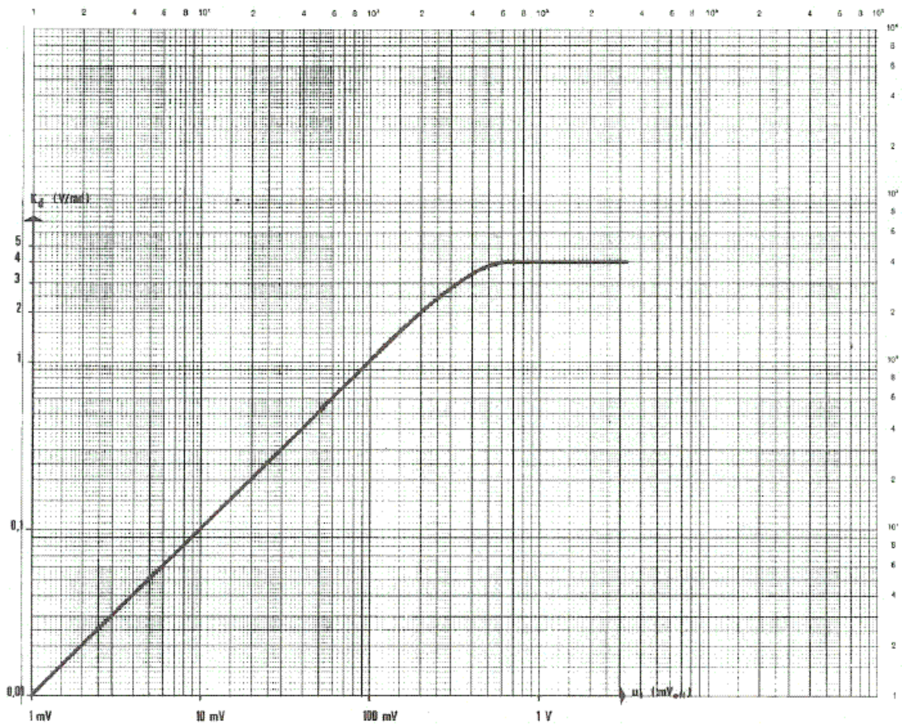
The above measurement assumes that $\theta_e = 90°$ so that $\overline{v_d} = K_d \sin 90° = K_d$

For a sinewave input, the dc output of the PD is $2/\pi$ of the peak sinusoidal voltage.

If a dc voltage is applied at the PD input of $(\sqrt{2}/1)\text{x}(2/\pi)\ V_{peak} \approx 0.9V_{peak}$, then K_d is simply 1/2 of the peak-to-peak output of the phase detector.

Measurement of K_d – Continued

v_1 = 10, 20, 30 and 40 mV (rms)



More values indicate that the PD saturates at 0.4V (rms)

Measurement of the Hold Range, $\Delta\omega_H$, and the Pull In Range, $\Delta\omega_p$

Measurement Circuit:

The measurement requires a full functional PLL. The loop filter must be added which in this case consists of both on-chip and external components.

