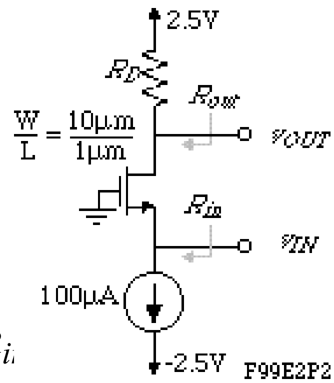
Homework # 5

Problem 1

The following questions relate to the MOSFET circuit shown. Assume the MOSFET is saturated and has the large signal parameters of $K_N' = 24\ \mu A/V^2$, $V_{TN} = 0.75$ V, and $\lambda_N = 0.01$ V^{-1}.



(a.) What is the dc value of v_{IN}?

(b.) What is the largest value of R_D for which the MOSFET is saturated?

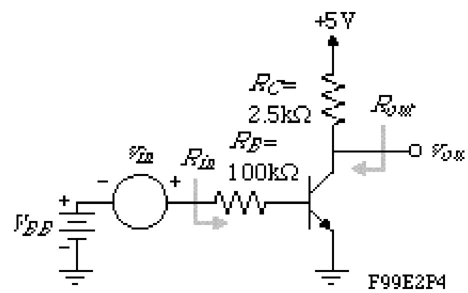
(c.) If R_D = 10kΩ, what is the numerical value of the small-signal voltage gain, v_{out}/v_{in}?

(d.) What is the numerical value of the small-signal input resistance, R_i

(e.) What is the numerical value of the small-signal output resistance, R_{out}?

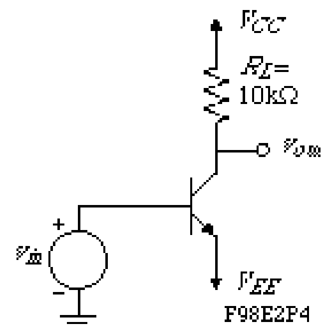
Problem 2

A BJT amplifier is shown. (a.) Find the value of V_{BB} that will give I_C = 1mA. (b.) Assume that I_C = 1mA and find the small signal input resistance, R_{in}, output resistance, R_{out}, and the midband voltage gain, $A_v(0)$. (c.) If C_π = 10pF and C_μ = 1pF, use Miller's approximation to find the -3dB frequency of this amplifier in Hertz.



Problem 3

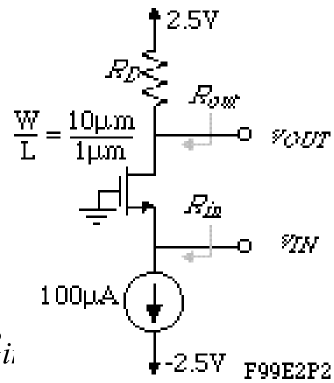
Find the -3dB frequency of the circuit shown in Hz. The values of the hybrid-pi model are r_b = 100Ω, r_π = 10kΩ, g_m = 10mS, r_o = 100kΩ, C_μ = 1pF, and f_T = 100MHz. (Hint: Use the Miller approximation on C_μ and ignore the output pole.)



Homework # 5 - Solutions

Problem 1

The following questions relate to the MOSFET circuit shown. Assume the MOSFET is saturated and has the large signal parameters of $K_N' = 24\ \mu A/V^2$, $V_{TN} = 0.75$ V, and $\lambda_N = 0.01$ V^{-1}.



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(a.) What is the dc value of v_{IN}?

(b.) What is the largest value of R_D for which the MOSFET is saturated?

(c.) If R_D = 10kΩ, what is the numerical value of the small-signal voltage gain, v_{out}/v_{in}?

(d.) What is the numerical value of the small-signal input resistance, R_i

(e.) What is the numerical value of the small-signal output resistance, R_{out}?

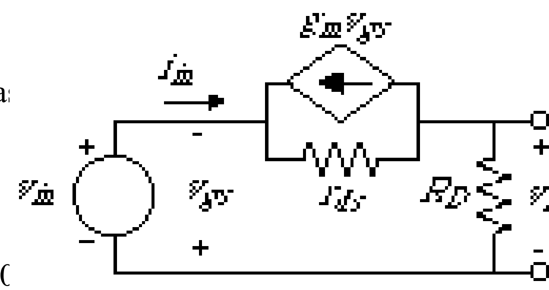
Solution:

(a.) $V_{IN} = -V_{GS} = -\sqrt{\dfrac{2I_D}{K'(W/L)}} - V_T = -0.913-0.75 = -1.663\text{V} \rightarrow \boxed{V_{IN} = -1.663\text{V}}$

(b.) Remember that the drain can be a value of V_T below the gate when the transistor is on the edge of saturation.