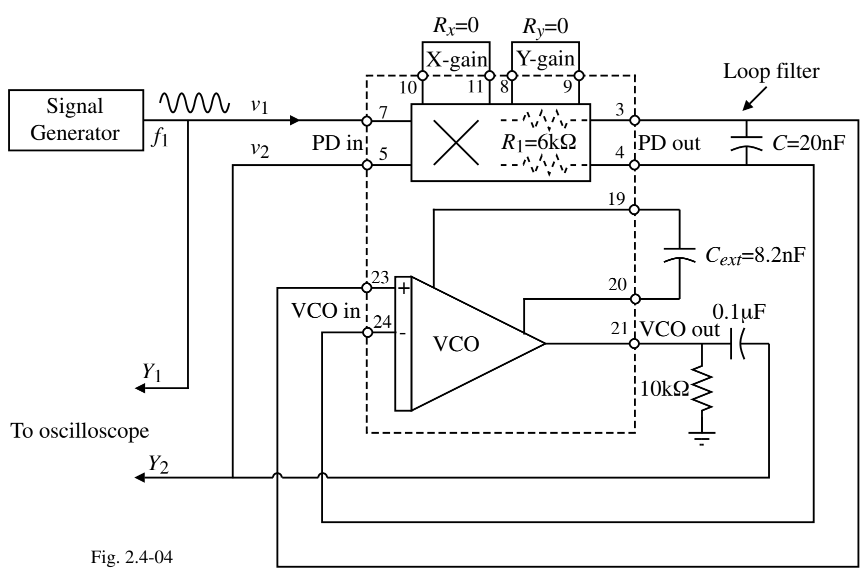


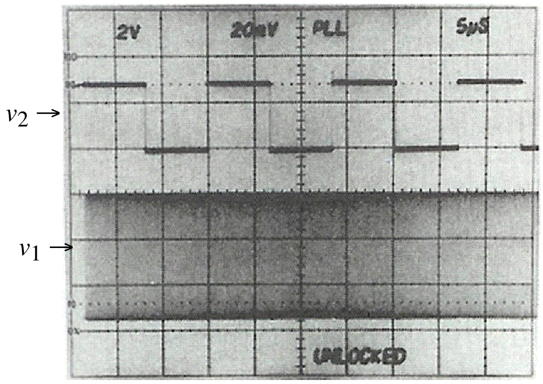
Fig. 2.4-04

1.) To measure Δf_H, start with a value of f_1 where the loop is locked and slowly vary f_1 to find the upper and lower values where the system unlocks.

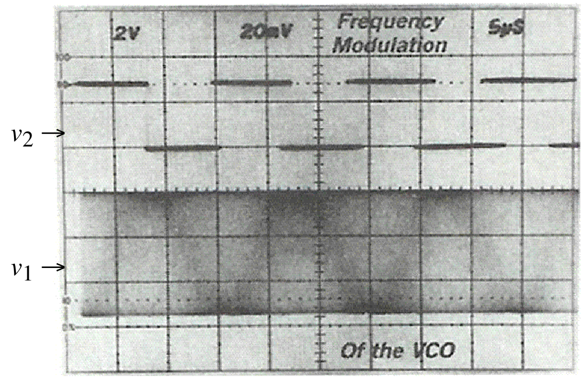
2.) To measure Δf_p, start with f_1 at approximately the center frequency, then increase f_1 until the loop locks out. Decrease f_1 until the loop pulls in. The difference between this value of f_1 and f_o is Δf_p.

Measurement of $\Delta\omega_H$ and $\Delta\omega_p$ – Continued

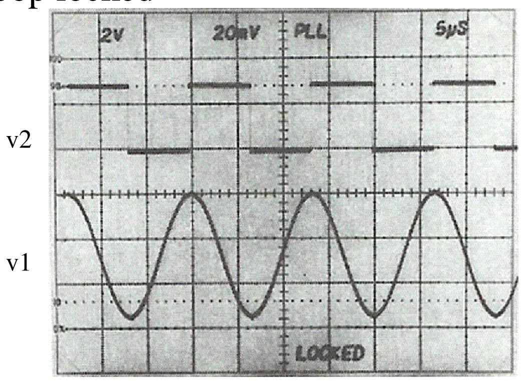
Loop out of lock



Loop on the threshold of lock



Loop locked



Measurement of ω_n, ζ, and the Lock Range $\Delta\omega_L$

Test circuit:

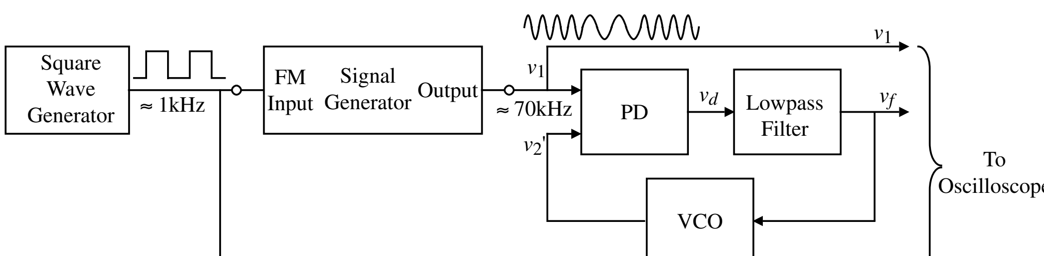
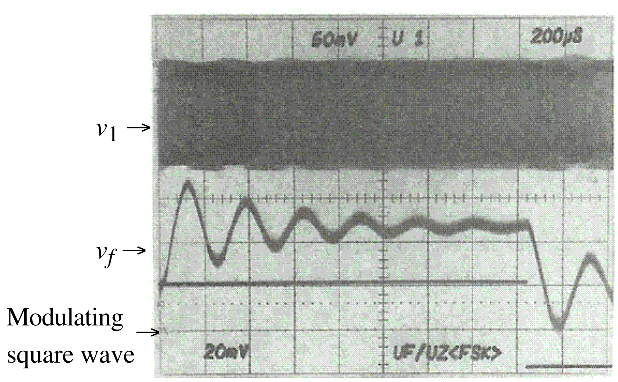
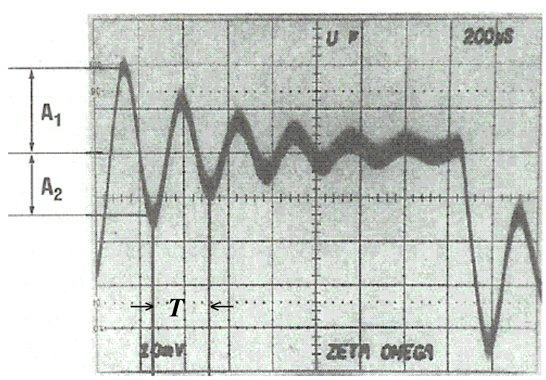


Fig. 2.4-05

Waveforms:





Parameter extraction:

With A_1=1.9, A_2=1.4, $\zeta = \dfrac{\ln(A_1/A_2)}{\sqrt{\pi^2 + [\ln(A_1/A_2)]^2}}$ and $\omega_n = \dfrac{2\pi}{T\sqrt{1-\zeta^2}} \Rightarrow \zeta = 0.8$ and $f_n = 4.1\text{kHz}$

Measurement of ω_n, ζ, and the Lock Range $\Delta\omega_L$ – Continued

Measurement of $\Delta\omega_L$:

1.) The signal generator is adjusted to generate two frequencies, ω_{high} and ω_{low} such that,

$$\omega_{high} > \omega_o + \Delta\omega_p$$

2.) Set $\omega_{low} = \omega_{high}$ (the amplitude of the square wave generator will be zero)

3.) Decrease ω_{low}.

4.) When $\omega_{low} \approx \omega_o + \Delta\omega_L$, the PLL will lock.

$$\therefore\ \Delta\omega_L \approx \omega_{low} - \omega_o$$

Measurement of the Phase Transfer Function, $H(j\omega)$

Since most signal generators are not phase modulated, use a frequency modulated signal generator instead as follows.

Test circuit:

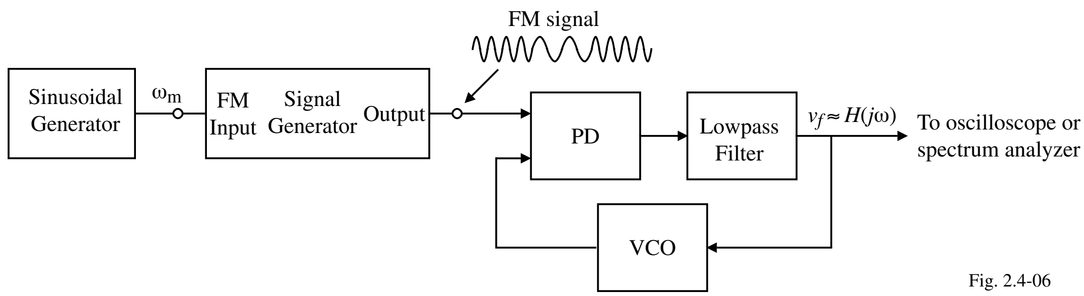


Fig. 2.4-06

Principle:

$$\omega_1 = \omega_o + \Delta\omega \sin\omega_m t \quad\rightarrow\quad \theta_1(t) = \int_0^t \omega_1 dt = -\frac{\Delta\omega}{\omega_m}\cos\omega_m t \quad\rightarrow\quad |\theta_1(j\omega)| = \frac{\Delta\omega}{\omega_m}$$

$$H(j\omega) = \frac{\theta_2(j\omega)}{\theta_1(j\omega)} \quad \text{and} \quad \text{VCO gain at } \omega_m \rightarrow \theta_2(j\omega) = \frac{K_o}{j\omega_m} V_f(j\omega_m)$$

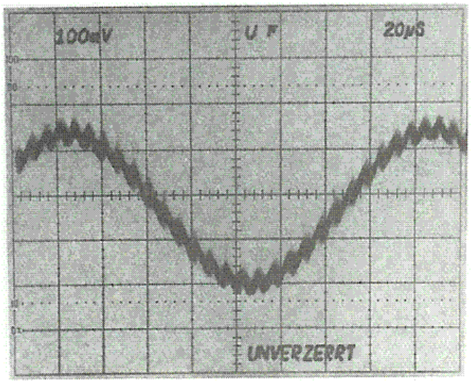
$$\therefore \quad |H(j\omega)| = \frac{K_o V_f(j\omega_m)}{\Delta\omega}$$

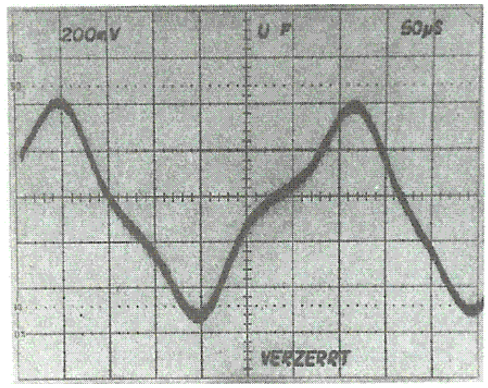
What about $\Delta\omega$?