$$\therefore\ R_D = \frac{2.5-(-0.75)}{100\mu A} = 32\text{k}\Omega \rightarrow \boxed{R_D \le 32\text{k}\Omega}$$

(c.) Small-signal model: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Noting that $v_{gs} = -v_{in}$, we can sum the currents at the output node a

$$g_m v_{in} + g_{ds}(v_{in} - v_{out}) = G_D v_{out} \qquad \rightarrow \qquad \frac{v_{out}}{v_{in}} = \frac{g_m + g_{ds}}{G_D + g_{ds}}$$

$g_m = \sqrt{2 \cdot 100 \cdot 24 \cdot 10} = 219\mu S$, $g_{ds} = 100 \cdot 0.01 = 1\mu S$, and $G_D = 10($

$$\therefore \qquad \frac{v_{out}}{v_{in}} = +2.18\text{V/V} \qquad \boxed{\frac{v_{out}}{v_{in}} = +2.18\text{V/V}}$$

(d.) Summing the currents at the input gives $i_{in} = g_m v_{in} + g_{ds}(v_{in} - v_{out})$

Noting that $v_{out} = i_{in} R_D$ gives

$$i_{in}(1+g_{ds}R_D) = (g_m+g_{ds})v_{in} \rightarrow \qquad \frac{v_{in}}{i_{in}} = \frac{1+g_{ds}R_D}{g_m+g_{ds}} = \frac{1+0.1}{219+1}\text{x}10^6 = 4591\Omega$$

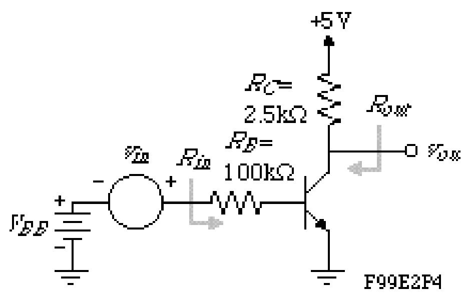
∴ $R_{in} = 4591\Omega$

(e.) The output resistance is simplified if we recall that $v_{in} = 0$ which means that $v_{gs} = 0$. Therefore the output resistance is simply

$$R_{out} = \frac{1}{g_{ds}+G_D} = \frac{10^6}{1+100} = 9.9\text{k}\Omega$$

Problem 2

A BJT amplifier is shown. (a.) Find the value of V_{BB} that will give I_C = 1mA. (b.) Assume that I_C = 1mA and find the small signal input resistance, R_{in}, output resistance, R_{out}, and the midband voltage gain, $A_v(0)$. (c.) If C_π = 10pF and C_μ = 1pF, use Miller's approximation to find the -3dB frequency of this amplifier ir



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Solution:

(a.) $$V_{BB} = \frac{R_B I_C}{\beta} + V_t ln\left(\frac{I_C}{I_s}\right) = \frac{100\cdot 1}{100} + 0.656 = 1.656\text{V}$$

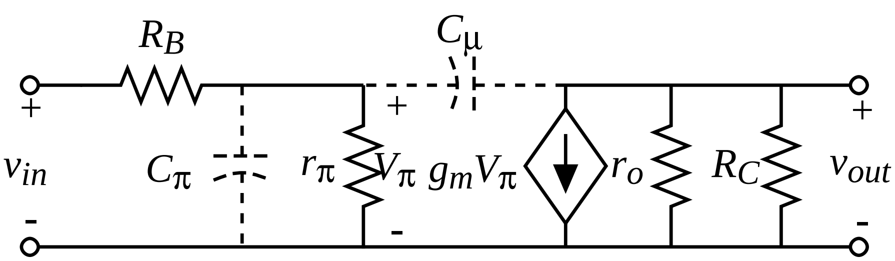
→ $V_{BB} = 1.656\text{V}$

(b.) Small-signal model:

$$g_m = \frac{I_C}{V_t} = \frac{1\text{mA}}{25.9\text{mV}} = 38.6\text{mS},$$

$$r_\pi = \frac{1+\beta}{g_m} = \frac{101}{0.0386} = 2.616\text{k}\Omega,$$

$$r_o = \frac{V_A}{I_C} = \frac{100\text{V}}{1\text{mA}} = 100\text{k}\Omega$$



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First: $R_{in} = R_B + r_\pi = 102.62\text{k}\Omega$ and $R_{out} = r_o || R_C = 100\text{k}\Omega || 2.5\text{k}\Omega = 2.44\text{k}\Omega$

Next: $$A_v(0) = \frac{V_{out}}{V_{in}} = \frac{-g_m r_\pi R_{out}}{r_\pi + R_B} = \frac{-0.0386(2.44\text{x}10^3)2.616}{102.616} = -2.4\text{V/V}$$ → $A_v(0) = -2.4\text{V/V}$