As long as $\Delta\omega$ is small enough, the PD operates in its linear region and $v_f(t)$ is an undistorted sinewave (see next slide).

Measurement of $H(j\omega)$ – Continued

$\Delta\omega$ small enough for linear operation. $\Delta\omega$ too large for linear operation.





Implementation:

1.) Can plot the frequency response point-by-point.

2.) Use a spectrum analyzer

- Sweep generator rate << Spectrum analyzer sweep rate
- Watch out that resonance peaks in the response don't cause nonlinear operation.

Typical results: →

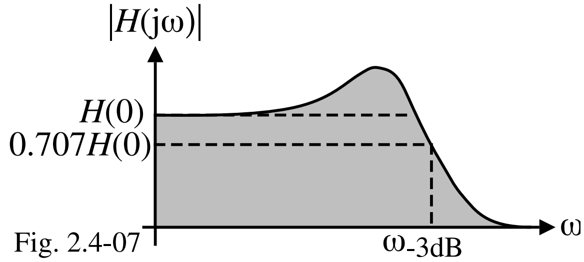


Fig. 2.4-07

Measurement of $H(j\omega)$ – Continued

Timing relationship between the sweep generator and the spectrum analyzer:

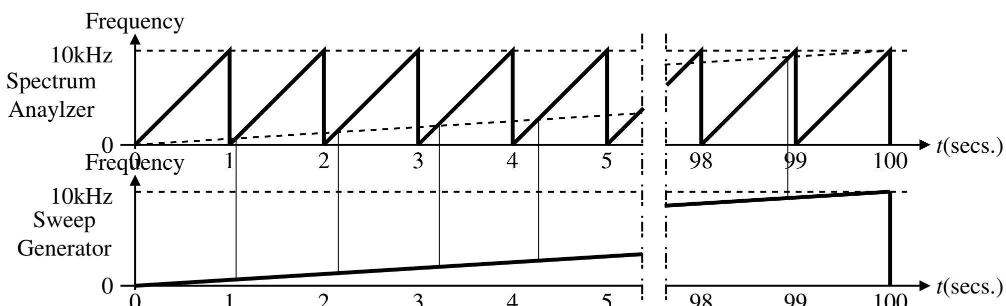
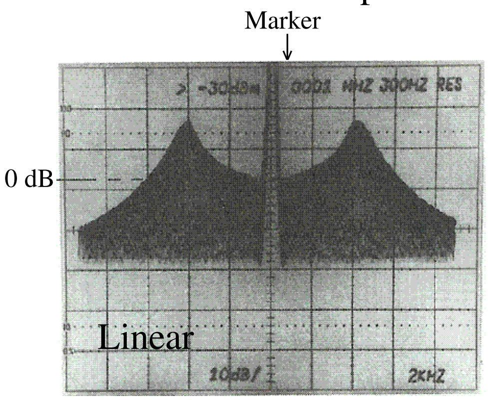
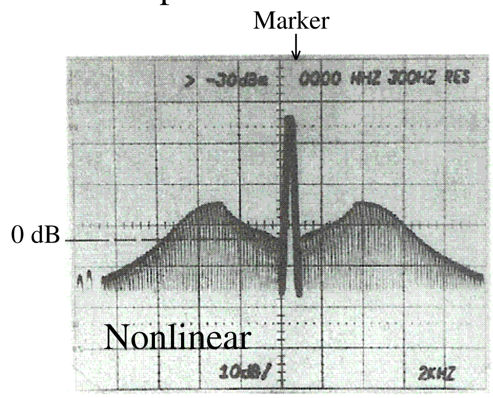


Fig. 2.4-08

Basically, f_m should approximate a constant during one sweep period of the analyzer.

Problem due to resonance peaks that cause nonlinear operation:





SUMMARY

- PLL Design Equations

  Basic design equations for
  - Type-I, first-order loop
  - Type-I, second-order loop
  - Type-II, second-order loop
- Design of a 450-475 MHz DPLL Frequency Synthesizer

  PFD plus Charge Pump

  Design of active PI filter

  Stability
- Measurements of PLL Performance

  How to experimentally measure the various performance parameters of a PLL