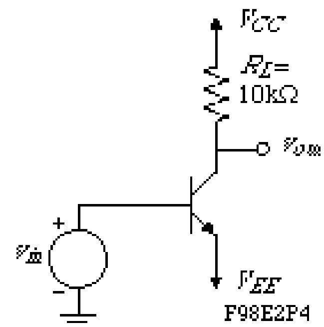
(c.) With the capacitors included in the model,

$$\omega_{-3\text{dB}} = \frac{1}{(R_B||r_\pi)[C_\pi + C_\mu(1+g_m R_{out})]} = \frac{10^{12}}{(2.549\text{x}10^3)[10+1(1+93.18)]}$$

$$= 3.766\text{x}10^6 \text{ rads/sec.} \rightarrow$$ $f_{-3\text{dB}} = 0.599$ MHz

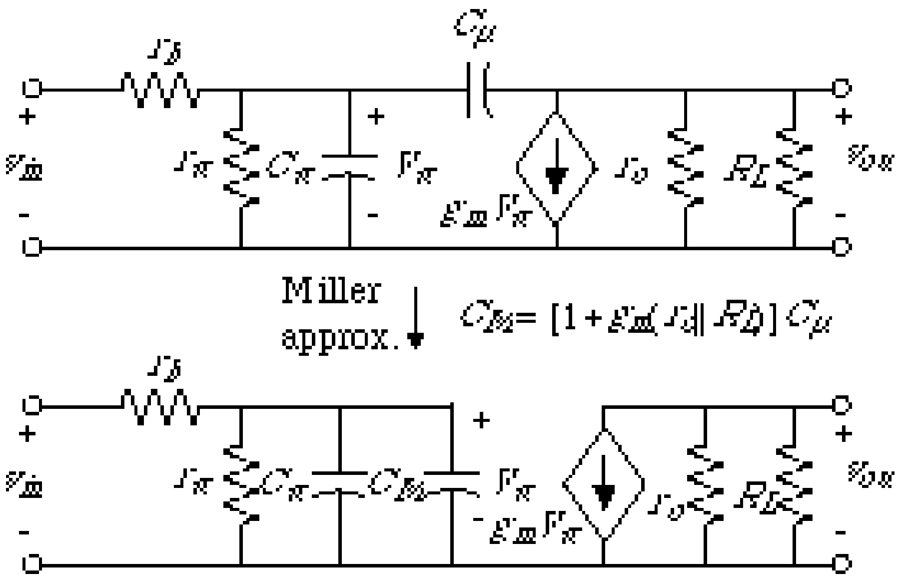
Problem 3

Find the -3dB frequency of the circuit shown in Hz. The values of the hybrid-pi model are r_b = 100Ω, r_π = 10kΩ, g_m = 10mS, r_o = 100kΩ, C_μ = 1pF, and f_T = 100MHz. (Hint: Use the Miller approximation on C_μ and ignore the output pole.)



Solution:

Small signal model:



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$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{10\text{x}10^{-3}}{2\pi\text{x}10^8} - 1\text{pF} = 14.9\text{pF}$$

Method 1 –Miller Approx.: $C_M = [1 + 10^{-2}(100\text{k}\Omega \| 10\text{k}\Omega)]1\text{pF} = (1+90.9)1\text{pF} = 91.9\text{pf}$

Thus, $$\omega_{-3\text{dB}} \approx \frac{1}{(r_b \| r_\pi)(C_\pi + C_M)} = \frac{1}{99\Omega \cdot 106.8\text{pF}} = 94.6\text{x}10^6 \text{ rads/sec.}$$

∴ $$\boxed{f_{-3\text{dB}} \approx \frac{94.6\text{x}10^6}{2\pi} = 15.1\text{MHz}}$$

Method 2 – Classical Approach (two-node problem):

$V_{out}(g_o+G_L+sC_\mu) = (sC_\mu - g_m)V_{out}$ & $g_b V_{in} = (g_\pi+g_b+sC_\pi+sC_\mu)V_\pi - sC_\mu V_{out}$

Solving: $$\frac{V_{out}}{V_{in}} = \frac{g_b(sC_\mu - g_m)}{s^2 C_\mu C_\pi + s[(C_\mu+C_\pi)(g_o+G_L)+(g_\pi+g_b)C_\mu+g_m C_\mu] + (g_\pi+g_b)(g_o+G_L)}$$

& denominator is: $D(s) = 14.9\text{x}10^{-24}(s^2 + 14.66\text{x}10^8 s + 7.4564\text{x}10^{16})$, which has two roots.

The smallest is -527.6×10^6 rads/sec which gives $\boxed{f_{-3dB} = \frac{527 \times 10^6}{2\pi} = 8.4\text{MHz}}$